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1 Electrical Circuits

1.1 Introduction

In practice, the electrical circuits may consist of one or more sources of energy and number of electrical parameters, connected in different ways. The different electrical parameters or elements are resistors, capacitors and inductors. The combination of such elements alongwith various sources of energy gives rise to complicated electrical circuits, generally referred as **networks**. The terms **circuit** and **network** are used synonymously in the electrical literature. The d.c. circuits consist of only resistances and d.c. sources of energy. And the circuit analysis means to find a current through or voltage across any branch of the circuit.

1.2 The Structure of Matter

In the understanding of fundamentals of electricity, the knowledge of the structure of matter plays an important role. The matter which occupies the space may be soild, liquid or gaseous. The molecules and atoms, of which all substances are composed are not at all elemental, but are themselves made up of simpler entities. We know this because we, up to certain extent, are successful in breaking atoms and studying the resulting products. For instance, such particles are obtained by causing ultraviolet light to fall on cold metal surfaces, such particles are spontaneously ejected from the radioactive elements. So these particles are obtained from many different substances under such widely varying conditions. It is believed that such particles are one of the elemental constituents of all matter, called **electrons.**

Infact, according to the **modern electron theory**, atom is composed of the three fundamental particles, which are invisible to bare eyes. These are the **neutron**, the **proton** and the **electron**. The proton is defined as positively charged while the electron is defined as negatively charged. The neutron is uncharged i.e. neutral in nature possessing no charge. The mass of neutron and proton is same while the electron is very light, almost $1/1840^{th}$ the mass of the neutron and proton. The following table gives information about these three particles.

Fundamental particles of matter	Symbol	Nature of charge possessed	Mass in kg.
Neutron	n	0	1.675×10^{-27}
Proton	p+	+	1.675×10^{-27}
Electron	e^-	–	9.107×10^{-31}

Table 1.1

1.2.1 Structure of an Atom

All of the protons and neutrons are bound together into a compact nucleus. **Nucleus** may be thought of as a central sun, about which electrons revolve in a particular fashion. This structure surrounding the nucleus is referred as the **electron cloud.**

In the normal atom the number of protons equal to the number of electrons. An atom as a whole is electrically neutral. The electrons are arranged in different **orbits.** The nucleus exerts a force of attraction on the revolving electrons and hold them together. All these different orbits are called **shells** and possess certain energy. Hence these are also called **energy shells or quanta.** The orbit which is closest to the nucleus is always under the tremendous force of attraction while the orbit which is farthest from the nucleus is under very weak force of attraction.

Key Point : *The electron or the electrons revolving in farthest orbit are hence loosely held to the nucleus. Such a shell is called the* ***valence shell****. And such electrons are called* ***valence electrons.***

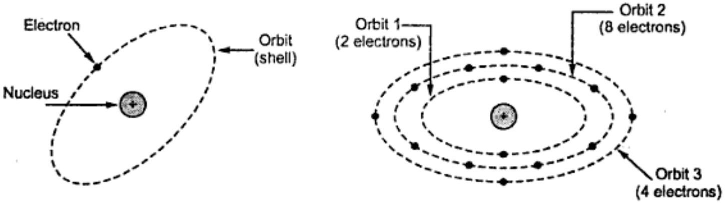
In some atoms such valence electrons are so loosely bound to the nucleus that at room temperature the additional energy imparted to the valence electrons causes them to escape from the shell and exist as **free electrons.** Such free electrons are basically responsible for the flow of electric current through metals.

Key Point : *More the number of free electrons, better is the metal for the conduction of the current. For example, copper has* 8.5×10^{28} *free electrons per cubic meter and hence it is a good conductor of electricity.*

The electrons which are revolving round the nucleus, not revolve in a single orbit. Each orbit consists of fixed number of electrons. In general, an orbit can contain a maximum of $2n^2$ electrons where n is the number of orbit. So first orbit or shell can occupy maximum of 2×1^2 i.e. 2 electrons while the second shell can occupy maximum of 2×2^2 i.e. 8 electrons and so on. The exception to this rule is that the valence shell can occupy maximum 8 electrons irrespective of its number. Let us see the structure of two different atoms.

This atom consists of one proton and one electron revolving around the nucleus. This is the simplest atom. This is shown in the Fig. 1.1 (a). The dot represents an electron while nucleus is represented by a circle with the positive sign inside it.

2) Silicon : This atom consists of 14 electrons. These revolve around the nucleus in three orbits. The first orbit has maximum 2 electrons, the second has maximum 8 electrons and the third orbit has remaining 4 electrons. This is shown in the Fig. 1.1 (b).



(a) Hydrogen atom **(b) Silicon atom**

Fig. 1.1

The 4 electrons located in the farthest shell are loosely held by the nucleus and generally available as free electrons. If by any means some of the electrons are removed, the negative charge of that atom decreases while positively charged protons remain same. The resultant charge on the atom remains more positive in nature and such element is called **positively charged.** While if by any means the electrons are added, then the total negative charge increases than positive and such element is called **negatively charged.**

1.3 Concept of Charge

In all the atoms, there exists number of electrons which are very loosely bound to its nucleus. Such electrons are free to wonder about, through the space under the influence of specific forces. Now when such electrons are removed from an atom it becomes positively charged. This is because of loosing negatively charged particles i.e. electrons from it. As against this, if excess electrons are added to the atom it becomes negatively charged.

> **Key Point** : *Thus total deficiency or addition of excess electrons in an atom is called its charge and the element is said to be charged.*

The following table shows the different particles and charge possessed by them.

Particle	Charge possessed in Coulomb	Nature
Neutron	0	Neutral
Proton	1.602×10^{-19}	Positive
Electron	1.602×10^{-19}	Negative

Table 1.2

1.3.1 Unit of Charge

As seen from the Table 1.2 that the charge possessed by the electron is very very small hence it is not convenient to take it as the unit of charge.

The unit of the measurement of the charge is **Coulomb.**

The charge on one electron is 1.602×10^{-19}, so one coulomb charge is defined as the charge possessed by total number of ($1/\ 1.602 \times 10^{-19}$) electrons i.e. 6.24×10^{18} number of electrons.

Thus, $\boxed{1 \text{ coulomb} = \text{charge on } 6.24 \times 10^{18} \text{ electrons}}$

From the above discussion it is clear that if an element has a positive charge of one coulomb then that element has a deficiency of 6.24×10^{18} number of electrons.

Key Point: *Thus, addition or removal of electrons causes the change in the nature of the charge possessed by the element.*

1.4 Concept of Electromotive Force and Current

It has been mentioned earlier that the free electrons are responsible for the flow of electric current. Let us see how it happens.

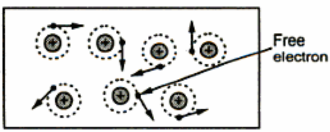


Fig. 1.2 Inside the piece of a conductor

To understand this, first we will see the enlarged view of the inside of a piece of a conductor. A conductor is one which has abundant free electrons. The free electrons in such a conductor are always moving in random directions as shown in the Fig. 1.2.

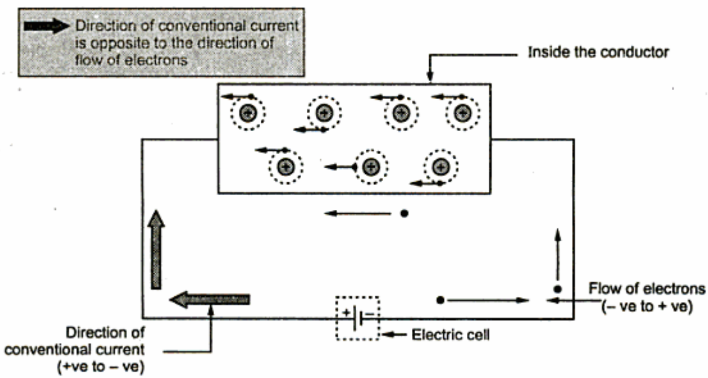


Fig. 1.3 The flow of current

The small electrical effort, externally applied to such conductor makes all such free electrons to drift along the metal in a definite particular direction. This direction depends on how the external electrical effort is applied to the conductor. Such an electrical effort may be an electrical cell, connected across the two ends of a conductor. Such physical phenomenon is represented in the Fig. 1.3.

Key Point : *An electrical effort required to drift the free electrons in one particular direction, in a conductor is called* ***Electromotive Force*** *(e.m.f.)*

The metal consists of particles which are charged. The like charges repel while unlike charges attract each other. But as external electric effort is applied, the free electrons as are negatively charged, get attracted by positive of the cell connected. And this is the reason why electrons get aligned in one particular direction under the influence of an electromotive force.

Key Point : *The electric effort i.e. e.m.f. is maintained across the positive and negative electrodes of the cell, due to the chemical action inside the solution contained in the cell.*

Atoms, when they loose or gain electrons, become charged accordingly and are called **ions**. Now when free electron gets dragged towards positive from an atom it becomes positively charged ion. Such positive ion drags a free electron from the next atom. This process repeats from atom to atom along the conductor. So there is flow of electrons from negative to positive of the cell, externally through the conductor across which the cell is connected. This movement of electrons is called an **Electric Current**.

The movement of electrons is always from negative to positive while movement of current is always assumed as from positive to negative. This is called **direction of conventional current**.

Key Point : *Direction of conventional current is from positive to negative terminal while direction of flow of electrons is always from negative to positive terminal, through the external circuit across which the e.m.f. is applied.*

We are going to follow direction of the conventional current throughout this book. i.e. from positive to negative terminal, of the battery through the external circuit.

1.5 Relation between Charge and Current

The current is flow of electrons. Thus current can be measured by measuring how many electrons are passing through material per second. This can be expressed in terms of the charge carried by those electrons in the material per second. So the **flow of charge per unit time** is used to quantify an electric current.

Key Point : *So current can be defined as rate of flow of charge in an electric circuit or in any medium in which charges are subjected to an external electric field.*

The charge is indicated by **Q** coulombs while current is indicated by **I**. The unit for the current is **Amperes** which is nothing but coulombs/sec. Hence mathematically we can write the relation between the charge (Q) and the electric current (I) as,

$$\boxed{I = \frac{Q}{t} \quad \text{Amperes}}$$

where I = Average current flowing

Q = Total charge transferred

t = Time required for transfer of charge.

Definition of 1 Ampere : *A current of 1 Ampere is said to be flowing in the conductor when a charge of one coulomb is passing any given point on it in one second.*

Now 1 coulomb is 6.24×10^{18} number of electrons. So 1 ampere current flow means flow of 6.24×10^{18} electrons per second across a section taken any where in the circuit.

$$\boxed{\text{1 Ampere current} = \text{Flow of } 6.24 \times 10^{8} \text{ electrons per second}}$$

1.6 Concept of Electric Potential and Potential Difference

When two similarly charged particles are brought near, they try to repel each other while dissimilar charges attract each other. This means, every charged particle has a tendency to do work.

Key Point: *This ability of a charged particle to do the work is called its* ***electric potential.*** *The unit of electric potential is* ***volt.***

The electric potential at a point due to a charge is one volt if one joule of work is done in bringing a unit positive charge i.e. positive charge of one coulomb from infinity to that point.

Mathematically it is expressed as,

$$\boxed{\text{Electrical Potential} = \frac{\text{Workdone}}{\text{Charge}} = \frac{W}{Q}}$$

Let us define now the potential difference.

It is well known that, flow of water is always from higher level to lower level, flow of heat is always from a body at higher temperature to a body at lower temperature. Such a level difference which causes flow of water, heat and so on, also exists in electric circuits. In electric circuits flow of current is always from higher electric potential to lower electric potential. So we can define potential difference as below :

Key Point : *The difference between the electric potentials at any two given points in a circuit is known as Potential Difference (p.d.). This is also called voltage between the two points and measured in volts. The symbol for voltage is V.*

For example, let the electric potential of a charged particle A is say V_1 while the electric potential of a charged particle B is say V_2. Then the potential difference between the two particles A and B is $V_1 - V_2$. If $V_1 - V_2$ is positive we say that A is at higher potential than B while if $V_1 - V_2$ is negative we say that B is at higher potential than A.

Key Point: *The potential difference between the two points is one volt if one joule of work is done in displacing unit charge (1 coulomb) from a point of lower potential to a point of higher potential.*

Consider two points having potential difference of V volts between them, as shown in the Fig. 1.4. The point A is at higher potential than B. As per the definition of volt, the V joules of work is to be performed to move unit charge from point B to point A.

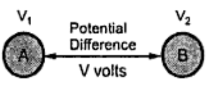


Fig. 1.4

Thus, when such two points, which are at different potentials are joined together with the help of wire, the electric current flows from higher potential to lower potential i.e. the electrons start flowing from lower potential to higher potential. Hence, to maintain the flow of electrons i.e. flow of electric current, there must exist a potential difference between the two points.

Key Point: *No current can flow if the potential difference between the two points is zero.*

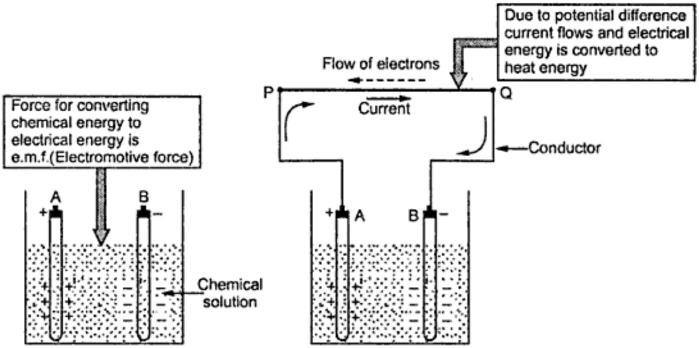
1.7 Electromotive Force and Potential Difference

Earlier we have seen the concept of e.m.f. The e.m.f. is that force which causes the flow of electrons i.e. flow of current in the given circuit. Let us understand its meaning more clearly.

Consider a simple cell shown in Fig. 1.5 (a). Due to the chemical reaction in the solution the terminal 'A' has acquired positive charge while terminal 'B' has acquired negative charge.

If now a piece of conductor is connected between the terminals A and B then flow of electrons starts through it. This is nothing but the flow of current through the conductor. This is shown in the Fig. 1.5 (b). The electrons will flow from terminal B to A and hence direction of current is from A to B i.e. positive to negative as shown.

One may think that once the positive charge on terminal A gets neutralised due to the electrons, then flow of electrons will stop. Both the terminals may get neutralised after some time. But this does not happen practically. This is because chemical reaction in the solution maintains terminal A positively charged and terminal B as negatively charged. This maintains the flow of current. The chemical reaction converts chemical energy into electric energy which maintains flow of electrons.



(a) Cell

(b) Current due to a cell

Fig. 1.5

Key Point: *This force which requires to keep electrons in motion i.e. to convert chemical or any other form of energy into electric energy is known as* ***Electromotive Force (e.m.f.) denoted by*** *E. The unit of e.m.f. is* **volt** *and unless and untill there is some e.m.f. present in the circuit, a continuous flow of current is not possible.*

Consider two points P and Q as shown in the Fig. 1.5 (b), then the current is flowing from point P to point Q. This means there exists a potential difference between the points P and Q. This potential difference is called voltage denoted as V and measured in volts.

In other words we can explain the difference between e.m.f. and p.d. as below. In the cell two energy transformations are taking place simultaneously. The one is chemical energy because of solution in cell is getting converted to electrical energy which is basic cause for flow of electrons and hence current. The second is when current flows, the piece of metal gets heated up i.e. electrical energy is getting converted to heat energy, due to flow of current.

In the first transformation electrical energy is generated from other form of energy. The force involved in such transformation is electromotive force. When current flows, due to which metal gets heated up i.e. due to existence of potential difference between two points, voltage is existing. And in such case electrical energy gets converted to other form of energy. The force involved in such transformation is nothing but the potential difference or voltage. Both e.m.f. and potential difference are in generally referred as voltage.

1.8 Network Terminology

In this section, we shall define some of the basic terms which are commonly associated with a network.

1.8.1 Network

Any arrangement of the various electrical energy sources along with the different circuit elements is called an **electrical network.** Such a network is shown in the Fig. 1.6.

1.8.2 Network Element

Any individual circuit element with two terminals which can be connected to other circuit element, is called a **network element.**

Network elements can be either active elements or passive elements. Active elements are the elements which supply power or energy to the network. Voltage source and current source are the examples of active elements. Passive elements are the elements which either store energy or dissipate energy in the form of heat. Resistor, inductor and capacitor are the three basic passive elements. Inductors and capacitors can store energy and resistors dissipate energy in the form of heat.

1.8.3 Branch

A part of the network which connects the various points of the network with one another is called **a branch.** In the Fig. 1.6, A-B, B-C, C-D, D-A, D-E, C-F and E-F are the various branches. A branch may consist more than one element.

1.8.4 Junction Point

A point where three or more branches meet is called a **junction point.** Point D and C are the junction points in the network shown in the Fig. 1.6.

1.8.5 Node

A point at which two or more elements are joined together is called **node.** The junction points are also the nodes of the network. In the network shown in the Fig. 1.6, A, B, C, D, E and F are the nodes of the network.

1.8.6 Mesh (or Loop)

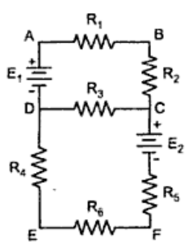


Fig. 1.6 An electrical network

Mesh (or Loop) is a set of branches forming a closed path in a network in such a way that if one branch is removed then remaining branches do not form a closed path. A loop also can be defined as a closed path which originates from a particular node, terminating at the same node, travelling through various other nodes, without travelling through any node twice. In the Fig. 1.6 paths A-B-C-D-A, A-B-C-F-E-D-A, D-C-F-E-D etc. are the loops of the network.

1.9 Classification of Electrical Networks

The behaviour of the entire network depends on the behaviour and characteristics of its elements. Based on such characteristics electrical network can be classified as below :

i) Linear Network : A circuit or network whose parameters i.e. elements like resistances, inductances and capacitances are always constant irrespective of the change in time, voltage, temperature etc. is known as **linear network.** The Ohm's law can be applied to such network. The mathematical equations of such network can be obtained by using the law of superposition. The response of the various network elements is linear with respect to the excitation applied to them.

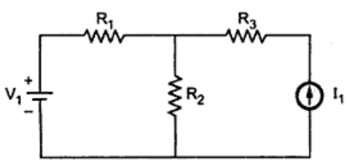
ii) Non Linear Network : A circuit whose parameters change their values with change in time, temperature, voltage etc. is known as **non linear network .** The Ohm's law may not be applied to such network. Such network does not follow the law of superposition. The response of the various elements is not linear with respect to their excitation. The best example is a circuit consisting of a diode where diode current does not vary linearly with the voltage applied to it.

iii) Bilateral Network : A circuit whose characteristics, behaviour is same irrespective of the direction of current through various elements of it, is called **bilateral network.** Network consisting only resistances is good example of bilateral network.

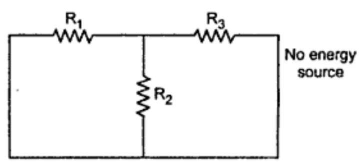
iv) Unilateral Network : A circuit whose operation, behaviour is dependent on the direction of the current through various elements is called **unilateral network.** Circuit consisting diodes, which allows flow of current only in one direction is good example of unilateral circuit.

v) Active Network : A circuit which contains at least one source of energy is called **active.** An energy source may be a voltage or current source.

vi) Passive Network : A circuit which contains no energy source is called **passive circuit.** This is shown in the Fig. 1.7.



(a) Active network



(b) Passive network

Fig. 1.7

vii) Lumped Network : A network in which all the network elements are physically separable is known as **lumped network.** Most of the electric networks are lumped in nature, which consists elements like R, L, C, voltage source etc.

viii) Distributed Network : A network in which the circuit elements like resistance, inductance etc. cannot be physically separable for analysis purposes, is called **distributed network**. The best example of such a network is a transmission line where resistance, inductance and capacitance of a transmission line are distributed all along its length and cannot be shown as a separate elements, any where in the circuit.

The classification of networks can be shown as,

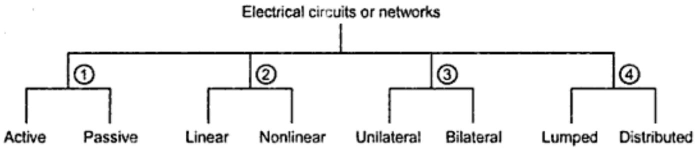


Fig. 1.8 Classification of networks

1.10 Basic Circuit Components

Let us take a brief review of three basic elements namely resistance, capacitance and inductance.

1.10.1 Resistance

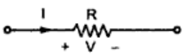


Fig. 1.9

It is the property of the material by which it opposes the flow of current through it. The resistance of element is denoted by the symbol 'R'. Resistance is measured in ohms (Ω).

The resistance of a given material depends on the physical properties of that material and given by,

$$\boxed{R = \frac{\rho l}{a}}$$

where l = Length in metres

a = Cross-sectional area in square metres

ρ = Resistivity in ohms-metres

R = Resistance in ohms

We can define unit ohm as below.

> **Key Point**: *1 Ohm : The resistance of a circuit, in which a current of 1 ampere generates the heat at the rate of 1 joules per second is said to be 1 ohm.*

Now $\boxed{4.186 \text{ Joules} = 1 \text{ Calorie}}$

hence $\boxed{1 \text{ Joule} = 0.24 \text{ Calorie}}$

Thus unit 1 ohm can be defined as that resistance of the circuit if it develops 0.24 calories of heat, when one ampere current flows through the circuit for one second.

The unit ohm also can be defined as, one ohm resistance is that which allows one ampere current to flow through it when one volt voltage is impressed across it.

The relation between voltage and current for a resistance is given by **Ohm's law** as,

$$v = R\,i$$

$$\therefore \quad \boxed{R = \frac{v}{i}}$$

The power absorbed by a resistance is given by,

$$\therefore \quad \boxed{p(t) = vi = \frac{v^2}{R} = i^2R \text{ watts}}$$

while the amount of energy converted to heat energy in time t is given by,

$$\therefore \quad \boxed{w = \int_{-\infty}^{t} p\,dt = \int_{-\infty}^{t} i^2\,R\,dt = \int_{-\infty}^{t} vi\,dt}$$

Key Point: *As i^2 term is always positive, the energy absorbed by the resistance is always positive.*

If the voltage across resistance is constant V and the currrent through it is constant I then the energy for $t \geq 0$ is given by,

$$W = \int_{0}^{t} VI\,dt = VI\,t \text{ joules}$$

while, $$P = VI = \frac{V^2}{R} = I^2R \text{ watts}$$

1.10.2 Inductance

An inductance is the element in which energy is stored in the form of electromagnetic field. The inductance is denoted as 'L' and is measured in henries (H).

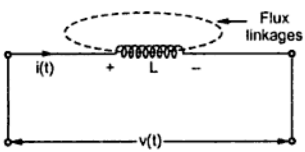


Fig. 1.10 Inductance

The Fig. 1.10 shows an inductance.

The time varying voltage v(t) is the voltage across it. It carries a current i(t) which is also time varying.

Key Point: *For an inductance, the voltage across it is proportional to the rate of change of current passing through it.*

$$\therefore \quad v(t) \propto \frac{di(t)}{dt}$$

The constant of proportionality in the above equation is the inductance L.

$$\therefore \quad \boxed{v(t) = L\frac{di(t)}{dt}}$$

If the voltage v (t) is known across an inductor then the current is given by,

$$\therefore \quad \boxed{i(t) = \frac{1}{L}\int_{-\infty}^{t} v(t)\,dt}$$

If the inductance has N turns and the flux ϕ produced by the current i(t) entirely links with the coil of N turns then according to Faraday's law,

$$v(t) = N\frac{d\phi}{dt}$$

The total flux linkages $N\phi$ are thus proportional to the current through the coil.

$$\therefore \quad N\phi = Li$$

$$\therefore \quad \boxed{L = \frac{N\phi}{i}}$$

The power in the inductor is given by,

$$\boxed{p(t) = vi = L\,i(t)\frac{di(t)}{dt}}$$

The energy stored in the inductor in the form of an electromagnetic field is,

$$w = \int p(t)\,dt = \int L\,i(t)\frac{d\,i(t)}{dt}\,dt$$

$$w = \int L\,i(t)\,di(t) = L\frac{i^2(t)}{2}$$

$$\therefore \quad \boxed{w = \frac{1}{2}L\,i^2(t)\ \text{joules}}$$

1.10.3 Capacitance

An element in which energy is stored in the form of an electrostatic field is known as capacitance. It is made up of two conducting plates separated by a dielectric material. It is denoted as 'C' and is measured in farads (F).

The Fig. 1.11 shows a capacitor. The voltage across it is time varying v(t) and current through it is also time varying i(t).

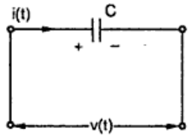


Fig. 1.11 Capacitor

Key Point: *For a capacitor, the current through it is proportional to the rate of change of voltage across it.*

$$i(t) \propto \frac{dv(t)}{dt}$$

The constant of proportionality is the capacitor C.

$$\therefore \qquad \boxed{i(t) = C\frac{dv(t)}{dt}}$$

While the ratio of the charge stored to the voltage across the capacitor is known as the capacitance C.

$$\therefore \qquad \boxed{C = \frac{q}{v}}$$

The voltage across the capacitor is given by,

$$\boxed{v(t) = \frac{1}{C}\int_{-\infty}^{t} i(t)\,dt}$$

The power in the capacitor is given by,

$$\boxed{p(t) = v\,i = C\,v(t)\frac{dv(t)}{dt}}$$

The energy stored in the capacitor is given by,

$$w = \int p(t)\,dt = \int C\,v(t)\frac{dv(t)}{dt}\,dt$$

$$w = \int C\,v(t)\,dv(t) = C\frac{v^2(t)}{2}$$

$$\therefore \qquad \boxed{w = \frac{1}{2}C\,v^2(t)\ \text{joules}}$$

1.11 Voltage Current Relationships for Passive Elements

The voltage current relationships for the passive elements resistance (R), inductance (L) and capacitor (C) are given in the Table 1.3.

Element	Basic relation	Voltage across, if current known	Current through, if voltage known	Energy
R	$R = \frac{v}{i}$	$v_R(t) = R\, i_R(t)$	$i_R(t) = \frac{1}{R} v_R(t)$	$w = \int_{-\infty}^{t} i_R(t)\, v_R(t)\, dt$
L	$L = \frac{N\phi}{i}$	$v_L(t) = L\frac{di_L(t)}{dt}$	$i_L(t) = \frac{1}{L}\int_{-\infty}^{t} v_L(t)\, dt$	$w = \frac{1}{2} L\, i^2(t)$
C	$C = \frac{q}{v}$	$v_C(t) = \frac{1}{C}\int_{-\infty}^{t} i_C(t)\, dt$	$i_C(t) = C\frac{dv_C(t)}{dt}$	$w = \frac{1}{2} C\, v^2(t)$

Table 1.3

Note that in the Table 1.3, v_R, v_L and v_C are the voltages across R, L and C respectively while i_R, i_L and i_C are the currents through R, L and C respectively.

Example 1.1 : *A current waveform flowing through an inductor of 1 mH is shown in the Fig. 1.12. Obtain and sketch the waveform of the voltage across the inductor.*

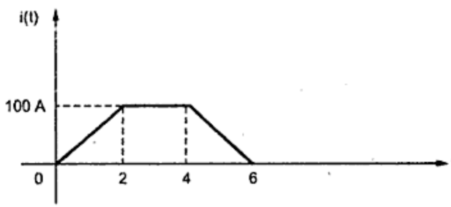


Fig. 1.12

Solution : From the given waveform,

For $0 < t < 2$, i(t) is a straight line of slope $= \frac{100}{2} = 50$

$$\therefore \quad i(t) = 50\,t \text{ and } \frac{di(t)}{dt} = 50 \qquad \ldots\ 0 < t < 2$$

$$\text{For } 2 < t < 4,\ i(t) = 100 \text{ and } \frac{di(t)}{dt} = 0$$

For $4 < t < 6$, i(t) is a straight line of slope $= -\frac{100}{2} = -50$

$\therefore \quad i(t) = -50t \text{ and } \frac{di(t)}{dt} = -50 \qquad \ldots 4 < t < 6$

Now $\quad v_L(t) = L\frac{di(t)}{dt}$

$= 1 \times 10^{-3} \times 50 = \mathbf{0.05\ V} \qquad \ldots 0 < t < 2$

$= 1 \times 10^{-3} \times 0 = \mathbf{0\ V} \qquad \ldots 2 < t < 4$

$= 1 \times 10^{-3} \times (-50) = \mathbf{-0.05\ V} \qquad \ldots 4 < t < 6$

The waveform is shown in the Fig. 1.12 (a).

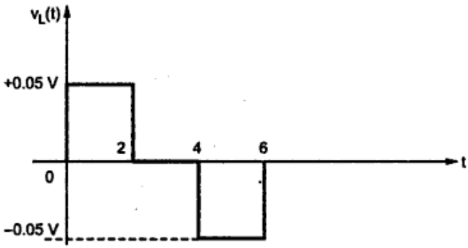


Fig. 1.12 (a)

1.12 Ohm's Law

This law gives relationship between the potential difference (V), the current (I) and the resistance (R) of a d.c. circuit. **Dr. Ohm** in 1827 discovered a law called **Ohm's Law.** It states,

Ohm's Law : *The current flowing through the electric circuit is directly proportional to the potential difference across the circuit and inversely proportional to the resistance of the circuit, provided the temperature remains constant.*

Mathematically, $\boxed{I \propto \frac{V}{R}}$

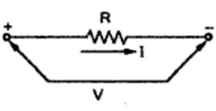


Fig. 1.13 Ohm's law

Where I is the current flowing in amperes, the V is the voltage applied and R is the resistance of the conductor, as shown in the Fig. 1.13.

Now $I = \frac{V}{R}$

The unit of potential difference is defined in such a way that the constant of proportionality is unity.

Ohm's law is,

$$I = \frac{V}{R} \quad \text{amperes}$$

$$V = I\,R \quad \text{volts}$$

$$\frac{V}{I} = \text{constant} = R \quad \text{ohms}$$

The Ohm's law can be defined as,

The ratio of potential difference (V) between any two points of a conductor to the current (I) flowing between them is constant, provided that the temperature of the conductor remains constant.

Key Point: *Ohm's law can be applied either to the entire circuit or to the part of a circuit. If it is applied to entire circuit, the voltage across the entire circuit and resistance of the entire circuit should be taken into account. If the Ohm's law is applied to the part of a circuit, then the resistance of that part and potential across that part should be used.*

1.12.1 Limitations of Ohm's Law

The limitations of the Ohm's law are,

1) It is not applicable to the non linear devices such as diodes, zener diodes, voltage regulators etc.
2) It does not hold good for non-metallic conductors such as silicon carbide. The law for such conductors is given by,

$$V = k\,I^m \quad \text{where k, m are constants.}$$

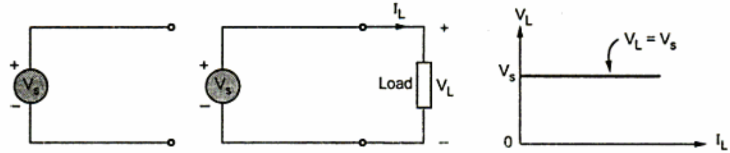
1.13 Energy Sources

There are basically two types of energy sources ; voltage source and current source. These are classified as i) Ideal source and ii) Practical source.

Let us see the difference between ideal and practical sources.

1.13.1 Voltage Source

Ideal voltage source is defined as the energy source which gives constant voltage across its terminals irrespective of the current drawn through its terminals. The symbol for ideal voltage source is shown in the Fig. 1.14 (a). This is connected to the load as shown in Fig. 1.14 (b). At any time the value of voltage at load terminals remains same. This is indicated by V- I characteristics shown in the Fig. 1.14 (c).

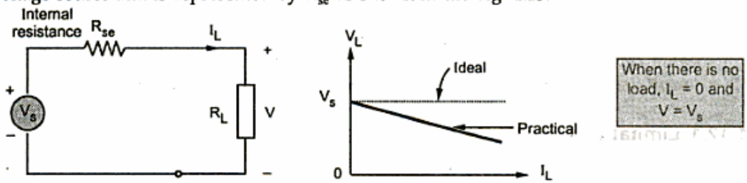


(a) Symbol (b) Circuit (c) Characteristics

Fig. 1.14 Ideal voltage source

Practical voltage source :

But practically, every voltage source has small internal resistance shown in series with voltage source and is represented by R_{se} as shown in the Fig. 1.15.



(a) Circuit (b) Characteristics

Fig. 1.15 Practical voltage source

Because of the R_{se}, voltage across terminals decreases slightly with increase in current and it is given by expression,

$$\boxed{V_L = -(R_{se})\, I_L + V_S = V_S - I_L\, R_{se}}$$

Key Point: *For ideal voltage source,* $R_{se} = 0$

Voltage sources are further classified as follows,

i) Time Invariant Sources :

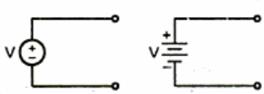


Fig. 1.16 (a) D. C. source

The sources in which voltage is not varying with time are known as **time invariant voltage sources** or **D.C. sources**. These are denoted by capital letters. Such a source is represented in the Fig. 1.16 (a).

ii) Time Variant Sources :

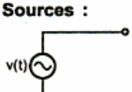
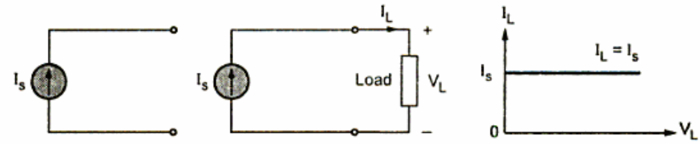


Fig. 1.16 (b) A. C. source

The sources in which voltage is varying with time are known as **time variant voltage sources** or **A.C. sources**. These are denoted by small letters. This is shown in the Fig. 1.16 (b).

1.13.2 Current Source

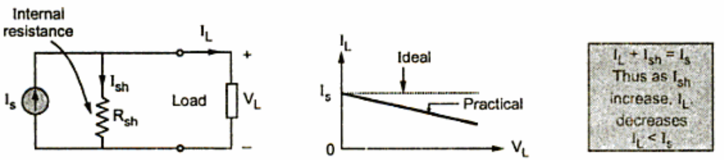
Ideal current source is the source which gives constant current at its terminals irrespective of the voltage appearing across its terminals. The symbol for ideal current source is shown in the Fig. 1.17 (a). This is connected to the load as shown in the Fig. 1.17 (b). At any time, the value of the current flowing through load I_L is same i.e. is irrespective of voltage appearing across its terminals. This is explained by V-I characteristics shown in the Fig. 1.17 (c).



(a) Symbol **(b) Circuit** **(c) Characteristics**

Fig. 1.17 Ideal current source

But practically, every current source has high internal resistance, shown in parallel with current source and it is represented by R_{sh}. This is shown in the Fig. 1.18.



(a) Circuit **(b) Characteristic**

Fig. 1.18 Practical current source

Because of R_{sh}, current through its terminals decreases slightly with increase in voltage at its terminals.

Key Point: *For ideal current source,* $R_{sh} = \infty$.

Similar to voltage sources, current sources are classified as follows :

i) Time Invariant Sources :

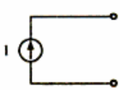


Fig. 1.19 (a) D.C. source

The sources in which current is not varying with time are known as **time invarient current sources** or **D.C. sources**. These are denoted by capital letters.

Such a current source is represented in the Fig. 1.19 (a).

ii) Time Variant Sources :

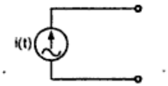


Fig. 1.19 (b) A.C. source

The sources in which current is varying with time are known as **time variant current sources** or **A.C. sources**. These are denoted by small letters.

Such a source is represented in the Fig. 1.19(b).

The sources which are discussed above are independent sources because these sources does not depend on other voltages or currents in the network for their value. These are represented by a circle with a polarity of voltage or direction of current indicated inside.

1.13.3 Dependent Sources

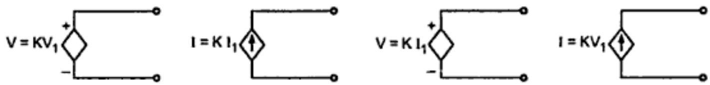
Dependent sources are those whose value of source depends on voltage or current in the circuit. Such sources are indicated by diamond as shown in the Fig. 1.20 and further, classified as,

i) Voltage Dependent Voltage Source : It produces a voltage as a function of voltages elsewhere in the given circuit. This is called **VDVS.** It is shown in the Fig. 1.20 (a).

ii) Current Dependent Current Source : It produces a current as a function of currents elsewhere in the given circuit. This is called **CDCS.** It is shown in the Fig. 1.20 (b).

iii) Current Dependent Voltage Source : It produces a voltage as a function of current elsewhere in the given circuit. This is called **CDVS.** It is shown in the Fig. 1.20 (c).

iv) Voltage Dependent Current Source : It produces a current as a function of voltage elsewhere in the given circuit. This is called **VDCS.** It is shown in the Fig. 1.20 (d).



(a) **(b)** **(c)** **(d)**

Fig. 1.20

K is constant and V_1 and I_1 are the voltage and current respectively, present elsewhere in the given circuit. The dependent sources are also known as controlled sources.

1.14 Series Circuit

A series circuit is one in which several resistances are connected one after the other. Such connection is also called **end to end** connection or **cascade** connection. There is only one path for the flow of current.

1.14.1 Resistors in Series

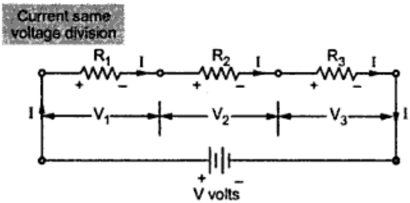


Fig. 1.21 A series circuit

Consider the resistances shown in the Fig. 1.21.

The resistance R_1, R_2 and R_3 are said to be in series. The combination is connected across a source of voltage V volts. Naturally the current flowing through all of them is same indicated as I amperes. e.g. the chain of small lights, used for the decoration purposes is good example of series combination.

Now let us study the **voltage distribution.**

Let V_1, V_2 and V_3 be the voltages across the terminals of resistances R_1, R_2 and R_3 respectively

Then, $$V = V_1 + V_2 + V_3$$

Now according to Ohm's law, $$V_1 = I R_1, \; V_2 = I R_2, \; V_3 = I R_3$$

Current through all of them is same i.e. I

$$\therefore \quad V = I R_1 + I R_2 + I R_3 = I(R_1 + R_2 + R_3)$$

Applying Ohm's law to overall circuit,

$$V = I R_{eq}$$

where R_{eq} = Equivalent resistance of the circuit. By comparison of two equations,

$$R_{eq} = R_1 + R_2 + R_3$$

i.e. total or **equivalent resistance** of the series circuit is arithmetic sum of the resistances connected in series.

For n resistances in series, $$\boxed{R = R_1 + R_2 + R_3 + \ldots\ldots + R_n}$$

1.14.1.1 Characteristics of Series Circuits

1) The same current flows through each resistance.
2) The supply voltage V is the sum of the individual voltage drops across the resistances.

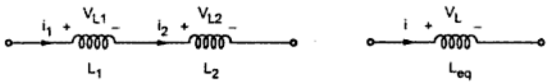
$$V = V_1 + V_2 + \ldots.. + V_n$$

3) The equivalent resistance is equal to the sum of the individual resistances.
4) The equivalent resistance is the largest of all the individual resistances.

i.e $$R > R_1, \; R > R_2, \; \ldots.. R > R_n$$

1.14.2 Inductors in Series

Consider the Fig. 1.22 (a). Two inductors L_1 and L_2 are connected in series. The currents flowing through L_1 and L_2 are i_1 and i_2 while voltages developed across L_1 and L_2 are V_{L1} and V_{L2} respectively. The equivalent circuit is shown in the Fig. 1.22 (b).



(a) Fig. 1.22 (b)

We have, $V_{L1} = L_1 \frac{di_1}{dt}$ and $V_{L2} = L_2 \frac{di_2}{dt}$ while $V_L = L_{eq} \frac{di}{dt}$

For series combination,

$$i = i_1 = i_2$$

and $$V_L = V_{L1} + V_{L2}$$

$\therefore$ $$L_{eq} \frac{di}{dt} = L_1 \frac{di}{dt} + L_2 \frac{di}{dt}$$

$\therefore$ $$L_{eq} \frac{di}{dt} = (L_1 + L_2) \frac{di}{dt}$$

$\therefore$ $$L_{eq} = L_1 + L_2$$

That means, equivalent inductance of the series combination of two inductances is the sum of inductances connected in series.

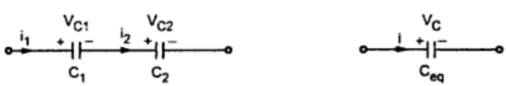
The total equivalent inductance of the series circuit is sum of the inductances connected in series.

For n inductances in series,

$$\boxed{L_{eq} = L_1 + L_2 + L_3 + \ldots + L_n}$$

1.14.3 Capacitor in Series

Consider the Fig. 1.23 (a). Two capacitors C_1 and C_2 are connected in series. The currents flowing through and voltages developed across C_1 and C_2 are i_1, i_2 and V_{C1} and V_{C2} respectively. The equivalent circuit is shown in the Fig. 1.23 (b).



(a) Fig. 1.23 (b)

We have, $V_{C1} = \frac{1}{C_1}\int_{-\infty}^{t} i_1 dt$, $V_{C2} = \frac{1}{C_2}\int_{-\infty}^{t} i_2 dt$ while $V = \frac{1}{C_{eq}}\int_{-\infty}^{t} i dt$

For series combination,

$$i = i_1 = i_2 \quad \text{and}$$

$$V_C = V_{C1} + V_{C2}$$

$$\frac{1}{C_{eq}}\int_{-\infty}^{t} i dt = \frac{1}{C_1}\int_{-\infty}^{t} i_1 dt + \frac{1}{C_2}\int_{-\infty}^{t} i_2 dt$$

But $i = i_1 = i_2$

$$\therefore \quad \frac{1}{C_{eq}}\int_{-\infty}^{t} i dt = \left(\frac{1}{C_1}+\frac{1}{C_2}\right)\int_{-\infty}^{t} i\, dt$$

$$\therefore \quad \frac{1}{C_{eq}} = \frac{1}{C_1}+\frac{1}{C_2}$$

$$\therefore \quad C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$

That means, reciprocal of equivalent capacitor of the series combination is the sum of the reciprocal of individual capacitances.

The reciprocal of the total equivalent capacitor of the series combination is the sum of the reciprocols of the individual capacitors, connected in series.

For n capacitors in series,

$$\boxed{\frac{1}{C_{eq}} = \frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}+\ldots\frac{1}{C_n}}$$

1.15 Parallel Circuits

The **parallel circuit** is one in which several resistances are connected across one another in such a way that one terminal of each is connected to form a junction point while the remaining ends are also joined to form another junction point.

1.15.1 Resistors in Parallel

Consider a parallel circuit shown in the Fig. 1.24.

In the parallel connection shown, the three resistances R_1 , R_2 and R_3 are connected in parallel and combination is connected across a source of voltage 'V '.

Voltage same current division

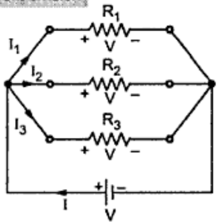


Fig. 1.24 A parallel circuit

In parallel circuit current passing through each resistance is different. Let total current drawn is say ' I ' as shown. There are 3 paths for this current, one through R_1, second through R_2 and third through R_3. Depending upon the values of R_1 , R_2 and R_3 the appropriate fraction of total current passes through them. These individual currents are shown as I_1 , I_2 and I_3. While the voltage across the two ends of each resistances R_1 , R_2 and R_3 is the same and equals the supply voltage V.

Now let us study current distribution. Apply Ohm's law to each resistance.

$$V = I_1 R_1 , \quad V = I_2 R_2 , \quad V = I_3 R_3$$

$$I_1 = \frac{V}{R_1} , \quad I_2 = \frac{V}{R_2} , \quad I_3 = \frac{V}{R_3}$$

$$I = I_1 + I_2 + I_3 = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

$$= V \left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right] \quad \text{... (1)}$$

For overall circuit if Ohm's law is applied,

$$V = I R_{eq}$$

and $$I = \frac{V}{R_{eq}} \quad \text{... (2)}$$

where R_{eq} = Total or **equivalent resistance** of the circuit.

Comparing the two equations,

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

where R is the equivalent resistance of the parallel combination.

In general if 'n' resistances are connected in parallel,

$$\boxed{\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots\ldots + \frac{1}{R_n}}$$

Conductance (G) :

It is known that, $\frac{1}{R} = G$ (conductance) hence,

$$\therefore \quad \boxed{G = G_1 + G_2 + G_3 + ... + G_n} \quad \text{... For parallel circuit}$$

Important result :

Now if n = 2, two resistances are in parallel then,

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\therefore \quad \boxed{R = \frac{R_1 R_2}{R_1 + R_2}}$$

This formula is directly used hereafter, for two resistances in parallel.

1.15.1.1 Characteristics of Parallel Circuits

1) The same potential difference gets across all the resistances in parallel.
2) The total current gets divided into the number of paths equal to the number of resistances in parallel. The total current is always sum of all the individual currents.

$$I = I_1 + I_2 + I_3 + ... + I_n$$

3) The reciprocal of the equivalent resistance of a parallel circuit is equal to the sum of the reciprocal of the individual resistances.
4) The equivalent resistance is the smallest of all the resistances.

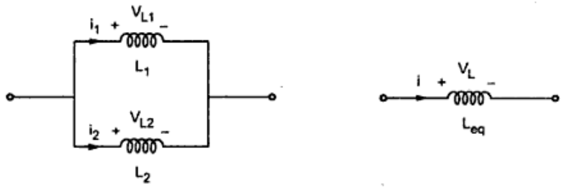
$$R < R_1, \quad R < R_2, ..., R < R_n$$

5) The equivalent conductance is the arithmetic addition of the individual conductances.

Key Point : *The equivalent resistance is smaller than the smallest of all the resistances connected in parallel.*

1.15.2 Inductors in Parallel

Consider the Fig. 1.25 (a). Two inductors L_1 and L_2 are connected in parallel. The currents flowing through L_1 and L_2 are i_1 and i_2 respectively. The voltage developed across L_1 and L_2 are V_{L1} and V_{L2} respectively. The equivalent circuit is shown in Fig. 1.25 (b)



(a) **Fig. 1.25** (b)

For inductor we have,

$$i_1 = \frac{1}{L_1}\int_{-\infty}^{t} V_{L1}\,dt, \qquad i_2 = \frac{1}{L_2}\int_{-\infty}^{t} V_{L2}\,dt, \quad \text{while } i = \frac{1}{L_{eq}}\int_{-\infty}^{t} V_L\,dt$$

For parallel combination,

$$V_L = V_{L1} = V_{L2} \quad \text{and}$$

$$i = i_1 + i_2$$

$$\therefore \quad \frac{1}{L_{eq}}\int_{-\infty}^{t} V_L dt = \frac{1}{L_1}\int_{-\infty}^{t} V_L\,dt + \frac{1}{L_2}\int_{-\infty}^{t} V_L\,dt = \left(\frac{1}{L_1}+\frac{1}{L_2}\right)\int_{-\infty}^{t} V_L\,dt$$

$$\therefore \quad \frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2}$$

That means, reciprocal of equivalent inductance of the parallel combination is the sum of reciprocals of the individual inductances.

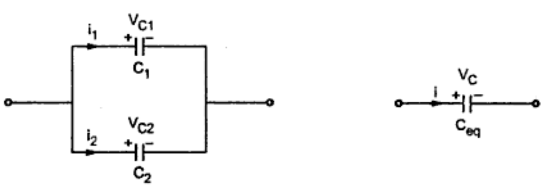
For n inductances in parallel,

$$\therefore \quad \boxed{\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \ldots \frac{1}{L_n}}$$

1.15.3 Capacitors in Parallel

Consider the Fig. 1.26 (a). Two capacitors C_1 and C_2 are connected in parallel. The currents flowing through C_1 and C_2 are i_1 and i_2 respectively and voltages developed across C_1, C_2 are V_{C1} and V_{C2} respectively.

The equivalent circuit is shown in the Fig. 1.26 (b).



(a) **Fig. 1.26** (b)

For capacitor we have, $i_1 = C_1 \frac{dV_{C1}}{dt}$, $i_2 = C_2 \frac{dV_{C2}}{dt}$, while $i = C_{eq} \frac{dV_C}{dt}$

For parallel combination,

$$V_{C1} = V_{C2} = V_C \quad \text{and}$$

$$i = i_1 + i_2$$

$$C_{eq} \frac{dV_C}{dt} = C_1 \frac{dV_{C1}}{dt} + C_2 \frac{dV_{C2}}{dt}$$

$$\therefore \quad C_{eq} \frac{dV_C}{dt} = (C_1 + C_2) \frac{dV_C}{dt}$$

$$\therefore \quad C_{eq} = C_1 + C_2$$

That means, equivalent capacitance of the parallel combination of the capacitances is the sum of the individual capacitances connected in series.

For n capacitors in parallel,

$$\therefore \quad \boxed{C_{eq} = C_1 + C_2 + \ldots + C_n}$$

The Table 1.4 gives the equivalent at 'n' basic elements is series,

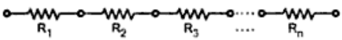
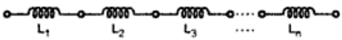
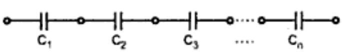
Element	Equivalent
'n' Resistances in series 	$R_{eq} = R_1 + R_2 + R_3 + \ldots + R_n$
'n' Inductors in series 	$L_{eq} = L_1 + L_2 + L_3 + \ldots + L_n$
'n' Capacitors in series 	$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_n}$

Table 1.4 Series combinations of elements

The Table 1.5 gives the equivalent of 'n' basic elements in parallel,

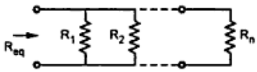
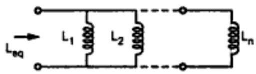
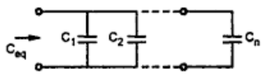
Element	Equivalent
'n' Resistances in parallel 	$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n}$
'n' Inductors in parallel 	$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \ldots + \frac{1}{L_n}$
'n' Capacitors in parallel 	$C_{eq} = C_1 + C_2 + \ldots + C_n$

Table 1.5 Parallel combinations of elements

Key Point : *The current through series combination remains same and voltage gets divided while in parallel combination voltage across combination remains same and current gets divided.*

➡ **Example 1.2 :** *Find the equivalent resistance between the two points A and B shown in the Fig. 1.27.*

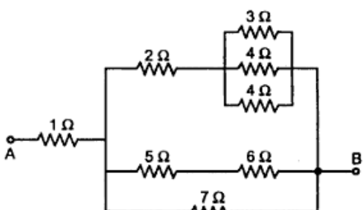


Fig. 1.27

Solution : Identify combinations of series and parallel resistances.

The resistances 5 Ω and 6 Ω are in series, as going to carry same current.

So equivalent resistance is 5 + 6 = 11 Ω

While the resistances 3 Ω , 4 Ω, and 4 Ω are in parallel, as voltage across them same but current divides.

$\therefore$ Equivalent resistance is,
$$\frac{1}{R} = \frac{1}{3}+\frac{1}{4}+\frac{1}{4} = \frac{10}{12}$$

$$\therefore \qquad R = \frac{12}{10} = 1.2\ \Omega$$

Replacing these combinations redraw the figure as shown in the Fig. 1.28 (a).

Now again 1.2 Ω and 2 Ω are in series so equivalent resistance is 2 + 1.2 = 3.2 Ω while 11 Ω and 7 Ω are in parallel.

Using formula $\frac{R_1\ R_2}{R_1+R_2}$ equivalent resistance is $\frac{11\times 7}{11+7} = \frac{77}{18} = 4.277\ \Omega$.

Replacing the respective combinations redraw the circuit as shown in the Fig. 1.28 (b).

Now 3.2 and 4.277 are in parallel.

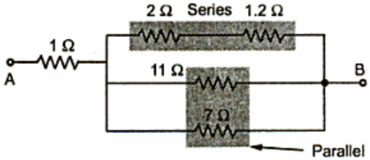


Fig. 1.28 (a)

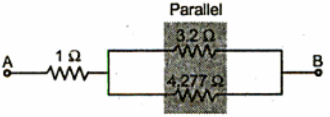


Fig. 1.28 (b)

$$\therefore \quad \text{Replacing them by} \quad \frac{3.2\times 4.277}{3.2+4.277} = 1.8304\ \Omega$$

$$\therefore \qquad R_{AB} = 1 + 1.8304 = \mathbf{2.8304\ \Omega}$$

1.16 Short and Open Circuits

In the network simplification, short circuit or open circuit existing in the nework plays an important role.

1.16.1 Short Circuit

When any two points in a network are joined directly to each other with a thick metalic conducting wire, the two points are said to be short circuited. **The resistance of such short circuit is zero.**

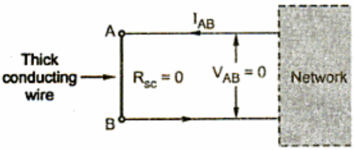


Fig. 1.29 Short circuit

The part of the network, which is short circuited is shown in the Fig. 1.29. The points A and B are short circuited. The resistance of the branch A-B is $R_{sc} = 0\ \Omega$.

The curent I_{AB} is flowing through the short circuited path.

According to Ohm's law,

$$V_{AB} = R_{sc} \times I_{AB} = 0 \times I_{AB} = 0 \text{ V}$$

Key Point : *Thus, voltage across short circuit is always zero though current flows through the short circuited path.*

1.16.2 Open Circuit

When there is no connection between the two points of a network, having some voltage across the two points then the two points are said to be open circuited.

As there is no direct connection in an open circuit, **the resistance of the open circuit is ∞.**

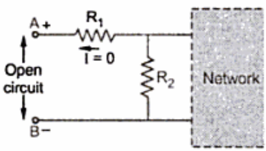


Fig. 1.30 Open circuit

The part of the network which is open circuited is shown in the Fig. 1.30. The points A and B are said to be open circuited. The resistance of the branch AB is $R_{oc} = \infty\,\Omega$.

There exists a voltage across the points AB called open circuit voltage, V_{AB} but $R_{oc} = \infty\ \Omega$.

According to Ohm's law,

$$I_{oc} = \frac{V_{AB}}{R_{oc}} = \frac{V_{AB}}{\infty} = 0 \text{ A}$$

Key Point : *Thus, current through open circuit is always zero though there exists a voltage across open circuited terminals.*

1.16.3 Redundant Branches and Combinations

The redundant means excessive and unwanted.

Key Point : *If in a circuit there are branches or combinations of elements which do not carry any current then such branches and combinations are called redundant from circuit point of view.*

The redundant branches and combinations can be removed and these branches do not affect the performance of the circuit.

The two important situations of **redundancy** which may exist in practical circuits are,

Situation 1 : Any branch or combination across which there exists a short circuit, becomes redundant as it does not carry any current.

If in a network, there exists a direct short circuit across a resistance or the combination of resistances then that resistance or the entire combination of resistances becomes **inactive** from the circuit point of view. Such a combination is redundant from circuit point of view.

To understand this, consider the combination of resistances and a short circuit as shown in the Fig. 1.31 (a) and (b).

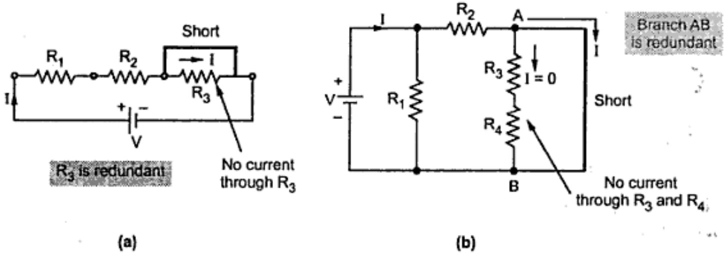


Fig. 1.31 Redundant branches

In Fig. 1.31 (a), there is short circuit across R_3. The current always prefers low resistance path hence entire current I passes through short circuit and hence resistance R_3 becomes redundant from the circuit point of view.

In Fig. 1.31 (b), there is short circuit across combination of R_3 and R_4. The entire current flows through short circuit across R_3 and R_4 and no current can flow through combination of R_3 and R_4. Thus that combination becomes meaningless from the circuit point of view. **Such combinations can be eliminated while analysing the circuit.**

Situation 2 : If there is open circuit in a branch or combination, it can not carry any current and becomes redundant.

In Fig. 1.32 as there exists open circuit in branch BC, the branch BC and CD can not carry any current and are become redundant from circuit point of view.

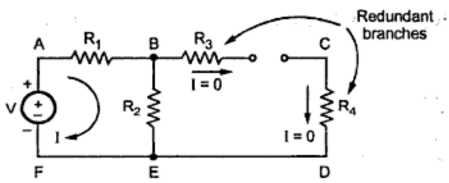


Fig. 1.32 Redundant branches due to open circuit

1.17 Voltage Division in Series Circuit of Resistors

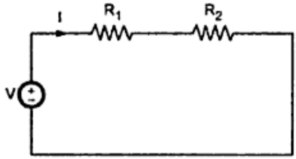


Fig. 1.33

Consider a series circuit of two resistors R_1 and R_2 connected to source of V volts.

As two resistors are connected in series, the current flowing through both the resistors is same, i.e. I. Then applying KVL, we get,

$$V = I R_1 + I R_2$$

$$\therefore \quad I = \frac{V}{R_1 + R_2}$$

Total voltage applied is equal to the sum of voltage drops V_{R1} and V_{R2} across R_1 and R_2 respectively.

$$\therefore \quad V_{R1} = I \,.\, R_1$$

$$\therefore \quad \boxed{V_{R1} = \frac{V}{R_1 + R_2} \cdot R_1 = \left[\frac{R_1}{R_1 + R_2}\right] V}$$

Similarly, $V_{R2} = I \,.\, R_2$

$$\therefore \quad \boxed{V_{R2} = \frac{V}{R_1 + R_2} \cdot R_2 = \left[\frac{R_2}{R_1 + R_2}\right] V}$$

So this circuit is a **voltage divider circuit.**

Key Point : *So in general, voltage drop across any resistor, or combination of resistors, in a series circuit is equal to the ratio of that resistance value to the total resistance, multiplied by the source voltage.*

1.18 Current Division in Parallel Circuit of Resistors

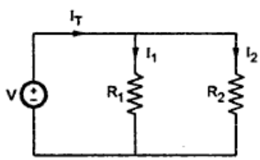


Fig. 1.34

Consider a parallel circuit of two resistors R_1 and R_2 connected across a source of V volts.

Current through R_1 is I_1 and R_2 is I_2, while total current drawn from source is I_T.

$$\therefore \quad I_T = I_1 + I_2$$

But $I_1 = \frac{V}{R_1}, \quad I_2 = \frac{V}{R_2}$

i.e. $V = I_1 R_1 = I_2 R_2$

$$\therefore \quad I_1 = I_2 \left(\frac{R_2}{R_1}\right)$$

Substituting value of I_1 in I_T,

$$\therefore \quad I_T = I_2\left(\frac{R_2}{R_1}\right) + I_2 = I_2\left[\frac{R_2}{R_1}+1\right] = I_2\left[\frac{R_1+R_2}{R_1}\right]$$

$$\therefore \quad \boxed{I_2 = \left[\frac{R_1}{R_1+R_2}\right] I_T}$$

Now $$I_1 = I_T - I_2 = I_T - \left[\frac{R_1}{R_1+R_2}\right] I_T$$

$$\therefore \quad I_1 = \left[\frac{R_1+R_2-R_1}{R_1+R_2}\right] I_T$$

$$\therefore \quad \boxed{I_1 = \left[\frac{R_2}{R_1+R_2}\right] I_T}$$

Key Point : *In general, the current in any branch is equal to the ratio of opposite branch resistance to the total resistance value, multiplied by the total current in the circuit.*

Example 1.3 : *Find the voltage across the three resistances shown in the Fig. 1.35.*

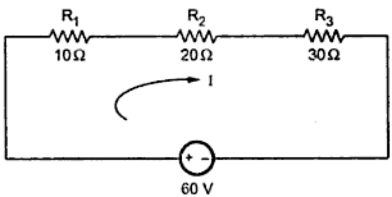


Fig. 1.35

Solution :

$$I = \frac{V}{R_1+R_2+R_3} \quad \text{... series circuit}$$

$$= \frac{60}{10+20+30} = 1\text{ A}$$

$$\therefore \quad V_{R1} = IR_1 = \frac{V \times R_1}{R_1+R_2+R_3} = 1\times 10 = \mathbf{10\ V}$$

$$\therefore \quad V_{R2} = IR_2 = \frac{V \times R_2}{R_1+R_2+R_3} = 1\times 20 = \mathbf{20\ V}$$

and $$V_{R3} = IR_3 = \frac{V \times R_3}{R_1+R_2+R_3} = 1\times 30 = \mathbf{30\ V}$$

Key Point : *It can be seen that voltage across any resistance of series circuit is ratio of that resistance to the total resistance, multiplied by the source voltage.*

Example 1.4 : *Find the magnitudes of total current, current through R_1 and R_2 if, $R_1 = 10\ \Omega$, $R_2 = 20\ \Omega$, and $V = 50$ V.*

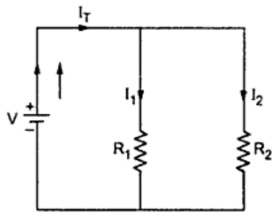


Fig. 1.36

Solution : The equivalent resistance of two is,

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2} = \frac{10 \times 20}{10+20} = 6.67\ \Omega$$

$$\therefore \quad I_T = \frac{V}{R_{eq}} = \frac{50}{6.67} = \mathbf{7.5\ A}$$

As per the current distribution in parallel circuit,

$$I_1 = I_T\left(\frac{R_2}{R_1+R_2}\right) = 7.5 \times \left(\frac{20}{10+20}\right) = \mathbf{5\ \ A}$$

and

$$I_2 = I_T\left(\frac{R_1}{R_1+R_2}\right) = 7.5 \times \left(\frac{10}{10+20}\right) = \mathbf{2.5\ \ A}$$

It can be verified that $I_T = I_1 + I_2$.

1.19 Kirchhoff's Laws

In 1847, a German Physicist, **Kirchhoff,** formulated two fundamental laws of electricity. These laws are of tremendous importance from network simplification point of view.

1.19.1 Kirchhoff's Current Law (KCL)

Consider a junction point in a complex network as shown in the Fig. 1.37.

At this junction point if $I_1 = 2$ A, $I_2 = 4$ A and $I_3 = 1$ A then to determine I_4 we write, total current entering is $2 + 4 = 6$ A while total current leaving is $1 + I_4$ A

And hence, $I_4 = 5$ A.

This analysis of currents entering and leaving is nothing but the application of Kirchhoff's Current Law. The law can be stated as,

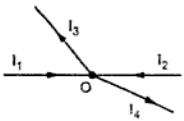


Fig. 1.37 Junction point

The total current flowing towards a junction point is equal to the total current flowing away from that junction point.

Another way to state the law is,

The algebraic sum of all the current meeting at a junction point is always zero.

The word algebraic means considering the signs of various currents.

$$\boxed{\sum I \quad \text{at junction point} = 0}$$

Sign convention : Currents flowing towards a junction point are assumed to be positive while currents flowing away from a junction point assumed to be negative.

e.g. Refer to Fig. 1.37, currents I_1 and I_2 are positive while I_3 and I_4 are negative.

Applying KCL, $\sum I$ at junction O $= 0$

$$I_1 + I_2 - I_3 - I_4 = 0 \text{ i.e. } I_1 + I_2 = I_3 + I_4$$

The law is very helpful in network simplification.

1.19.2 Kirchhoff's Voltage Law (KVL)

"In any network, the algebraic sum of the voltage drops across the circuit elements of any closed path (or loop or mesh) is equal to the algebraic sum of the e.m.f. s in the path"

In other words, "The algebraic sum of all the branch voltages, around any closed path or closed loop is always zero."

$$\boxed{\text{Around a closed path } \sum V = 0}$$

The law states that if one starts at a certain point of a closed path and goes on tracing and noting all the potential changes (either drops or rises), in any one particular direction, till the starting point is reached again, he must be at the same potential with which he started tracing a closed path.

Sum of all the potential rises must be equal to sum of all the potential drops while tracing any closed path of the circuit. The total change in potential along a closed path is always zero.

This law is very useful in loop analysis of the network.

1.19.3 Sign Conventions to be Followed while Applying KVL

When current flows through a resistance, the voltage drop occurs across the resistance. The polarity of this voltage drop always depends on direction of the current. The current always flows from higher potential to lower potential.

(a) **Fig. 1.38** (b)

In the Fig. 1.38 (a), current I is flowing from right to left, hence point B is at higher potential than point A, as shown.

In the Fig. 1.38 (b), current I is flowing from left to right, hence point A is at higher potential than point B, as shown.

Once all such polarities are marked in the given circuit, we can apply KVL to any closed path in the circuit.

Now while tracing a closed path, if we go from – ve marked terminal to + ve marked terminal, that voltage must be taken as positive. This is called **potential rise**.

For example, if the branch AB is traced from A to B then the drop across it must be considered as rise and must be taken as + IR while writing the equations.

While tracing a closed path, if we go from +ve marked terminal to – ve marked terminal, that voltage must be taken as negative. This is called **potential drop**.

For example, in the Fig. 1.38 (a) only, if the branch is traced from B to A then it should be taken as negative, as – IR while writing the equations.

Similarly in the Fig. 1.38 (b), if branch is traced from A to B then there is a voltage drop and term must be written negative as – IR while writing the equation. If the branch is traced from B to A, it becomes a rise in voltage and term must be written positive as + IR while writing the equation.

Key Point :

1) ***Potential rise*** *i.e. travelling from negative to positively marked terminal, must be considered as* ***Positive.***
2) ***Potential drop*** *i.e. travelling from positive to negatively marked terminal, must be considered as* ***Negative.***
3) *While tracing a closed path, select any one direction clockwise or anticlockwise. This selection is totally independent of the directions of currents and voltages of various branches of that closed path.*

1.19.4 Application of KVL to a Closed Path

Consider a closed path of a complex network with various branch currents assumed as shown in the Fig. 1.39 (a).

As the loop is assumed to be a part of complex network, the branch currents are assumed to be different from each other.

Due to these currents the various voltage drops taken place across various resistances are marked as shown in the Fig. 1.39 (b).

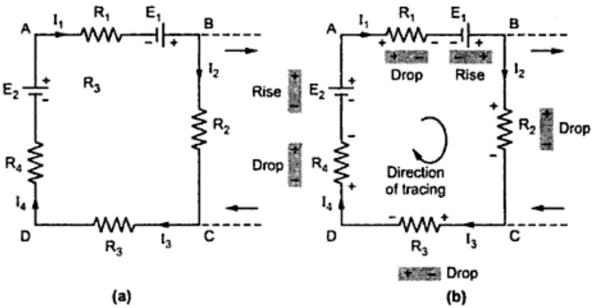


Fig. 1.39 (a), (b) Closed loop of a complex network

The polarity of voltage drop along the current direction is to be marked as positive (+) to negative (–).

Let us **trace** this closed path in clockwise direction i.e. **A-B-C-D-A.**

Across R_1 there is voltage drop $I_1 R_1$ and as getting traced from +ve to –ve, it is drop and must be taken as negative while applying KVL.

Battery E_1 is getting traced from negative to positive i.e. it is a rise hence must be considered as positive.

Across R_2 there is a voltage drop $I_2 R_2$ and as getting traced from +ve to –ve, it is drop and must be taken negative.

Across R_3 there is a drop $I_3 R_3$ and as getting traced from +ve to –ve, it is drop and must be taken as negative.

Across R_4 there is drop $I_4 R_4$ and as getting traced from +ve to –ve, it is drop must be taken as negative.

Battery E_2 is getting traced from –ve to +ve, it is rise and must be taken as positive.

∴ We can write an equation by using KVL around this closed path as,

$$-I_1 R_1 + E_1 - I_2 R_2 - I_3 R_3 - I_4 R_4 + E_2 = 0 \qquad \text{... Required KVL equation}$$

i.e. $$E_1 + E_2 = I_1 R_1 + I_2 R_2 + I_3 R_3 + I_4 R_4$$

If we trace the closed loop in opposite direction i.e. along **A-D-C-B-A** and follow the same sign convention, the resulting equation will be same as what we have obtained above.

Key Point : *So while applying KVL, **direction in which loop is to be traced is not important but following the sign convention is most important**.*

The same sign convention is followed in this book to solve the problems.

1.19.5 Steps to Apply Kirchhoff's Laws to Get Network Equations

The steps are stated based on the branch current method.

Step 1 : Draw the circuit diagram from the given information and insert all the values of sources with appropriate polarities and all the resistances.

Step 2 : Mark all the branch currents with some assumed directions using KCL at various nodes and junction points. Kept the number of unknown currents minimum as far as possible to limit the mathematical calculations required to solve them later on.

Assumed directions may be wrong, in such case answer of such current will be mathematically negative which indicates the correct direction of the current. A particular current leaving a particular source has some magnitude, then same magnitude of current should enter that source after travelling through various branches of the network.

Step 3 : Mark all the polarities of voltage drops and rises as per directions of the assumed branch currents flowing through various branch resistances of the network. This is necessary for application of KVL to various closed loops.

Step 4 : Apply KVL to different closed paths in the network and obtain the corresponding equations. Each equation must contain some element which is not considered in any previous equation.

> **Key Point** : ***KVL must be applied to sufficient number of loops such that each element of the network is included atleast once in any of the equations.***

Step 5 : Solve the simultaneous equations for the unknown currents. From these currents unknown voltages and power consumption in different resistances can be calculated.

What to do if current source exists ?

> **Key Point**: *If there is current source in the network then complete the current distribution considering the current source. But while applying KVL, the loops should not be considered involving current source. The loop equations must be written to those loops which do not include any current source. This is because drop across current source is unknown.*

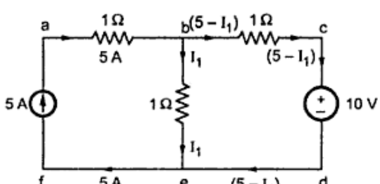


Fig. 1.40

For example, consider the circuit shown in the Fig. 1.40. The current distribution is completed interms of current source value. Then KVL must be applied to the loop b-c-d-e-b, which does not include current source. **The loop a-b-e-f-a should not be used for KVL application**, as it includes current source. Its effect is already considered at the time of current distribution.

1.20 Cramer's Rule

If the network is complex, the number of equations i.e. unknowns increases. In such case, the solution of simultaneous equations can be obtained by **Cramer's Rule** for determinants.

Let us assume that set of simultaneous equations obtained is, as follows,

$$\therefore \quad \boxed{\begin{aligned} a_{11} x_1 + a_{12} x_2 + \ldots\ldots + a_{1n} x_n &= C_1 \\ a_{21} x_1 + a_{22} x_2 + \ldots\ldots + a_{2n} x_n &= C_2 \\ &\vdots \\ a_{n1} x_1 + a_{n2} x_2 + \ldots\ldots + a_{nn} x_n &= C_n \end{aligned}}$$

where $C_1, C_2, \ldots\ldots\ldots\ldots, C_n$ are constants.

Then Cramer's rule says that form a system determinant Δ or D as,

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix} = D$$

Then obtain the subdeterminants D_j by replacing j^{th} column of Δ by the column of constants existing on right hand side of equations i.e. $C_1, C_2, \ldots C_n$;

$$D_1 = \begin{vmatrix} C_1 & a_{12} & \cdots & a_{1n} \\ C_2 & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ C_n & a_{n2} & \cdots & a_{nn} \end{vmatrix}, \qquad D_2 = \begin{vmatrix} a_{11} & C_1 & \cdots & a_{1n} \\ a_{21} & C_2 & \cdots & a_{2n} \\ \vdots & & & \\ a_{n1} & C_n & \cdots & a_{nn} \end{vmatrix}$$

and

$$D_n = \begin{vmatrix} a_{11} & a_{12} & \cdots & C_1 \\ a_{21} & a_{22} & \cdots & C_2 \\ \vdots & & \cdots & \vdots \\ a_{n1} & a_{n2} & \cdots & C_n \end{vmatrix}$$

The unknowns of the equations are given by Cramer's rule as,

$$X_1 = \frac{D_1}{D}, \; X_2 = \frac{D_2}{D}, \ldots\ldots\ldots\ldots, X_n = \frac{D_n}{D}$$

where $D_1, D_2, \ldots, D_n$ and D are values of the respective determinants.

Example 1.5 : *Apply Kirchhoff's current law and voltage law to the circuit shown in the Fig. 1.41.*

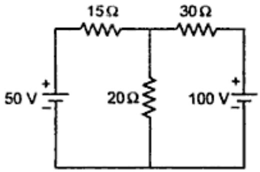


Fig. 1.41

Indicate the various branch currents.

Write down the equations relating the various branch currents.

Solve these equations to find the values of these currents.

Is the sign of any of the calculated currents negative ?

If yes, explain the significance of the negative sign.

Solution : Application of Kirchhoff's law :

Step 1 and 2 : Draw the circuit with all the values which are same as the given network. Mark all the branch currents starting from +ve of any of the source, say +ve of 50 V source.

Step 3 : Mark all the polarities for different voltages across the resistances. This is combined with step 2 shown in the network below in Fig. 1.41 (a).

Step 4 : Apply KVL to different loops.

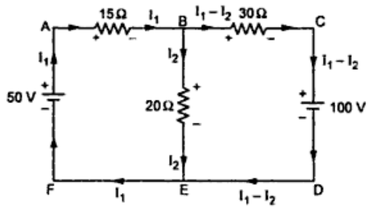


Fig. 1.41 (a)

Loop 1 : A-B-E-F-A.,

$$-15\,I_1 - 20\,I_2 + 50 = 0 \qquad \dots (1)$$

Loop 2 : B-C-D-E-B,

$$-30\,(I_1 - I_2) - 100 + 20\,I_2 = 0 \qquad \dots (2)$$

Rewriting all the equations, taking constants on one side.

$$15\,I_1 + 20\,I_2 = 50 \quad \dots(1) \quad \text{and} \quad -30\,I_1 + 50\,I_2 = 100 \quad \dots(2)$$

Apply Cramer's rule,
$$D = \begin{vmatrix} 15 & 20 \\ -30 & 50 \end{vmatrix} = 1350$$

Calculating D_1,
$$D_1 = \begin{vmatrix} 50 & 20 \\ 100 & 50 \end{vmatrix} = 500$$

$$I_1 = \frac{D_1}{D} = \frac{500}{1350} = \mathbf{0.37\ A}$$

Calculating D_2, $D_2 = \begin{vmatrix} 15 & 50 \\ -30 & 100 \end{vmatrix} = 3000$

$$I_2 = \frac{D_2}{D} = \frac{3000}{1350} = \mathbf{2.22\ A}$$

For I_1 and I_2, as answer is positive, assumed direction is correct.

$\therefore$ For I_1 answer is 0.37 A. For I_2 answer is 2.22 A

$$I_1 - I_2 = 0.37 - 2.22 = -\mathbf{1.85\ A}$$

Negative sign indicates assumed direction is wrong.

i.e. $I_1 - I_2 = 1.85$ A flowing in opposite direction to that of the assumed direction.

Example 1.6 : *Find i_1 and i_2 using KCL and KVL.* **[April - 2003 (Set-3)]**

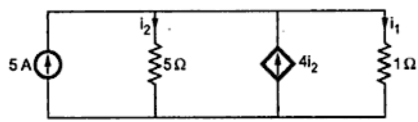


Fig. 1.42

Solution : The current distribution using KCL is as shown,

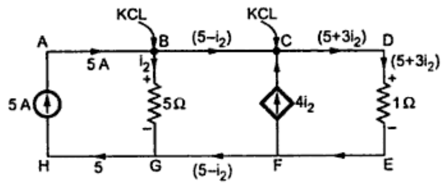


Fig. 1.42 (a)

Key Point *: KVL should not be applied to the loop consisting current source.*

From branch DE,

$$i_1 = 5 + 3i_2 \qquad \text{... (1)}$$

Applying KVL to the loop BCDEFGB without current source,

$$-1 \times (5 + 3i_2) + 5i_2 = 0 \qquad \text{... (2)}$$

$$\therefore \quad 2i_2 = 5$$

$$\therefore \quad i_2 = \mathbf{2.5\ A}$$

from equation (1) $i_1 = \mathbf{12.5\ A}$

1.21 Source Transformation

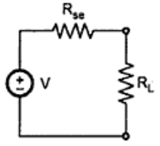


Fig. 1.43 (a) Voltage source

Consider a practical voltage source shown in the Fig. 1.43 (a) having internal resistance R_{se}, connected to the load having resistance R_L.

Now we can replace voltage source by equivalent current source.

Key Point: *The two sources are said to be* ***equivalent****, if they supply equal load current to the load, with same load connected across its terminals*

The current delivered in above case by voltage source is,

$$I = \frac{V}{(R_{se} + R_L)}, \qquad R_{se} \text{ and } R_L \text{ in series} \qquad \ldots(1)$$

If it is to be replaced by a current source then load current must be $\frac{V}{(R_{se} + R_L)}$

Consider an equivalent current source shown in the Fig. 1.43 (b).

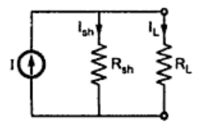


Fig. 1.43 (b) Current source

The total current is 'I'.

Both the resistances will take current proportional to their values.

From the current division in parallel circuit we can write,

$$I_L = I \times \frac{R_{sh}}{(R_{sh} + R_L)} \qquad \ldots(2)$$

Now this I_L and $\frac{V}{R_{se} + R_L}$ must be same, so equating (1) and (2),

$$\therefore \qquad \frac{V}{R_{se} + R_L} = \frac{I \times R_{sh}}{R_{sh} + R_L}$$

Let internal resistance be, $R_{se} = R_{sh} = R$ say.

Then, $\qquad V = I \times R_{sh} = I \times R$

or $\qquad I = \frac{V}{R_{sh}}$

$$\therefore \qquad \boxed{I = \frac{V}{R} = \frac{V}{R_{se}}}$$

Key Point: *If voltage source is converted to current source, then current source* $I = \dfrac{V}{R_{se}}$ *with parallel internal resistance equal to* R_{se}.

Key Point: *If current source is converted to voltage source, then voltage source* $V = I\,R_{sh}$ *with series internal resistance equal to* R_{sh}.

The direction of current of equivalent current source is always from **–ve to + ve, internal to the source.** While converting current source to voltage source, polarities of voltage is always as +ve terminal at top of arrow and –ve terminal at bottom of arrow, as direction of current is from –ve to +ve, internal to the source. **This ensures that current flows from positive to negative terminal in the external circuit.**

Note the directions of transformed sources, shown in the Fig. 1.43 (a), (b), (c) and (d).

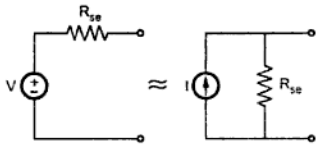


Fig. 1.44 (a) $I = \dfrac{V}{R_{se}}$

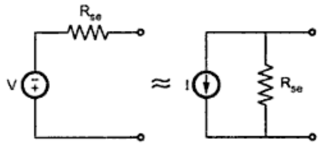


Fig 1.44 (b) $I = \dfrac{V}{R_{se}}$

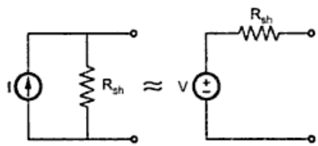


Fig. 1.44 (c) $V = I \times R_{sh}$

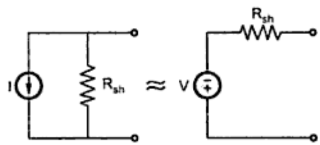


Fig. 1.44 (d) $V = I \times R_{sh}$

Example 1.7 : *Transform a voltage source of 20 volts with an internal resistance of 5 Ω to a current source.*

Solution: Refer to the Fig. 1.45 (a).

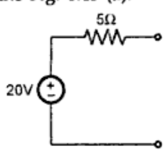


Fig. 1.45 (a)

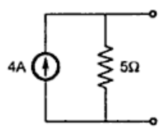


Fig. 1.45 (b)

Then current of current source is, $I = \dfrac{V}{R_{se}} = \dfrac{20}{5} = 4$ A with internal parallel resistance same as R_{se}.

$\therefore$ Equivalent current source is as shown in the Fig. 1.45 (b).

Example 1.8 : *Convert the given current source of 50 A with internal resistance of 10 Ω to the equivalent voltage source.*

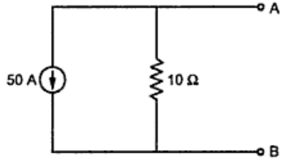


Fig. 1.46

Solution : The given values are, I = 50 A and R_{sh} = 10 Ω

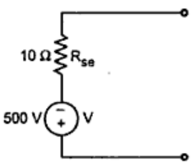


Fig. 1.46 (a)

For the equivalent voltage source,

$$V = I \times R_{sh} = 50 \times 10$$
$$= 500 \text{ V}$$
$$R_{se} = R_{sh} = 10\ \Omega \text{ in series.}$$

The equivalent voltage source is shown in the Fig. 1.46 (a).

Note the polarities of voltage source, which are such that + ve at top of arrow and – ve at bottom.

1.22 Star and Delta Connection of Resistances

In the complicated networks involving large number of resistances, Kirchhoff's laws give us complex set of simultaneous equations. It is time consuming to solve such set of simultaneous equations involving large number of unknowns. In such a case application of Star-Delta or Delta-Star transformation, considerably reduces the complexity of the network and brings the network into a very simple form. This reduces the number of unknowns and hence network can be analysed very quickly for the required result. These transformations allow us to replace three star connected resistances of the network, by equivalent delta connected resistances, without affecting currents in other branches and vice-versa.

Let us see what is Star connection ?

If the three resistances are connected in such a manner that one end of each is connected together to form a junction point called **Star point**, the resistances are said to be connected in **Star**.

The Fig. 1.47 (a) and (b) show star connected resistances. The star point is indicated as S. Both the connections Fig. 1.47 (a) and (b) are exactly identical. The Fig. 1.47 (b) can be redrawn as Fig. 1.47 (a) or vice-versa, in the circuit from simplification point of view.

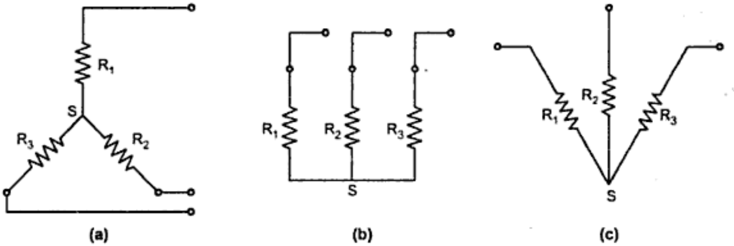


Fig. 1.47 Star connection of three resistances

Let us see what is delta connection ?

If the three resistances are connected in such a manner that one end of the first is connected to first end of second, the second end of second to first end of third and so on to complete a loop then the resistances are said to be connected in **Delta**.

Key Point: *Delta connection always forms a loop, closed path.*

The Fig. 1.48 (a) and (b) show delta connection of three resistances. The Fig. 1.48 (a) and (b) are exactly identical.

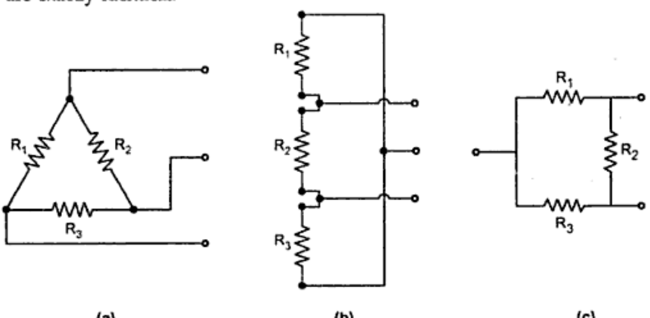
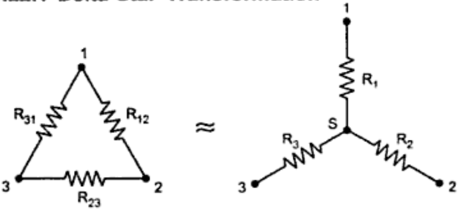


Fig. 1.48 Delta connection of three resistances

1.22.1 Delta-Star Transformation

Consider the three resistances R_{12}, R_{23}, R_{31} connected in Delta as shown in the Fig. 1.49. The terminals between which these are connected in Delta are named as 1, 2 and 3.

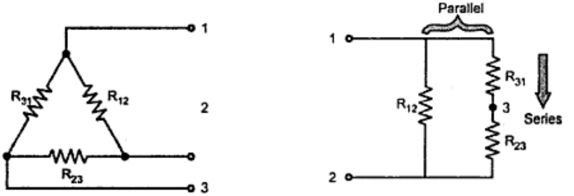


Given Delta **Fig. 1.49** **Equivalent Star**

Now it is always possible to replace these Delta connected resistances by three equivalent Star connected resistances R_1, R_2, R_3 between the same terminals 1, 2, and 3. Such a Star is shown inside the Delta in the Fig. 1.49 which is called **equivalent Star of Delta connected resistances.**

> **Key Point**: *Now to call these two arrangements as equivalent, the resistance between any two terminals must be same in both the types of connections.*

Let us analyse Delta connection first, shown in the Fig. 1.50 (a).



(a) Given Delta **(b) Equivalent between 1 and 2**

Fig. 1.50

Now consider the terminals (1) and (2). Let us find equivalent resistance between (1) and (2). We can redraw the network as viewed from the terminals (1) and (2), without considering terminal (3). This is shown in the Fig. 1.50 (b).

Now terminal '3' we are not considering, so between terminals (1) and (2) we get the combination as,

R_{12} parallel with $(R_{31}+R_{23})$ as R_{31} and R_{23} are in series.

$\therefore$ Between (1) and (2) the resistance is,

$$= \frac{R_{12}(R_{31}+R_{23})}{R_{12}+(R_{31}+R_{23})} \qquad \text{...(a)}$$

$$\left[\text{using } \frac{R_1\, R_2}{R_1+R_2} \text{ for parallel combination}\right]$$

Now consider the same two terminals of equivalent Star connection shown in the Fig. 1.51.

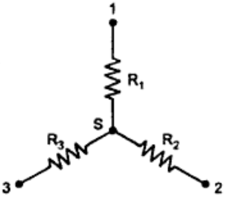


Fig. 1.51 Star connection

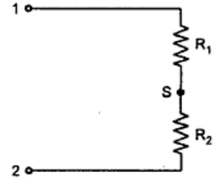


Fig. 1.52 Equivalent between 1 and 2

Now as viewed from terminals (1) and (2) we can see that terminal (3) is not getting connected anywhere and hence is not playing any role in deciding the resistance as viewed from terminals (1) and (2).

And hence we can redraw the network as viewed through the terminals (1) and (2) as shown in the Fig. 1.52.

$\therefore$ Between (1) and (2) the resistance is $= R_1 + R_2$ ... (b)

This is because, two of them found to be in series across the terminals 1 and 2 while 3 found to be open.

Now to call this Star as equivalent of given Delta it is necessary that the resistances calculated between terminals (1) and (2) in both the cases should be equal and hence equating equations (a) and (b),

$$\frac{R_{12}(R_{31}+R_{23})}{R_{12}+(R_{23}+R_{31})} = R_1 + R_2 \qquad \text{...(c)}$$

Similarly if we find the equivalent resistance as viewed through terminals (2) and (3) in both the cases and equating, we get,

$$\frac{R_{23}(R_{31}+R_{12})}{R_{12}+(R_{23}+R_{31})} = R_2 + R_3 \qquad \text{...(d)}$$

Similarly if we find the equivalent resistance as viewed through terminals (3) and (1) in both the cases and equating, we get,

$$\frac{R_{31}(R_{12}+R_{23})}{R_{12}+(R_{23}+R_{31})} = R_3 + R_1 \qquad \text{...(e)}$$

Now we are interested in calculating what are the values of R_1, R_2, R_3 interms of known values R_{12}, R_{23}, and R_{31}.

Subtracting euqation (d) from euqation (c),

$$\frac{R_{12}(R_{31}+R_{23}) - R_{23}(R_{31}+R_{12})}{(R_{12}+R_{23}+R_{31})} = R_1 + R_2 - R_2 - R_3$$

$$\therefore \qquad R_1 - R_3 = \frac{R_{12}\,R_{31} - R_{23}\,R_{31}}{R_{12}+R_{23}+R_{31}} \qquad \text{...(f)}$$

Adding euqation (f) and euqation (e),

$$\frac{R_{12}\,R_{31} - R_{23}\,R_{31} + R_{31}(R_{12}+R_{23})}{R_{12}+R_{23}+R_{31}} = R_1 + R_3 + R_1 - R_3$$

$$\therefore \qquad \frac{R_{12}\,R_{31} - R_{23}\,R_{31} + R_{31}\,R_{12} + R_{31}\,R_{23}}{R_{12}+R_{23}+R_{31}} = 2R_1$$

$$2R_1 = \frac{2\,R_{12}\,R_{31}}{R_{12}+R_{23}+R_{31}}$$

$$\boxed{R_1 = \frac{R_{12}\,R_{31}}{R_{12}+R_{23}+R_{31}}}$$

Similarly by using another combinations of subtraction and addition with equations (c), (d) and (e) we can get,

$$\boxed{R_2 = \frac{R_{12}\,R_{23}}{R_{12}+R_{23}+R_{31}}}$$

and

$$\boxed{R_3 = \frac{R_{23}\,R_{31}}{R_{12}+R_{23}+R_{31}}}$$

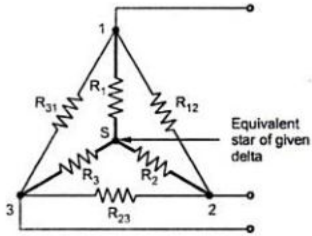


Fig. 1.53 Delta and equivalent Star

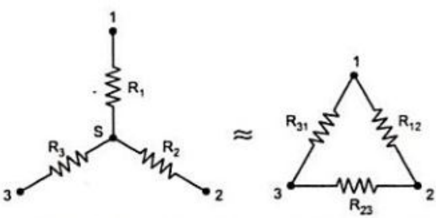
Easy way of remembering the result :

The equivalent star resistance between any terminal and star point is equal to the product of the two resistances in delta, which are connected to same terminal, divided by the sum of all three delta connected resistances.

So if we want equivalent resistance between terminal (2) and star point i.e. R_2 then it is the product of two resistances in delta which are connected to same terminal i.e. terminal (2) which are R_{12} and R_{23} divided by sum of all delta connected resistances i.e. R_{12}, R_{23} and R_{31}.

$$\therefore \qquad R_2 = \frac{R_{12}\,R_{23}}{R_{12}+R_{23}+R_{31}}$$

1.22.2 Star-Delta Transformation



Given Star Fig. 1.54 Equivalent Delta

Consider the three resistances R_1, R_2 and R_3 connected in Star as shown in Fig. 1.54.

Now by Star-Delta conversion, it is always possible to replace these Star connected resistances by three equivalent Delta connected resistances R_{12}, R_{23} and R_{31}, between the same terminals. This is called **equivalent Delta of the given star.**

Now we are interested in finding out values of R_{12}, R_{23} and R_{31} interms of R_1, R_2 and R_3.

For this we can use set of equations derived in previous article. From the result of Delta-Star transformation we know that,

$$R_1 = \frac{R_{12}\,R_{31}}{R_{12}+R_{23}+R_{31}} \quad \text{...(g)}$$

$$R_2 = \frac{R_{12}\,R_{23}}{R_{12}+R_{23}+R_{31}} \quad \text{...(h)}$$

$$R_3 = \frac{R_{23}\,R_{31}}{R_{12}+R_{23}+R_{31}} \quad \text{...(i)}$$

Now multiply (g) and (h), (h) and (i), (i) and (g) to get following three equations.

$$R_1R_2 = \frac{R_{12}^2\,R_{31}\,R_{23}}{(R_{12}+R_{23}+R_{31})^2} \quad \text{...(j)}$$

$$\therefore \quad R_2R_3 = \frac{R_{23}^2\,R_{12}\,R_{31}}{(R_{12}+R_{23}+R_{31})^2} \quad \text{...(k)}$$

$$R_3R_1 = \frac{R_{31}^2\,R_{12}\,R_{23}}{(R_{12}+R_{23}+R_{31})^2} \quad \text{...}(l)$$

Now add equations (j) ,(k) and (*l*)

$$\therefore \quad R_1R_2+R_2R_3+R_3R_1 = \frac{R_{12}^2R_{31}R_{23}+R_{23}^2R_{12}R_{31}+R_{31}^2R_{12}\,R_{23}}{(R_{12}+R_{23}+R_{31})^2}$$

$$\therefore \quad R_1R_2+R_2R_3+R_3R_1 = \frac{R_{12}R_{31}R_{23}(R_{12}+R_{23}+R_{31})}{(R_{12}+R_{23}+R_{31})^2}$$

$$\therefore \quad R_1R_2+R_2R_3+R_3R_1 = \frac{R_{12}R_{31}\,R_{23}}{R_{12}+R_{23}+R_{31}}$$

But $\dfrac{R_{12}R_{31}}{R_{12}+R_{23}+R_{31}} = R_1$ From equation (g)

$\therefore$ Substituting in above in R.H.S. we get,

$$\therefore \quad R_1R_2+R_2R_3+R_3R_1 = R_1\,R_{23}$$

$$\therefore \quad \boxed{R_{23} = R_2+R_3+\frac{R_2\,R_3}{R_1}}$$

Similarly substituting in R.H.S., remaining values, we can write relations for remaining two resistances.

$$\boxed{R_{12} = R_1 + R_2 + \frac{R_1 R_2}{R_3}}$$

and

$$\boxed{R_{31} = R_3 + R_1 + \frac{R_1 R_3}{R_2}}$$

Easy way of remembering the result :

The equivalent delta connected resistance to be connected between any two terminals is sum of the two resistances connected between the same two terminals and star point respectively in star, plus the product of the same two star resistances divided by the third star resistance.

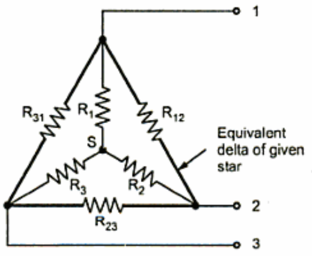


Fig. 1.55 Star and equivalent Delta

So if we want equivalent delta resistance between terminals (3) and (1), then take sum of the two resistances connected between same two terminals (3) and (1) and star point respectively i.e. terminal (3) to star point R_3 and terminal (1) to star point i.e. R_1. Then to this sum of R_1 and R_3, add the term which is the product of the same two resistances i.e. R_1 and R_3 divided by the third star resistance which is R_2.

$\therefore$ We can write, $R_{31} = R_1 + R_3 + \dfrac{R_1 R_3}{R_2}$ which is same as derived above.

Result for equal resistances in star and delta :

If all resistances in a Delta connection have same magnitude say R, then its equivalent Star will contain,

$$R_1 = R_2 = R_3 = \frac{R \times R}{R+R+R} = \frac{R}{3}$$

i.e. **equivalent Star contains three equal resistances, each of magnitude one third the magnitude of the resistances connected in Delta.**

If all three resistances in a Star connection are of same magnitude say R, then its equivalent Delta contains all resistances of same magnitude of ,

$$R_{12} = R_{31} = R_{23} = R+R+\frac{R \times R}{R} = 3R$$

i.e. **equivalent delta contains three resistances each of magnitude thrice the magnitude of resistances connected in Star.**

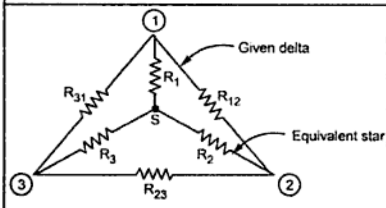
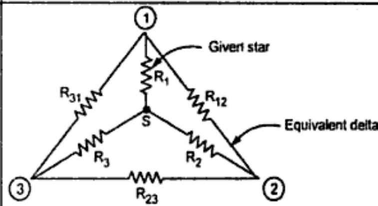
<table>
<tr><th>Delta-Star</th><th>Star-Delta</th></tr>
<tr><td>
</td><td>
</td></tr>
<tr><td>$R_1 = \frac{R_{12}R_{31}}{R_{12}+R_{23}+R_{31}}$</td><td>$R_{12} = R_1 + R_2 + \frac{R_1 R_2}{R_3}$</td></tr>
<tr><td>$R_2 = \frac{R_{12}R_{23}}{R_{12}+R_{23}+R_{31}}$</td><td>$R_{23} = R_2 + R_3 + \frac{R_2 R_3}{R_1}$</td></tr>
<tr><td>$R_3 = \frac{R_{23}R_{31}}{R_{12}+R_{23}+R_{31}}$</td><td>$R_{31} = R_3 + R_1 + \frac{R_1 R_3}{R_2}$</td></tr>
</table>

Table 1.6 Star-Delta and Delta-Star Transformations

➡ **Example 1.9 :** *Convert the given Delta in the Fig. 1.56 into equivalent Star.*

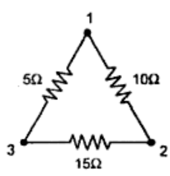


Fig. 1.56

Solution : Its equivalent star is as shown in the Fig. 1.57.

where

$$R_1 = \frac{10 \times 5}{5+10+15} = 1.67\ \Omega$$

$$R_2 = \frac{15 \times 10}{5+10+15} = 5\ \Omega$$

$$R_3 = \frac{5 \times 15}{5+10+15} = 2.5\ \Omega$$

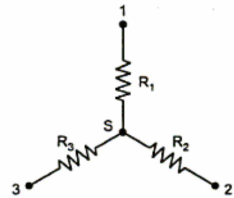


Fig. 1.57

Example 1.10 : *Find equivalent resistance between points A-B.*

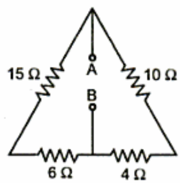


Fig. 1.58

Solution : Redrawing the circuit,

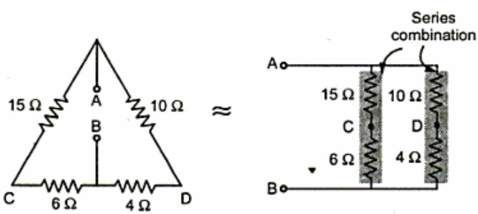
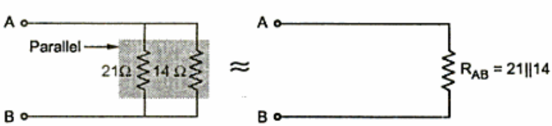


Fig. 1.58 (a)



$$R_{AB} = \frac{21 \times 14}{21+14} = 8.4\ \Omega$$

Fig. 1.58 (b)

Example 1.11 : *Find equivalent resistance between points A-B.*

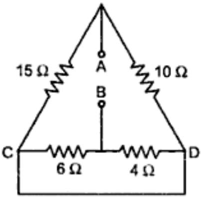


Fig. 1.59

Solution : Redraw the circuit,

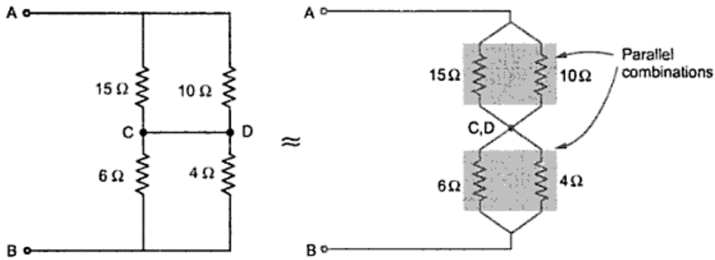


Fig. 1.59 (a)

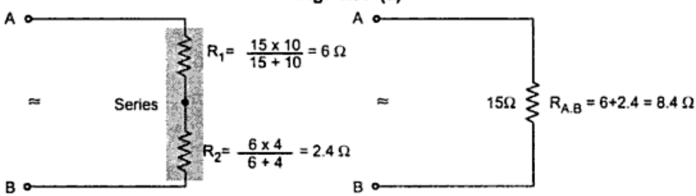


Fig. 1.59 (b)

$$\therefore \qquad R_{AB} = 8.4\ \Omega$$

1.23 Loop Analysis or Mesh Analysis

This method of analysis is specially useful for the circuits that have many nodes and loops. The difference between application of Kirchhoff's laws and loop analysis is, in loop analysis instead of branch currents, the loop currents are considered for writing the equations. The another difference is, in this method, each branch of the network may carry more than one current. The total branch current must be decided by the algebraic sum of all currents through that branch. While in analysis using Kirchhoff's laws, each branch carries only one current. The advantage of this method is that for complex networks the number of unknowns reduces which greatly simplifies calculation work.

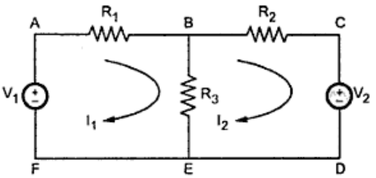


Fig. 1.60

Consider following network shown in the Fig. 1.60. There are two loops. So assuming two loop currents as I_1 and I_2.

Key Point : *While assume loop currents, consider the loops such that each element of the network will be included atleast once in any of the loops.*

Now branch B-E carries two currents; I_1 from B to E and I_2 from E to B. So net current through branch B-E will $(I_1 - I_2)$ and corresponding drop across R_3 must be as shown below in the Fig. 1.61.

Fig. 1.61

Consider loop A - B - E - F - A,

For branch B-E, polarities of voltage drops will be B +ve, E –ve for current I_1 while E +ve, B –ve for current I_2 flowing through R_3.

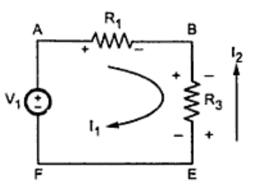


Fig. 1.62

Now while writing loop equations assume main loop current as positive and remaining loop current must be treated as negative for common branches.

Writing loop equations for the network shown in the Fig. 1.62.

For loop A - B - E - F - A,

$$-I_1 R_1 - I_1 R_3 + I_2 R_3 + V_1 = 0$$

For loop B - C - D - E - B

$$-I_2 R_2 - V_2 - I_2 R_3 + I_1 R_3 = 0$$

By solving above simultaneous equations any unknown branch current can be determined.

1.23.1 Points to Remember for Loop Analysis

1. While assuming loop currents make sure that atleast one loop current links with every element.
2. No two loops should be identical.
3. Choose minimum number of loop currents.
4. Convert current sources if present, into their equivalent voltage sources for loop analysis, whenever possible.
5. If current in a particular branch is required, then try to choose loop current in such a way that only one loop current links with that branch.

1.23.2 Supermesh

Key Point: *If there exists a current source in any of the branches of the network then a loop **cannot** be defined through the current source as drop across the current source is unknown, from KVL point of view.*

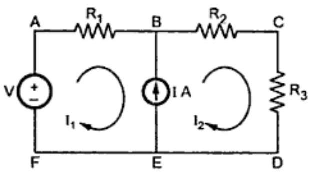


Fig. 1.63

For example, consider the network shown in the Fig. 1.63. In this circuit, branch B-E consists of a current source. So loop A-B-E-F-A cannot be defined as loop from KVL point of view, as drop across the current source is not known.

In such case, **to get the required equation interms of loop currents, analyse the branch consisting of a current source independently. Express the current source interms of the assumed loop currents**. For example, in the Fig. 1.63 analyse the branch BE. The current source is of IA in the direction of loop current I_2. So I_2 is more than I_1 and we can write an equation,

$$I = I_2 - I_1$$

So all such branches, consisting current sources must be analysed independently. Get the equations for current sources interms of loop currents. Then apply KVL to the remaining loops which are existing without involving the branches consisting of current sources. The loop existing, around a current source which is common to the two loops is called **supermesh**. In the Fig. 1.63, the loop A-B-C-D-E-F-A is supermesh.

1.23.3 Steps for the Loop Analysis

Step 1 :	Choose the various loops.
Step 2 :	Show the various loop currents and the polarities of associated voltage drops.

Step 3 : Before applying KVL, look for any current source. Analyse the branch consisting current source independently and express the current source value interms of assumed loop currents. Repeat this for all the current sources.

Step 4 : After the step 3, apply KVL to those **loops, which do not include any current source.** A loop cannot be defined through current source from KVL point of view. Follow the sign convention.

Step 5 : Solve the equations obtained in step 3 and step 4 simultaneously, to obtain required unknowns.

Example 1.12 : *For the circuit shown in the Fig. 1.64, find the current through 30 Ω resistance using mesh analysis.*

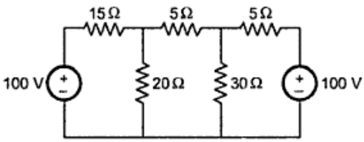


Fig. 1.64

Solution : The various loop currents are shown in the Fig. 1.64 (a).

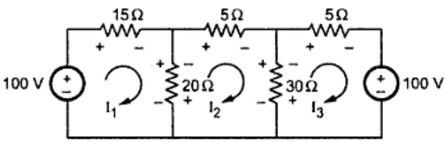


Fig. 1.64 (a)

Apply KVL to the various loops,

Loop 1, $\quad -15I_1 - 20I_1 + 20I_2 + 100 = 0$

$\therefore \quad +35I_1 - 20I_2 = 100 \quad \dots (1)$

Loop 2, $\quad -5I_2 - 30I_2 + 30I_3 - 20I_2 + 20I_1 = 0$

$\therefore \quad 20I_1 - 55I_2 + 30I_3 = 0 \quad \dots (2)$

Loop 3, $\quad -5I_3 - 100 - 30I_3 + 30I_2 = 0$

$\therefore \quad 30I_2 - 35I_3 = 100 \quad \dots (3)$

$$\therefore \quad D = \begin{vmatrix} 35 & -20 & 0 \\ 20 & -55 & 30 \\ 0 & 30 & -35 \end{vmatrix} = 21875$$

$$D_2 = \begin{vmatrix} 35 & 100 & 0 \\ 20 & 0 & 30 \\ 0 & 100 & -35 \end{vmatrix} = -35000$$

$$D_3 = \begin{vmatrix} 35 & -20 & 100 \\ 20 & -55 & 0 \\ 0 & 30 & 100 \end{vmatrix} = -92500$$

$$I_2 = \frac{D_2}{D} = \frac{-35000}{21875} = -1.6 \text{ A}$$

$$I_3 = \frac{D_3}{D} = \frac{-92500}{21875} = -4.2285 \text{ A}$$

$$\therefore \quad I_{30\Omega} = I_2 - I_3 = -1.6 - (-4.2285) = \mathbf{2.6285\ A} \downarrow$$

As $I_2 - I_3$ is positive, current flows in the assumed direction of I_2.

Example 1.13 : *In the circuit shown in the Fig. 1.65, use the loop analysis to find the power delivered to the 4 Ω resistor.*

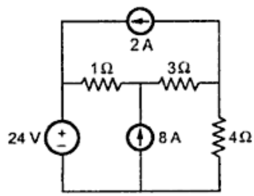


Fig. 1.65

Solution : The various loop currents are shown in the Fig. 1.65 (a). The problem consists of current sources hence follow supermesh steps.

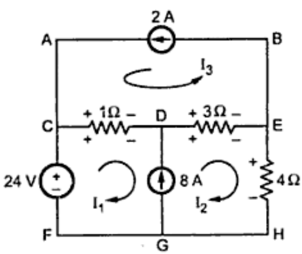


Fig. 1.65 (a)

Loops cannot be defined through current sources. So analyse the branches consisting of current sources first.

From branch A-B we can write,

$$I_3 = 2 \text{ A} \qquad \dots (1)$$

From branch DG we can write,

$$I_2 - I_1 = 8 \text{ A} \qquad \dots (2)$$

Now apply KVL to the loop **without current source** i.e.

loop C-D-E-H-G-F-C,

$$-1 \times I_3 - 1 \times I_1 - 3I_3 - 3I_2 - 4I_2 + 24 = 0$$

$$\therefore \quad 4I_3 + 7I_2 + I_1 = 24 \quad \text{... (3)}$$

Using equation (1) and equation (2) in (3) we get,

$$8 + 7I_2 + (I_2 - 8) = 24$$

$$\therefore \quad 8I_2 = 24$$

$$\therefore \quad I_2 = 3 \text{ A}$$

This is current through 4 Ω resistor. So power delivered to the 4 Ω resistor is,

$$P = I_2^2 \times 4 = 3^2 \times 4 = \mathbf{36\ W}$$

1.24 Node Analysis

This method is mainly based on **Kirchhoff's Current Law (KCL)**. This method uses the analysis of the different nodes of the network. We have already defined a node. Every junction point in a network, where two or more branches meet is called a **node**. One of the nodes is assumed as reference node whose potential is assumed to be zero. It is also called **zero potential node** or **datum node**. At other nodes the different voltages are to be measured with respect to this reference node. The reference node should be given a number zero and then the equations are to be written for all other nodes by applying KCL. The advantage of this method lies in the fact that we get (n – 1) equations to solve if there are 'n' nodes. This reduces calculation work.

Consider the following network shown in the Fig. 1.66.

Let voltages at node 1 and node 2 be V_1 and V_2. Mark various branch currents as shown in the Fig. 1.67. Now analyse each node using **KCL independently**.

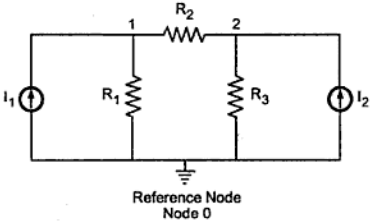


Fig. 1.66

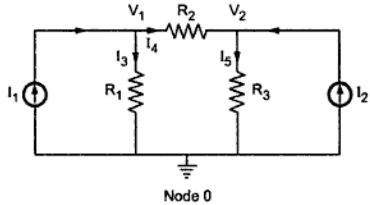


Fig. 1.67

Now applying KCL at node 1,

$$I_1 - I_3 - I_4 = 0 \quad \dots (1)$$

At node,

$$I_2 + I_4 - I_5 = 0 \quad \dots (2)$$

The currents in these equations can be expressed interms of node voltages as,

$$I_1 - \frac{V_1}{R_1} - \frac{(V_1 - V_2)}{R_2} = 0 \quad \dots (3)$$

and

$$I_2 + \frac{(V_1 - V_2)}{R_2} - \frac{V_2}{R_3} = 0 \quad \dots (4)$$

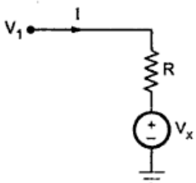


Fig. 1.68

As I_1 and I_2 are known, we get two equations (3) and (4) with the two unknowns V_1 and V_2. Solving these equations simultaneously, the node voltages V_1 and V_2 can be determined. Once V_1 and V_2 are known, current through any branch of the network can be determined. If there exists a voltage source in any of the branches as shown in the Fig. 1.68 then that must be considered while writing the equation for the current through that branch.

Now V_1 is at higher potential with respect to base, forcing current I downwards. While polarity of V_x is such that it tries to force current upwards. So in such a case equation of current becomes,

$$I = \frac{V_1 - V_x}{R} \quad \dots \text{ I leaving the node}$$

If the direction of current I is assumed entering the node then it is assumed that V_x is more than V_1 and hence equation for current I becomes,

$$I = \frac{V_x - V_1}{R} \quad \dots \text{ I entering the node}$$

1.24.1 Points to Remember for Nodal Analysis

1. While assuming branch currents, make sure that each unknown branch current is considered atleast once.
2. Convert the voltage source present into their equivalent current sources for node analysis, wherever possible.
3. Follow the same sign convention, currents entering at node are to be considered positive, while currents leaving the node are to be considered as negative.
4. As far as possible, select the directions of various branch currents leaving the respective nodes.

1.24.2 Supernode

Consider a circuit shown in the Fig. 1.69. In this circuit, the nodes labelled V_2 and V_3 are connected directly through a voltage source, without any circuit element. The region surrounding a voltage source which connects the two node directly is called **supernode**.

In such a case, the nodes in supernode region can be analysed seperately and the relation between such node voltages and a source voltage connecting them can be separately obtained. In the circuit shown in the Fig. 1.69 we can write,

$$V_2 = V_3 + V_x$$

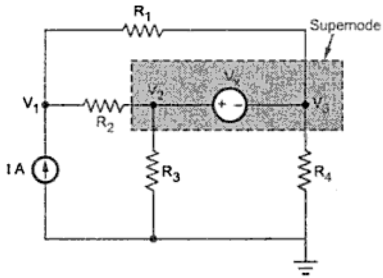


Fig. 1.69 Region of supernode

In addition to this equation, apply KCL to all the nodes assuming different branch currents at the nodes. The current through voltage source, connecting supernodes must be expressed interms of node voltages, using these KCL equations. Then the resulting equations and supernode equation are to be solved simultaneously to obtain the required unknown.

Now consider the loop including supernode as shown in the Fig. 1.70.

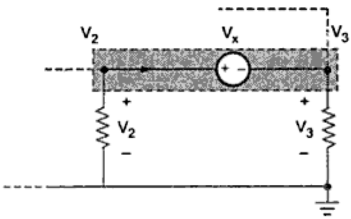


Fig. 1.70 Loop including supernode

Applying KVL to the loop,

$$-V_x - V_3 + V_2 = 0$$

$$\therefore \qquad \boxed{V_2 = V_x + V_3}$$

Thus the relationship between supernode voltages can be obtained using KVL.

Such equation can be written by inspection also. For the Fig. 1.71 shown, the equation is,

$$\boxed{V_3 = V_2 + V_x}$$

This is because V_x is helping V_2 to force current from node 2 to node 3.

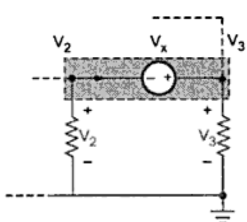


Fig. 1.71

1.24.3 Steps for the Node Analysis

Step 1 :	Choose the nodes and node voltages to be obtained.
Step 2 :	Choose the currents preferably leaving the node at each branch connected to each node.
Step 3 :	Apply KCL at each node with proper sign convention.
Step 4 :	If there are supernodes, obtain the equations directly interms of node voltages which are directly connected through voltage source.
Step 5 :	Obtain the equation for the each branch current interms of node voltages and substitute in the equations obtained in step 3.
Step 6 :	Solve all the equations obtained in step 4 and step 5 simultaneously to obtain the required node voltages.

Key Point : *If there are many number of branches in parallel in a network then node method is advantageous for the network analysis.*

Example 1.14 : *Find the current through each resistor of the circuit shown in the Fig. 1.72, using nodal analysis.*

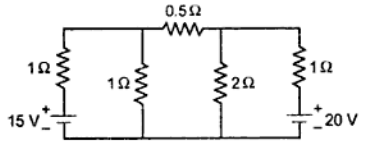


Fig. 1.72

Solution : The various node voltages and currents are shonw in the Fig. 1.72 (a).

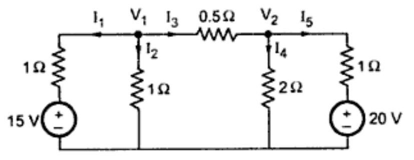


Fig. 1.72 (a)

At node 1, $\quad -I_1 - I_2 - I_3 = 0$

$$\therefore \quad -\left[\frac{V_1 - 15}{1}\right] - \left[\frac{V_1}{1}\right] - \left[\frac{V_1 - V_2}{0.5}\right] = 0$$

$$\therefore \quad -V_1 + 15 - V_1 - 2V_1 + 2V_2 = 0$$

$$\therefore \quad 4V_1 - 2V_2 = 15 \qquad \dots (1)$$

At node 2, $I_3 - I_4 - I_5 = 0$

$$\therefore \quad \frac{V_1 - V_2}{0.5} - \frac{V_2}{2} - \frac{V_2 - 20}{1} = 0$$

$$\therefore \quad 2V_1 - 2V_2 - 0.5\,V_2 - V_2 + 20 = 0$$

$$\therefore \quad 2V_1 - 3.5\,V_2 = -20 \qquad \ldots(2)$$

Multiplying equation (2) by 2 and subtracting from equation (1) we get,

$$5V_2 = 55$$

$$\therefore \quad V_2 = 11\text{ V}$$

and $V_1 = 9.25\text{ V}$

Hence the various currents are,

$$I_1 = \frac{V_1 - 5}{1} = 9.25 - 15 = -5.75\text{ A i.e. } 5.75\text{ A} \uparrow$$

$$I_2 = \frac{V_1}{1} = 9.25\text{ A}$$

$$I_3 = \frac{V_1 - V_2}{0.5} = -3.5\text{A i.e. } 3.5\text{ A} \leftarrow$$

$$I_4 = \frac{V_2}{2} = 5.5\text{ A}$$

$$I_5 = \frac{V_2 - 20}{1} = \frac{11 - 20}{1} = -9\text{ A i.e. } 9\text{ A} \uparrow$$

Example 1.15 : *Determine the node voltages for the network shown by nodal analysis.*
[May - 2004 (Set 1)]

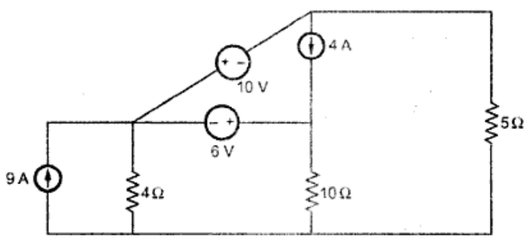


Fig. 1.73

Solution : The various node voltages are shown in the Fig. 1.73 (a).

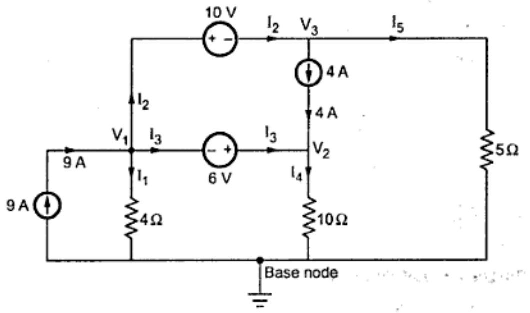


Fig. 1.73 (a)

The various branch currents are shown. Applying KCL at various nodes.

Node 1 : $$9 - I_1 - I_2 - I_3 = 0 \quad \text{... (1)}$$

Node 2 : $$I_3 - I_4 + 4 = 0 \quad \text{... (2)}$$

Node 3 : $$I_2 - 4 - I_5 = 0 \quad \text{... (3)}$$

Key Point: *Nodes V_1 and V_3 from supernode region and nodes V_1 and V_2 from super node region.*

Super node : $V_1 - 10 = V_3$ i.e. $V_1 - V_3 = 10$ ... (4)

Super node : $V_1 + 6 = V_2$ i.e. $V_1 - V_2 = -6$ ... (5)

From equation (2), $I_3 = I_4 - 4$ and from equation (3), $I_2 = I_5 + 4$

Using in equation (1), $9 - I_1 - I_5 - 4 - I_4 + 4 = 0$

i.e. $$I_1 + I_4 + I_5 = 9 \quad \text{... (6)}$$

But $I_1 = \frac{V_1}{4}$, $I_4 = \frac{V_2}{10}$, $I_5 = \frac{V_3}{5}$

$\therefore$ $$\frac{V_1}{4} + \frac{V_2}{10} + \frac{V_3}{5} = 9$$

i.e. $$0.25\, V_1 + 0.1 V_2 + 0.2\, V_3 = 9 \quad \text{...(7)}$$

Solving equations (4), (5) and (7) simultaneously,

$\mathbf{V_1 = 18.909\ V}$, $\mathbf{V_2 = 24.909\ V}$, $\mathbf{V_3 = 8.909\ V}$

1.25 A.C. through Pure Resistance

Consider a simple circuit consisting of a pure resistance 'R' ohms connected across a voltage $v = V_m \sin \omega t$. The circuit is shown in the Fig. 1.74.

According to Ohm's law, we can find the equation for the current i as,

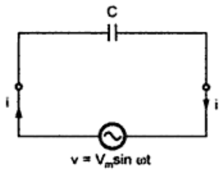


Fig. 1.74 Purely capacitive circuit

$$i = \frac{v}{R} = \frac{V_m \sin \omega t}{R} \quad \text{i.e.} \quad i = \left(\frac{V_m}{R}\right) \sin (\omega t)$$

This is the equation giving instantaneous value of the current.

Comparing this with standard equation,

$$i = I_m \sin (\omega t + \phi)$$

$$\boxed{I_m = \frac{V_m}{R} \quad \text{and} \quad \phi = 0}$$

So, maximum value of alternating current, i is $I_m = \frac{V_m}{R}$ while, as $\phi = 0$, it indicates that it is in phase with the voltage applied. There is no phase difference between the two. The current is going to achieve its maximum (positive and negative) and zero whenever voltage is going to achieve its maximum (positive and negative) and zero values.

Key Point: *In purely resistive circuit, the current and the voltage applied are in phase with each other.*

The waveforms of voltage and current and the corresponding phasor diagram is shown in the Fig. 1.75 (a) and (b).

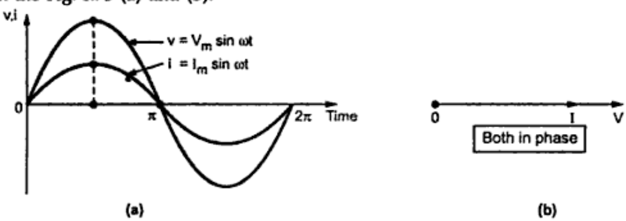


Fig. 1.75 A.C. through purely resistive circuit

In the phasor diagram, the phasors are drawn in phase and there is no phase difference in between them. Phasors represent the r.m.s. values of alternating quantities.

1.25.1 Power

The instantaneous power in a.c. circuits can be obtained by taking product of the instantaneous values of current and voltage.

$$P = v \times i = V_m \sin(\omega t) \times I_m \sin \omega t = V_m I_m \sin^2(\omega t)$$

$$= \frac{V_m I_m}{2}(1 - \cos 2\omega t)$$

$$\therefore \quad \boxed{P = \frac{V_m I_m}{2} - \frac{V_m I_m}{2}\cos(2\omega t)}$$

From the above equation, it is clear that the instantaneous power consists of two components,

1) Constant power component $\left(\frac{V_m I_m}{2}\right)$

2) Fluctuating component $\left[\frac{V_m I_m}{2}\cos(2\omega t)\right]$ having frequency, double the frequency of the applied voltage.

Now, the average value of the fluctuating cosine component of double frequency is zero, over one complete cycle. So, average power consumption over one cycle is equal to the constant power component i.e. $\frac{V_m I_m}{2}$.

$$\therefore \quad P_{av} = \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}}$$

$$\therefore \quad \boxed{P_{av} = V_{rms} \times I_{rms} \quad \text{watts}}$$

Generally, r.m.s. values are indicated by capital letters

$$\therefore \quad \boxed{P_{av} = V \times I \quad \text{watts} = I^2 R \quad \text{watts}}$$

The Fig. 1.76 shows the waveforms of voltage, current and power.

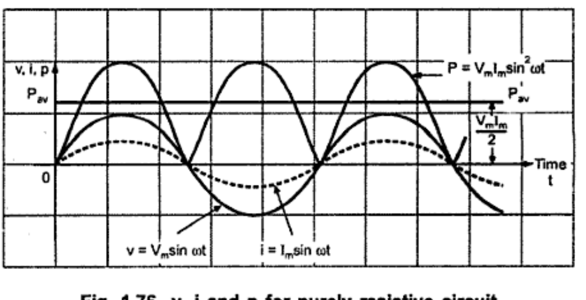


Fig. 1.76 v, i and p for purely resistive circuit

1.26 A.C. through Pure Inductance

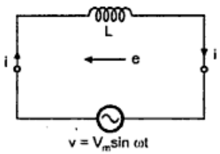


Fig. 1.77 Purely inductive circuit

Consider a simple circuit consisting of a pure inductance of L henries, connected across a voltage given by the equation, $v = V_m \sin \omega t$. The circuit is shown in the Fig. 1.77.

Pure inductance has zero ohmic resistance. Its internal resistance is zero. The coil has pure inductance of L henries (H).

When alternating current 'i' flows through inductance 'L', it sets up an alternating magnetic field around the inductance. This changing flux links the coil and due to self inductance, e.m.f. gets induced in the coil. This e.m.f. opposes the applied voltage.

The self induced e.m.f. in the coil is given by,

Self induced e.m.f., $$e = -L\frac{di}{dt}$$

At all instants, applied voltage, V is equal and opposite to the self induced e.m.f., e

$$\therefore \quad v = -e = -\left(-L\frac{di}{dt}\right)$$

$$\therefore \quad v = L\frac{di}{dt} \quad \text{i.e. } V_m \sin \omega t = L\frac{di}{dt}$$

$$\therefore \quad di = \frac{V_m}{L}\sin \omega t\, dt$$

$$\therefore \quad i = \int di = \int \frac{V_m}{L}\sin \omega t\, dt = \frac{V_m}{L}\left(\frac{-\cos \omega t}{\omega}\right)$$

$$= -\frac{V_m}{\omega L}\sin\left(\frac{\pi}{2}-\omega t\right) \quad \text{as } \cos \omega t = \sin\left(\frac{\pi}{2}-\omega t\right)$$

$$\therefore \quad i = \frac{V_m}{\omega L}\sin\left(\omega t-\frac{\pi}{2}\right) \quad \text{as } \sin\left(\frac{\pi}{2}-\omega t\right) = -\sin\left(\omega t-\frac{\pi}{2}\right)$$

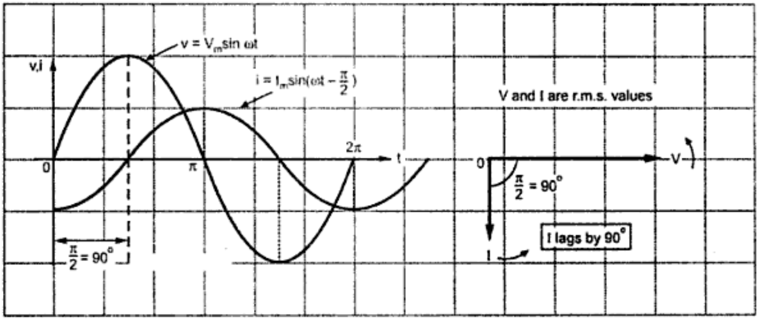
$$\therefore \quad \boxed{i = I_m \sin\left(\omega t-\frac{\pi}{2}\right)}$$

Where $$I_m = \frac{V_m}{\omega L} = \frac{V_m}{X_L}$$

Where $$\boxed{X_L = \omega L = 2\pi f L \ \Omega}$$

The above equation clearly shows that the current is purely sinusoidal and having phase angle of $-\frac{\pi}{2}$ radians i.e. $-90°$. This means that **the current lags voltage applied by 90°**. The negative sign indicates lagging nature of the current. If current is assumed as a reference, we can say that the voltage across inductance leads the current passing through the inductance by 90°.

The Fig. 1.78 shows the waveforms and the corresponding phasor diagram.



(a) Waveforms (b) Phasor diagram

Fig. 1.78 A.C. through purely inductive circuit

Key Point: *In purely inductive circuit, current lags voltage by 90°.*

1.26.1 Concept of Inductive Reactance

We have seen that in purely inductive circuit,

$$I_m = \frac{V_m}{X_L}$$

where $\boxed{X_L = \omega L = 2\pi f L\ \Omega}$

The term, X_L, is called **Inductive Reactance** and is measured in **ohms.**

So, **inductive reactance** is defined as the opposition offered by the inductance of a circuit to the flow of an alternating sinusoidal current.

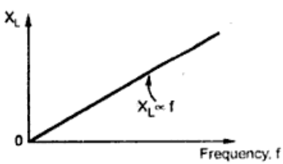


Fig. 1.79 X_L Vs f

It is measured in ohms and it depends on the frequency of the applied voltage.

The inductive reactance is directly proportional to the frequency for constant L.

$X_L \propto f$, for constant L

So, graph of X_L Vs f is a straight line passing through the origin as shown in the Fig. 1.79.

Key Point: *If frequency is zero, which is so for d.c. voltage, the inductive reactance is zero. Therefore, it is said that the inductance offers zero reactance for the d.c. or steady current.*

1.26.2 Power

The expression for the instantaneous power can be obtained by taking the product of instantaneous voltage and current.

$$\therefore \qquad P = v \times i = V_m \sin \omega t \times I_m \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$= -V_m I_m \sin(\omega t) \cos(\omega t) \quad \text{as } \sin\left(\omega t - \frac{\pi}{2}\right) = -\cos \omega t$$

$$\therefore \qquad \boxed{P = -\frac{V_m I_m}{2} \sin(2\,\omega t)} \quad \text{as } 2 \sin \omega t \cos \omega t = \sin 2\omega t$$

Key Point : *This power curve is a sine curve of frequency double than that of applied voltage.*

The average value of sine curve over a complete cycle is always zero.

$$P_{av} = \int_0^{2\pi} -\frac{V_m I_m}{2} \sin(2\,\omega t)\, d(\omega t) = 0$$

The Fig. 1.80 shows voltage, current and power waveforms.

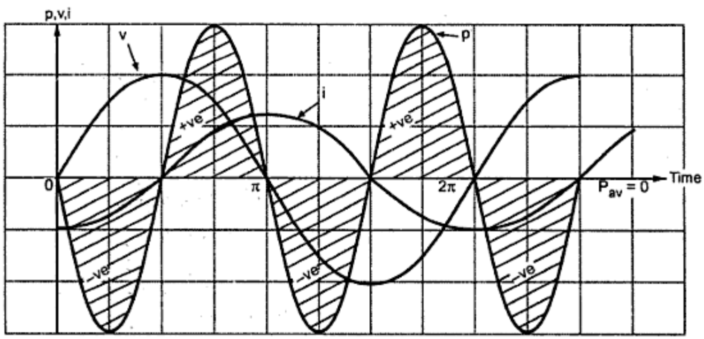


Fig. 1.80 Waveforms of voltage, current and power

It can be observed from it that when power curve is positive, energy gets stored in the magnetic field established due to the increasing current while during negative power curve, this power is returned back to the supply.

The areas of positive loop and negative loop are exactly same and hence, average power consumption is zero.

Key Point : *Pure inductance never consumes power.*

1.27 A.C. through Pure Capacitance

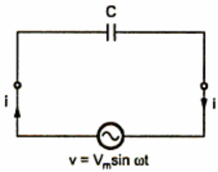


Fig. 1.81 Purely capacitive circuit

Consider a simple circuit consisting of a pure capacitor of C– farads, connected across a voltage given by the equation, $v = V_m \sin \omega t$. The circuit is shown in the Fig. 1.81.

The current i charges the capacitor C. The instantaneous charge 'q' on the plates of the capacitor is given by,

$$q = C v$$

$$\therefore \quad q = C V_m \sin \omega t$$

Now, current is rate of flow of charge.

$$\therefore \quad i = \frac{dq}{dt} = \frac{d}{dt}(C V_m \sin \omega t)$$

$$\therefore \quad i = C V_m \frac{d}{dt}(\sin \omega t) = C V_m \omega \cos \omega t$$

$$\therefore \quad i = \frac{V_m}{\left(\frac{1}{\omega C}\right)} \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$\therefore \quad i = I_m \sin\left(\omega t + \frac{\pi}{2}\right)$$

where $\boxed{I_m = \frac{V_m}{X_C}}$

where $\boxed{X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} \quad \Omega}$

The above equation clearly shows that the current is purely sinusoidal and having phase angle of $+\frac{\pi}{2}$ radians i.e. + 90°.

This means **current leads voltage applied by 90°.** The positive sign indicates leading nature of the current. If current is assumed reference, we can say that voltage across capacitor lags the current passing through the capacitor by 90°.

The Fig. 1.82 shows waveforms of voltage and current and the corresponding phasor diagram. The current waveform starts earlier by 90° in comparison with voltage waveform. When voltage is zero, the current has positive maximum value.

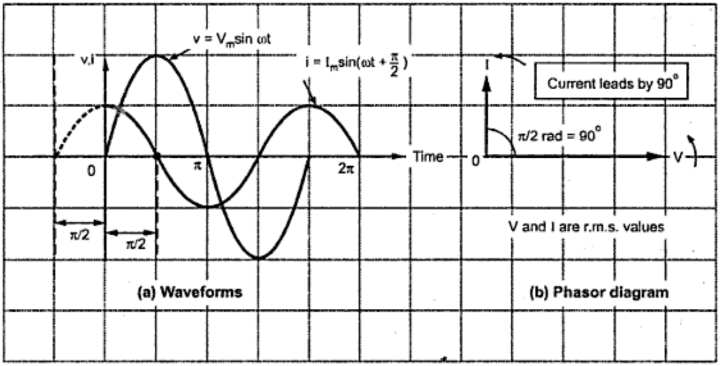


Fig. 1.82 A.C. through purely capacitive circuit

Key Point : *In purely capacitive circuit, current leads voltage by 90°.*

1.27.1 Concept of Capacitive Reactance

We have seen while expressing current equation in the standard form that,

$$I_m = \frac{V_m}{X_C} \quad \text{and} \quad X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}\ \Omega$$

The term X_C is called **Capacitive Reactance** and is measured in ohms.

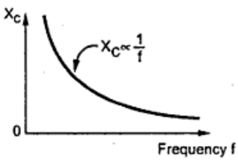


Fig. 1.83 X_C Vs f

So, **capacitive reactance** is defined as the opposition offered by the capacitance of a circuit to the flow of an alternating sinusoidal current.

X_C is measured in ohms and it depends on the frequency of the applied voltage.

The capacitive reactance is inversely proportional to the frequency for constant C.

$$X_C \propto \frac{1}{f} \quad \text{for constant C}$$

The graph of X_C Vs f is a rectangular hyperbola as shown in Fig. 1.83.

Key Point : *If the frequency is zero, which is so for d.c. voltage, the capacitive reactance is infinite. Therefore, it is said that the capacitance offers open circuit to the d.c. or it blocks d.c.*

1.27.2 Power

The expression for the instantaneous power can be obtained by taking the product of instantaneous voltage and current.

$$P = v \times i = V_m \sin(\omega t) \times I_m \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$= V_m I_m \sin(\omega t) \cos(\omega t) \qquad \text{as } \sin\left(\omega t + \frac{\pi}{2}\right) = \cos \omega t$$

$$\therefore \quad \boxed{P = \frac{V_m I_m}{2} \sin(2\omega t)} \qquad \text{as } 2 \sin \omega t \cos \omega t = \sin 2\omega t$$

Thus, power curve is a sine wave of frequency double that of applied voltage. The average value of sine curve over a complete cycle is always zero.

$$P_{av} = \int_0^{2\pi} \frac{V_m I_m}{2} \sin(2\omega t) \, d(\omega t) = 0$$

The Fig. 1.84 shows waveforms of current, voltage and power. It can be observed from the figure that when power curve is positive, in practice, an electrostatic energy gets stored in the capacitor during its charging while the negative power curve represents that the energy stored is returned back to the supply during its discharging. The areas of positive and negative loops are exactly the same and hence, average power consumption is zero.

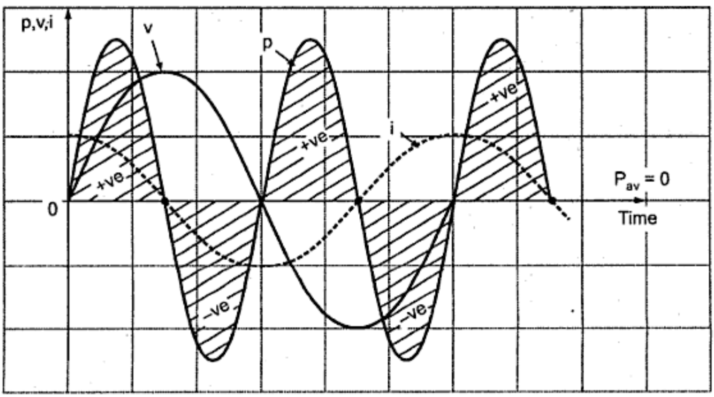


Fig. 1.84 Waveforms of voltage, current and power

Key Point : *Pure capacitance never consumes power.*

A.C. quantities are represented in equation form as,

$e = E_m \sin(\omega t+\phi)$ or $i = I_m \sin(\omega t+\phi)$

where E_m and I_m are maximum values and ϕ is phase. Practically their r.m.s. values are used given by,

$$E = \frac{E_m}{\sqrt{2}} \text{ and } I = \frac{I_m}{\sqrt{2}} \text{ where E, I are r.m.s. values.}$$

Example 1.16 : *A 50 Hz, alternating voltage of 150 V (r.m.s.) is applied independently to (1) Resistance of 10 Ω (2) Inductance of 0.2 H (3) Capacitance of 50 μF*

Find the expression for the instantaneous current in each case. Draw the phasor diagram in each case.

Solution : Case 1 : R = 10 Ω

$$V = V_m \sin \omega t$$

$$V_m = \sqrt{2}\, V_{r.m.s.} = \sqrt{2} \times 150 = 212.13 \text{ V}$$

$$I_m = \frac{V_m}{R} = \frac{212.13}{10} = 21.213 \text{ A}$$

In pure resistive circuit, current is in phase with the voltage.

$\therefore \quad \phi = \text{Phase Difference} = 0°$

$\therefore \quad i = I_m \sin \omega t = I_m \sin(2\pi f t)$

$\therefore \quad \mathbf{i = 21.213 \sin(100\pi t) \text{ A}}$

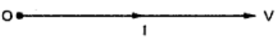


Fig. 1.85 (a)

The phasor diagram is shown in the Fig. 1.85 (a).

Case 2 : L = 0.2 Ω

Inductive reactance, $X_L = \omega L = 2\pi f L = 2\pi \times 50 \times 0.2 = 62.83\ \Omega$

$\therefore \quad I_m = \frac{V_m}{X_L} = \frac{212.13}{62.83} = 3.37 \text{ A}$

In pure inductive circuit, current lags voltage by 90°.

$\therefore \quad \phi = \text{Phase difference} = -90° = \frac{\pi}{2} \text{ rad}$

$\therefore \quad i = I_m \sin(\omega t - \phi)$ i.e. $\mathbf{i = 3.37 \sin\left(100\pi t - \frac{\pi}{2}\right) \text{ A}}$

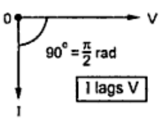


Fig. 1.85 (b)

The phasor diagram is shown in the Fig. 1.85 (b).

Case 3 : $C = 50\ \mu F$

Capacitive reactance, $$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 50 \times 10^{-6}} = 63.66\ \Omega$$

$$\therefore \quad I_m = \frac{V_m}{X_C} = \frac{212.13}{63.66} = 3.33\ A$$

In pure capacitive circuit, current leads voltage by 90°.

$$\therefore \quad \phi = \text{Phase Difference} = 90^\circ = \frac{\pi}{2}\ \text{rad}$$

$$\therefore \quad i = I_m \sin(\omega t + \phi)$$

$$\therefore \quad i = \mathbf{3.33\ sin\left(100\pi t + \frac{\pi}{2}\right)\ A}$$

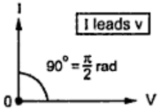


Fig. 1.85 (c)

The phasor diagram is shown in the Fig. 1.85 (c).

All the phasor diagrams represent r.m.s. values of voltage and current.

Examples with Solutions

Example 1.17 : *Find the voltage to be applied across AB in order to drive a current of 5 A into the circuit using star-delta transformations.* **[JNTU : May-2006, Set-1]**

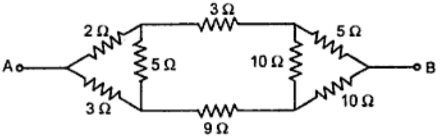


Fig. 1.86

Solution :

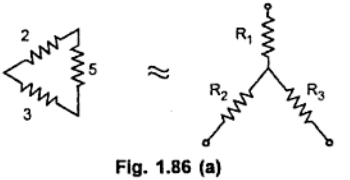


Fig. 1.86 (a)

$$R_1 = \frac{2 \times 5}{2+5+3} = 1\ \Omega$$

$$R_2 = \frac{3 \times 2}{2+5+3} = 0.6\ \Omega$$

$$R_3 = \frac{5 \times 3}{2+5+3} = 1.5\ \Omega$$

$$R_1 = \frac{10 \times 5}{10+5+10} = 2\ \Omega$$

$$R_2 = \frac{10\times10}{10+5+10} = 4\ \Omega$$

$$R_3 = \frac{5\times10}{10+5+10} = 2\Omega$$

The circuit reduces as shown in the Fig. 1.86 (c).

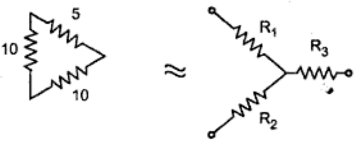


Fig. 1.86 (b)

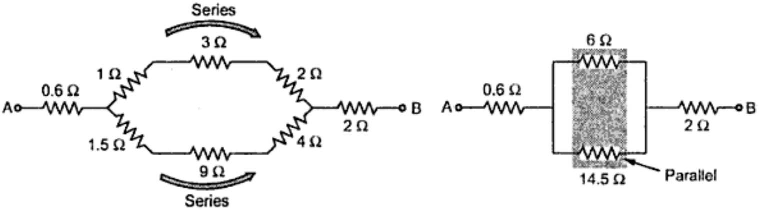


Fig. 1.86 (c) and (d)

$$R_{AB} = 0.6 + 4.2439 + 2$$

$$= 6.8439\ \Omega$$

$$I = \frac{V}{R_{AB}}$$

$$\therefore\ V = I\times R_{AB} = 5\times6.8439 = \mathbf{34.2195\ V}$$

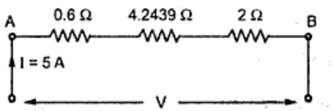


Fig. 1.86 (e)

Example 1.18 : *Find the equivalent resistance between the terminals Y and Z.*

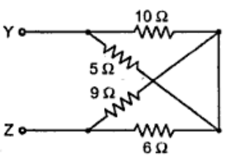
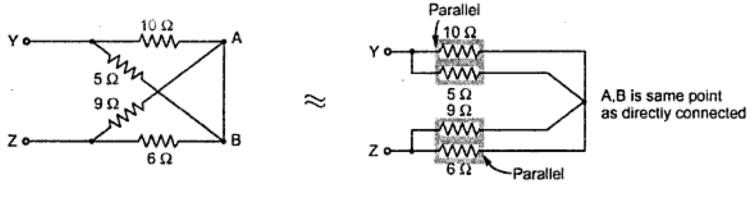


Fig. 1.87

[JNTU : June-2005, Set-3, Set-4]

Solution : Rearrange the circuit as shown below.



(a) (b)

Fig. 1.87

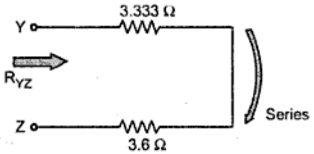


Fig. 1.87 (c)

$$\therefore\ R_{YZ} = 3.333 + 3.6 = \mathbf{6.9333\ \Omega}$$

➠ **Example 1.19 :** *Find the battery currents in all the sources of the network shown in the Fig. 1.88.* **[JNTU : Nov.-2005, June-2005, Set-1]**

Solution : Using the loop analysis, (Fig. 1.88 (a) see on next page)

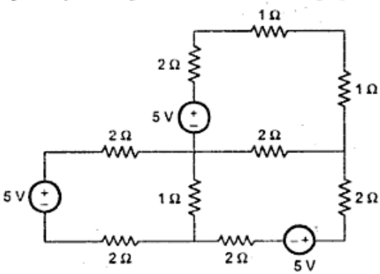


Fig. 1.88

Applying KVL to the three loops,

$-I_1 - I_1 - 2I_1 + 2I_3 + 5 - 2I_1 = 0$ i.e. $-6I_1 + 2I_3 = -5$ ... (1)

$-2I_3 + 2I_1 - 2I_3 - 5 - 2I_3 - I_3 + I_2 = 0$ i.e. $2I_1 + I_2 - 7I_3 = 5$ ... (2)

$-2I_2 - I_2 + I_3 - 2I_2 + 5 = 0$ i.e. $-5I_2 + I_3 = -5$ ... (3)

Solving equation (1), (2) and (3)

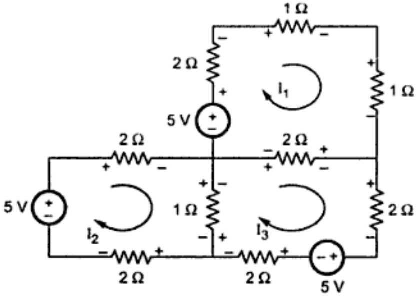


Fig. 1.88 (a)

$I_1 = 0.7065$ A, $I_2 = 0.9239$ A and $I_3 = -0.3804$ A

These are the currents in all the sources. I_3 is negative hence its direction is opposite to that assumed earlier.

Example 1.20 : *Calculate the unknown resistance R and the current flowing through it when the current in the branch OC is zero.*

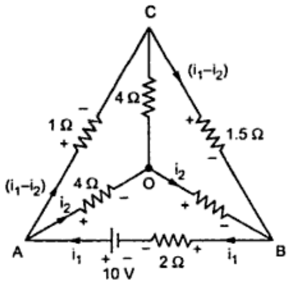


Fig. 1.89

[JNTU : March-2006 (set-1), Nov.-2005 (set-4), May-2005 (set-1)]

Solution : The various branch currents are shown in the Fig. 1.89. The current through branch OC is zero. Applying KVL to the various loops,

$$-4i_2 - Ri_2 - 2i_1 + 10 = 0$$

i.e. $$2i_1 + 4i_2 + Ri_2 = 10 \qquad \text{... Loop AOBA}$$

$$-(i_1 - i_2) - 1.5(i_1 - i_2) - 2i_1 + 10 = 0$$

i.e. $$+4.5i_1 - 2.5i_2 = 10 \qquad \text{... Loop ACBA}$$

$$-(i_1 - i_2) - 1.5(i_1 - i_2) + i_2R + 4i_2 = 0$$

i.e. $$-2.5i_1 + 6.5i_2 + i_2R = 0 \quad \text{... Loop ACBOA}$$

As current through branch O-C is zero, points O and C are **equipotential.** So drop across AO is same as drop acroos AC.

$\therefore$ $$4i_2 = 1(i_1 - i_2) \quad \text{i.e.} \quad i_1 = 5i_2$$

Using in loop A-C-B-A, $4.5(5i_2) - 2.5i_2 = 10$

$\therefore$ $$i_2 = 0.5 \text{ A}, \; i_1 = 2.5 \text{ A}$$

Using in loop A-O-B-A, $2 \times 2.5 + 4 \times 0.5 + 0.5R = 10$

$\therefore$ $$R = 6\,\Omega$$

And current through R is i_2 = **0.5 A**

Example 1.21 : *For the circuit shown in the Fig. 1.90 determine the current through 6 Ω resistor and the power supplied by the current source.*

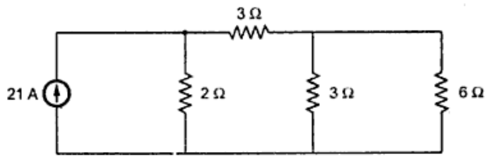


Fig. 1.90

[JNTU : Nov.-2007 (set-4)]

Solution : Use node analysis,

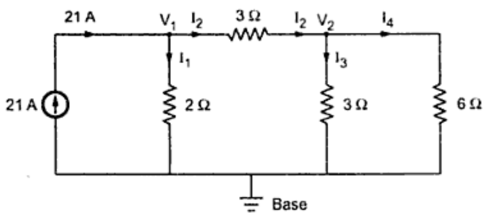


Fig. 1.91

Applying KCL at the two nodes,

$$21 - I_1 - I_2 = 0 \quad \text{... (1)}$$

$$I_2 - I_3 - I_4 = 0 \quad \text{... (2)}$$

Analysing various branches,

$$I_1 = \frac{V_1 - 0}{2}, \quad I_2 = \frac{V_1 - V_2}{3}, \quad I_3 = \frac{V_2 - 0}{3}, \quad I_4 = \frac{V_2 - 0}{6}$$

Using in the equations (1) and (2),

$$21 - \frac{V_1}{2} - \left[\frac{V_1 - V_2}{3}\right] = 0$$

i.e. $$0.8333\,V_1 - 0.333\,V_2 = 21 \qquad \text{... (3)}$$

$$\left[\frac{V_1 - V_2}{3}\right] - \frac{V_2}{3} - \frac{V_2}{6} = 0$$

i.e. $$0.3333\,V_1 - 0.8333\,V_2 = 0 \qquad \text{... (4)}$$

Solving equations (3) and (4), $V_1 = 30$ V, $V_2 = 12$ V

$$\therefore \quad I_{6\Omega} = I_4 = -\frac{V_2}{6} = 2\text{ A}\downarrow \qquad \text{... current through } 6\ \Omega$$

Voltage aross current source is the voltage across 2 Ω resistance, which is node voltage $V_1 = 30$ V.

$$\therefore \quad P = V_1 \times 21\text{ A} = 30 \times 21 = 630\text{ W} \qquad \text{... power supplied by source}$$

Example 1.22 : *Three resistors R_1, R_2 and R_3 are connected in series with a constant voltage source of V volts. The voltage across R_1 is 4 V, power loss in R_2 is 16 W and the value of R_3 is 6 Ω. If the current flowing through the circuit is 2 A, find the voltage V.*
[JNTU : Nov.-2007 (set-1)]

Solution : The arrangement is shown in the Fig. 1.92.

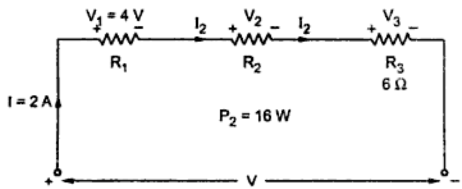


Fig. 1.92

$$P_2 = I^2 R_2$$

$$\therefore \quad 16 = (2)^2 R_2$$

$$\therefore \quad R_2 = 4\ \Omega$$

$$\therefore \quad V_2 = IR_2 = 8\text{ V}$$

$$V_3 = IR_3 = 2 \times 6 = 12\text{ V}$$

$$\therefore \quad V = V_1 + V_2 + V_3 = 4 + 8 + 12 = 24\text{ V}$$

Example 1.23 : *In the network shown in the Fig. 1.93, find all the branch currents and voltage drops across all resistors.*
[JNTU : Nov.-2007 (set-3)]

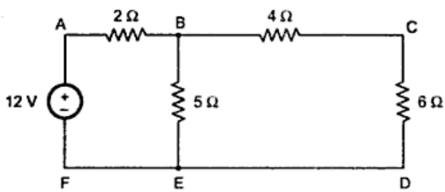


Fig. 1.93

Solution : The branch currents are shown in the Fig. 1.93 (a).

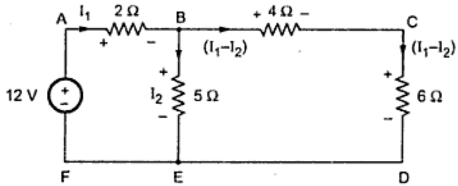


Fig. 1.93 (a)

Applying KVL to the two loops,

$$-2I_1 - 5I_2 + 12 = 0$$

i.e. $$2I_1 + 5I_2 = 12 \qquad \ldots (1)$$

$$-4(I_1 - I_2) - 6(I_1 - I_2) + 5I_2 = 0$$

i.e. $$-10I_1 + 15I_2 = 0$$

Solving equation (1) and (2), $I_1 = 2.25$ A, $I_2 = 1.5$ A

Branch	Current	Voltage drop
A-B	$I_1 = 2.25$ A	$2I_1 = 4.5$ V
B-C	$I_1 - I_2 = 0.75$ A	$4(I_1 - I_2) = 3$ V
C-D	$I_1 - I_2 = 0.75$ A	$6(I_1 - I_2) = 4.5$ V
B-E	$I_2 = 1.5$ A	$5I_2 = 7.5$ V
F-A	$I_1 = 2.25$ A ↑	12 V source

Example 1.24 : *Find the voltage drop across 1 Ω resistor and power loss across 2 Ω resistor in the Fig. 1.94.*

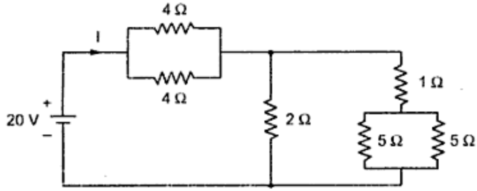


Fig. 1.94

[JNTU : Nov.-2008 (set-4)]

Solution :

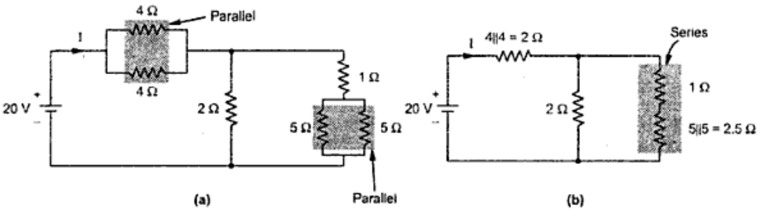


Fig. 1.95

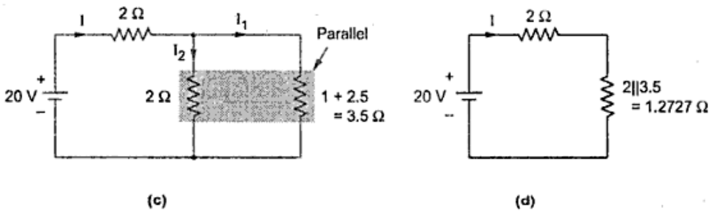


Fig. 1.96

$$\therefore \qquad I = \frac{20}{2 + 1.2727} = 6.1111 \text{ A}$$

By current division rule,

$$I_1 = I \times \frac{2}{2+3.5} = \frac{6.1111 \times 2}{5.5} = 2.222 \text{ A} \qquad \text{... current through } 1\,\Omega$$

$$\therefore \qquad V_{1\Omega} = I_1 \times 1\,\Omega = \mathbf{2.222\ V} \qquad \text{... voltage across } 1\,\Omega$$

$$I_2 = I \times \frac{3.5}{2+3.5} = \frac{6.1111 \times 3.5}{5.5} = 3.888 \text{ A} \qquad \text{... current through } 2\,\Omega$$

$$\therefore \qquad P_{2\Omega} = (I_2)^2 \times 2\,\Omega = (3.888)^2 \times 2 = \mathbf{30.233\ W} \qquad \text{... power across } 2\,\Omega$$

Example 1.25 : *In an a.c. circuit containing pure inductance, the voltage applied is 110 V, 50 Hz while the current is 10 A. Find the value of inductive reactance and inductance.*

[JNTU : Nov.-2008 (Set-2)]

Solution : The arrangement is shown in the Fig. 1.97.

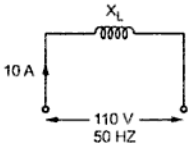


Fig. 1.97

$$|X_L| = \frac{|V|}{|I|} = \frac{110}{10} = \mathbf{11\ \Omega}$$

The inductive reactance is 11 Ω

$$X_L = 2\pi fL$$

$$\therefore \quad 11 = 2\pi \times 50 \times L$$

$$\therefore \quad L = \mathbf{35.014\ mH}$$

Example 1.26 : *Determine the reactance of 50 μF capacitor in a d.c. supply and also in an a.c. supply of 100 Hz.* **[JNTU : Nov.-2008 (Set-3)]**

Solution : For d.c. supply frequency is 0 Hz.

$$\therefore \quad X_C = \frac{1}{2\pi fC} = \frac{1}{0} = \infty$$

So capacitor gives **infinite** reactance in d.c. supply and acts as an open circuits.

In an a.c. supply of 100 Hz,

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 100 \times 50 \times 10^{-6}} = \mathbf{31.8309\ \Omega}$$

Example 1.27 : *Determine the value of resistance R and current in each branch when the total current taken by the circuit shown in the Fig. 1.98 is 6 A.*

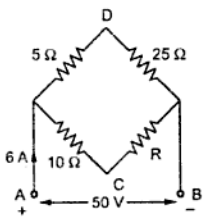


Fig. 1.98

[JNTU : Nov.-2008 (Set-1)]

Solution : The circuit can be redrawn as shown in the Fig. 1.99 (a).

(Fig. 1.99 (a, b) see on next page)

$$\therefore \quad R_{eq} = \frac{30(10 + R)}{30 + 10 + R} = \frac{300 + 30R}{40 + R}$$

$$\therefore \quad I = \frac{V}{R_{eq}} \quad \text{i.e.} \quad 6 = \frac{50}{\left[\frac{300 + 30R}{40 + R}\right]}$$

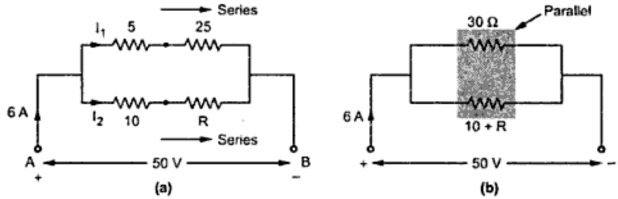


Fig. 1.99

$$\therefore \quad 6(300 + 30R) = 50(40 + R) \quad \text{i.e.} \quad 1800 + 180R = 2000 + 50R$$

$$\therefore \quad R = \mathbf{1.5384\ \Omega}$$

By current division rule,

$$I_1 = I_T \times \frac{(10 + R)}{(10 + R + 30)} = \frac{6 \times 11.5384}{41.5384} = \mathbf{1.667\ A}$$

$$I_2 = I_T \times \frac{30}{(10 + R + 30)} = \frac{6 \times 30}{41.5384} = \mathbf{4.333\ A}$$

Key Point: *Cross check* $I_1 + I_2 = 6$ A

Example 1.28 : *Find the equivalent resistance between points A and B in the network shown in the Fig. 1.100.*

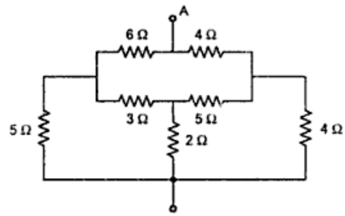


Fig. 1.100

[JNTU : Nov.-2008 (Set-1)]

Solution : Convert the delta of 2 Ω, 3 Ω and 5 Ω to equivalent star as shown in the Fig.1.101(a).

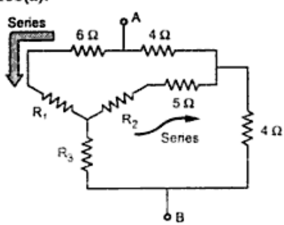


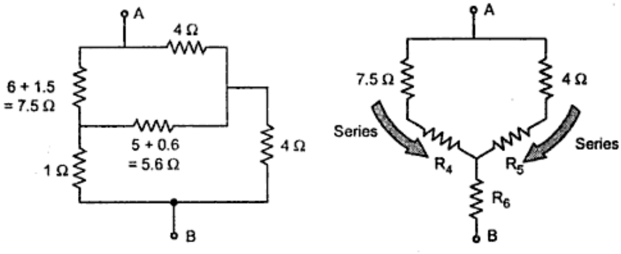
Fig. 1.101 (a)

$$R_1 = \frac{5 \times 3}{5 + 3 + 2} = 1.5\ \Omega$$

$$R_2 = \frac{3 \times 2}{5 + 3 + 2} = 0.6\ \Omega$$

$$R_3 = \frac{5 \times 2}{5 + 3 + 2} = 1\ \Omega$$

Convert the delta of 1 Ω, 5.6 Ω and 4 Ω to equivalent star as shown in the Fig. 101 (c).

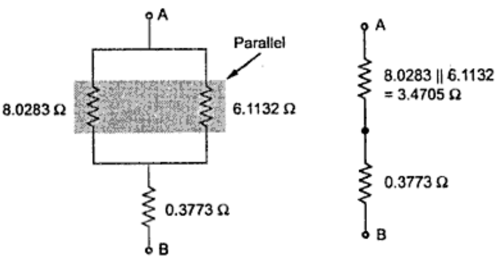


(b) (c)

Fig. 1.101

$$R_4 = \frac{1 \times 5.6}{1 + 5.6 + 4} = 0.5283\ \Omega, \quad R_5 = \frac{5.6 \times 4}{1 + 5.6 + 4} = 2.1132\ \Omega,$$

$$R_6 = \frac{1 \times 4}{1 + 5.6 + 4} = 0.3773\ \Omega$$



(d) (e)

$$\therefore\ R_{AB} = 3.4705 + 0.3773$$

$$= 3.8478\ \Omega$$

Fig. 1.101

Example 1.29. : *Find R_{ab} across the terminals a-b of the network shown in Fig. 1.102.*
[JNTU : Nov.-2008 (Set-2)]

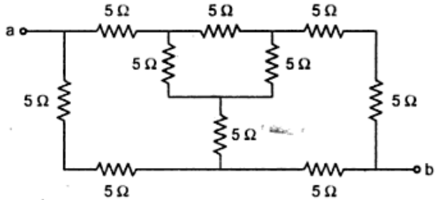


Fig. 1.102

Solution : Convert the inner delta of 5 Ω to equivalent star. As all the resistances of delta are same, all the resistances of equivalent star will be equal of value $R = \frac{5 \times 5}{5+5+5} = 1.667\ \Omega$

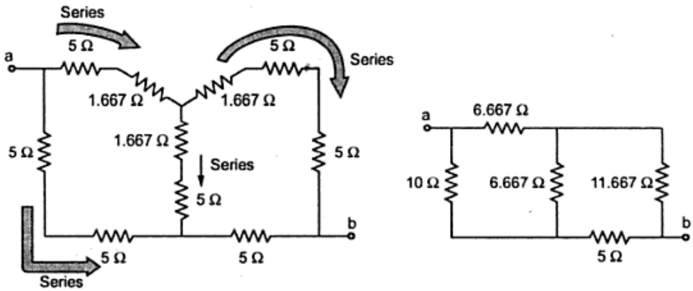


Fig. 1.103

Convert the delta of 6.667 Ω, 5 Ω and 11.667 Ω to equivalent star.

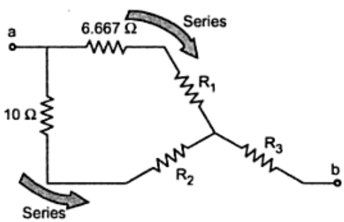


Fig. 1.103 (c)

$$R_1 = \frac{6.667 \times 11.667}{6.667 + 11.667 + 5} = 3.3333\ \Omega$$

$$R_2 = \frac{6.667 \times 5}{6.667 + 11.667 + 5} = 1.4286\ \Omega$$

$$R_3 = \frac{5 \times 11.667}{6.667 + 11.667 + 5} = 2.612\ \Omega$$

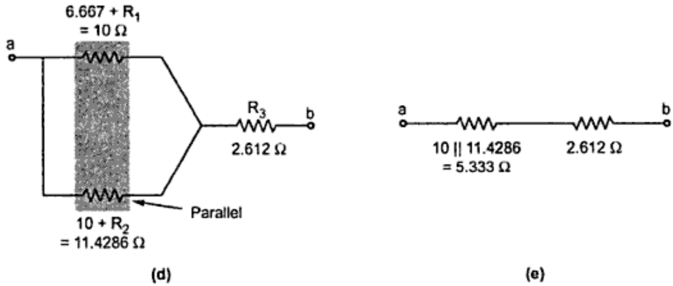


Fig. 1.103

$$\therefore \quad R_{ab} = 5.333 + 2.612 = 7.9452 \ \Omega$$

Example 1.30 : *A voltage waveform is shown in the Fig. 1.104 is applied to a pure resistor of* 40 Ω. *Sketch the waveform of the current passing through the resistance.*

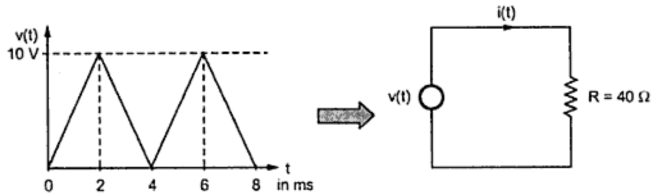


Fig. 1.104

[JNTU : Aug.-2008 (Set-1)]

Solution : Let us divide the voltage waveform into two sections.

For $0 \le t \le 2$, $v(t) = mt$ where $m = \frac{10-0}{2-0} = 5$

$$\therefore \quad i(t) = \frac{v(t)}{R} = \frac{5t}{40} = 0.125t \text{ A}$$

At $t = 2$, $v(t) = 10$ V, $i(t) = 0.125 \times 2 = 0.25$ A

For $2 < t \le 4$, $v(t) = mt + c$ where $m = \frac{0-10}{4-2} = -5$

$$\therefore \quad v(t) = -5t + C$$

Now at $t = 4$, $v(t) = 0$ i.e. $0 = -5 \times 4 + C$ i.e. $C = 20$

$$\therefore \qquad v(t) = -5t + 20$$

$$\therefore \qquad i(t) = \frac{v(t)}{R} = \frac{-5t + 20}{40} = -0.125\,t + 0.5$$

At t = 4, $v(t) = -5 \times 4 + 20 = 0$ V, $i(t) = -0.125 \times 4 + 0.5 = 0$ A

Hence the waveform of the current passing through the resistance is as shown in the Fig. 1.105.

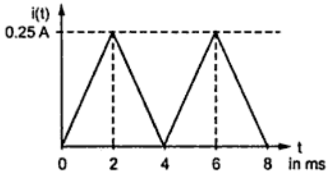


Fig. 1.105

Example 1.31 : *Obtain the currents in the various resistors of the network shown in the Fig. 1.106.*

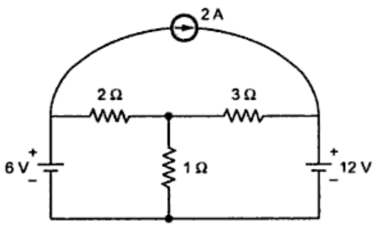


Fig. 1.106

[JNTU : Aug.-2008 (Set-1)]

Solution : Method I : Kirchhoff's laws

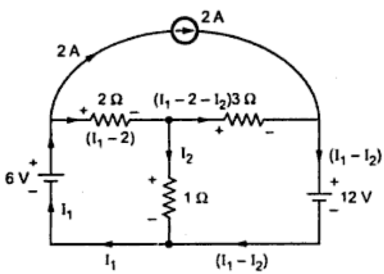


Fig. 1.107

Now apply KVL to the two loops without current source as effect of the current source is already considered while obtaining currents in various branches.

$$-2(I_1 - 2) - I_2 + 6 = 0 \quad \text{i.e.} \quad +2I_1 + I_2 = 10 \quad \dots (1)$$

$$-3(I_1 - 2 - I_2) - 12 + I_2 = 0 \quad \text{i.e.} \quad -3I_1 + 4I_2 = 6 \quad \dots (2)$$

Solving, $I_1 = 3.0909$ A, $I_2 = 3.8181$ A,

Currents through various resistances are,

$I_{2\Omega} = I_1 - 2 =$ **1.0909 A**, $I_{1\Omega} = I_2 =$ **3.8181 A**, $I_{3\Omega} = I_1 - 2 - I_2 =$ **− 2.7272 A.**

Current through 3 Ω is negative i.e. it is flowing in opposite direction to that assumed in the circuit.

Method II : Loop analysis

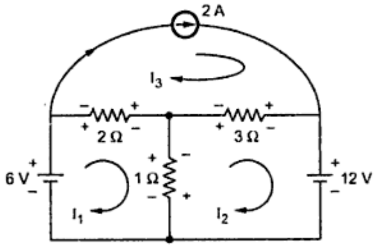


Fig. 1.108

From the current source branch,

$$I_3 = \mathbf{2\ A}$$

Applying KVL to other two loops **without current source,**

$$-2I_1 + 2I_3 - I_1 + I_2 + 6 = 0$$

i.e. $$-3I_1 + I_2 = -10 \quad \dots (1)$$

$$-3I_2 + 3I_3 - 12 - I_2 + I_1 = 0$$

i.e. $$I_1 - 4I_2 = 6 \quad \dots (2)$$

Solving, $I_1 = 3.0909$ A, $I_2 = -0.7272$ A,

Currents through various resistances are,

$I_{2\Omega} = I_1 - I_3 =$ **1.0909 A**, $I_{1\Omega} = I_1 - I_2 = 3.0909 - (-0.7272) =$ **3.8181 A**

$I_{3\Omega} = I_2 - I_3 = -0.7272 - 2 =$ **− 2.7272 A**

The currents are same as obtained by the method I.

Example 1.32 : *Calculate the current through the resistance of 5 Ω in the specified direction as shown in the Fig. 1.109.* **[JNTU : Aug.-2008 (Set-2)]**

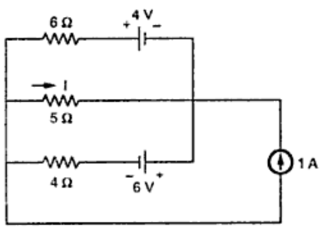


Fig. 1.109

Solution : Use the loop analysis

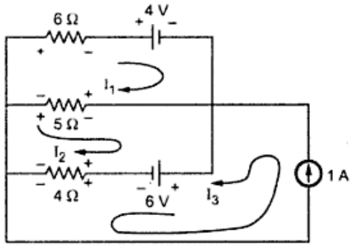


Fig. 1.110

From the current source branch,

$$I_3 = \mathbf{1\ A}$$

Applying KVL to the loops **without current source** we get,

$$-6I_1 - 4 - 5I_1 + 5I_2 = 0$$

i.e. $-11I_1 + 5I_2 = 4$ ... (1)

$$-5I_2 + 5I_1 - 6 - 4I_2 - 4I_3 = 0$$

i.e $5I_1 - 9I_2 = 10$ ... (2)

Solving, $I_1 = -1.1621$ A,

$I_2 = -1.7567$ A

Current through 5 Ω in specified direction is,

$$I_{5\Omega} = I_2 - I_1 = -1.7567 - (-1.1621) = -\ \mathbf{0.5946\ A}$$

As negative, current through 5 Ω flows in opposite direction to that specified in the circuit.

Review Questions

1. *What is charge ? What is the unit of measurement of charge ?*
2. *Explain the relation between charge and current.*
3. *What is the difference between e.m.f. and potential difference ?*
4. *Define the following terms :*
   i) Network ii) Network element iii) Branch iv) Mesh or loop v) Node
5. *Explain the classification of electrical networks.*
6. *Define : i) Passive and active networks ii) Ideal and practical voltage sources iii) Ideal and practical current sources.*
7. *Differentiate between :*
   i) Time invariant and time variant sources ii) Dependent and independent sources iii) Unilateral and bilateral networks
8. *State and explain Kirchhoff's Laws*
9. *Explain the voltage division in series circuit of two impedances.*

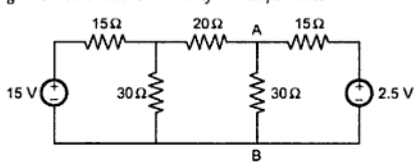


Fig. 1.111

10. *Explain the current division in parallel circuit of two impedances.*

11. *In the following circuit, determine*

    i) I_1, I_2 *and* I_3,

    ii) Value of R

    iii) Value of E_X

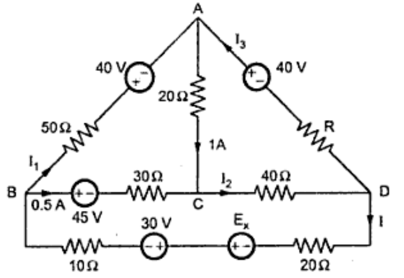


Fig. 1.112

(Ans. : I_1 = 0.3 A, I_2 = 1.5 A, E_x = 174 V, I_3 = 0.7 A, R = 71.4 Ω)

12. *A resistance R is connected in series with a parallel circuit comprising two resistances of 12 and 8 Ω. The total power dissipated in the circuit is 700 watts when the applied voltage is 200 V. Calculate the value of R.* **(Ans. : 52.3428 Ω)**

13. *In the series parallel circuit shown in the Fig. 1.113, find the*

    i) voltage drop across the 4 Ω resistance *ii) the supply voltage V*

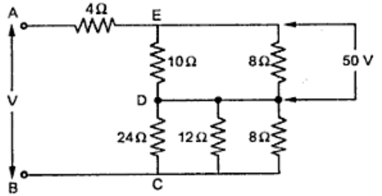


Fig. 1.113

(Ans. : 45 V, 140 V)

14. *Find the current in all the branches of the network shown in the Fig. 1.114.*

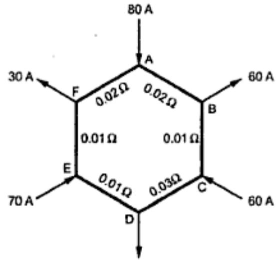


Fig. 1.114

(Ans. : 39 A, 21 A, 39 A, 81 A, 11 A, 41 A)

15. *If the total power dissipated in the circuit shown in the Fig. 1.115 is 18 watts, find the value of R and current through it.* **(Ans. : 12 Ω, 0.6 A)**

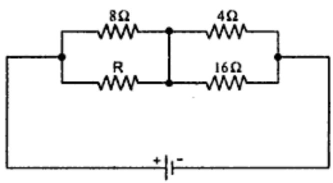


Fig. 1.115

16. *Explain with a suitable example how to obtain :*

    i) An equivalent current source from a given voltage source.

    ii) An equivalent voltage source from a given current source.

17. *Find the equivalent resistance across the terminals X-Y.*

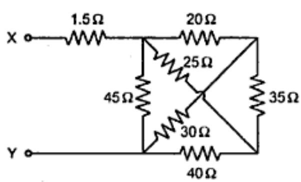


Fig. 1.116

18. *Find the equivalent resistance as viewed through the terminals,*
    i) B and C ii) A and N

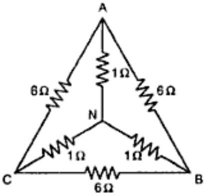


Fig. 1.117

(Ans. : i) 1.33 Ω ii) 0.77 Ω)

19. *Derive the relationship to express three star connected resistances into equivalent delta.*
20. *Derive the relationship to express three delta connected resistances into equivalent star.*
21. *Write notes on : i) Supernode ii) Supermesh iii) Combination of Sources*
22. *Explain the loop analysis of analysing a given network, with a suitable example.*
23. *Explain the nodal analysis of analysing a given network, with a suitable example.*
24. *Calculate the voltage across branch AB in circuit shown, using loop analysis.*

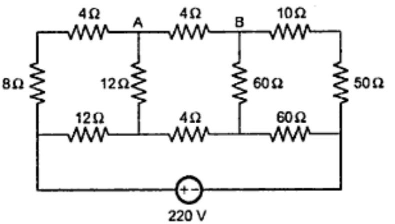


Fig. 1.118

25. *In the following circuit, determine : i) I_1, I_2 and I_3, ii) Value of R and iii) Value of E_X*

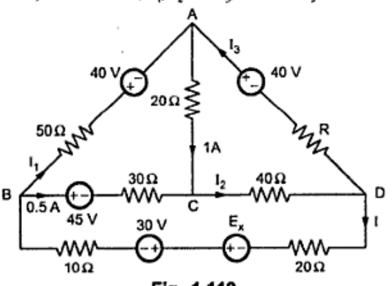


Fig. 1.119

(Ans. : I_1 = 0.3 A, I_2 = 1.5 A, E_x = 174 V, I_3 = 0.7 A, R = 71.4 Ω)

26. *Find potential between A and B by Nodal method.* **(Ans. : 0.375 V, B +ve)**

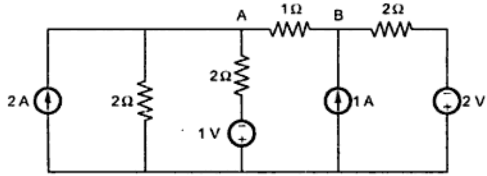


Fig. 1.120

27. *Show that current through purely resistive circuit is in phase with the applied voltage.*

28. *Show that current through pure inductance lags applied voltage by 90°.*

29. *Show that current through pure capacitor leads applied voltage by 90°.*

❑❑❑

2 D.C. Machines

2.1 Introduction

The study of the electrical engineering, basically involves the analysis of the energy transfer from one form to another. An electrical machine, deals with the energy transfer either from mechanical to electrical form or from electrical to mechanical form. This process is called **electromechanical energy conversion.**

An electrical machine which converts mechanical energy into an electrical energy is called an **electric generator.** While an electrical machine which converts an electrical energy into the mechanical energy is called an **electrical motor.**

Such electrical machines may be related to an electrical energy of an alternating type called **a.c. machines** or may be related to an electrical energy of direct type called **d.c. machines.**

The d.c. machines are classified as d.c. generators and d.c. motors. The construction of a d.c. machine basically remains same whether it is a generator or a motor. But before beginning the study of the d.c. machines, it is necessary to revise the basic concepts of magnetism and electromagnetism.

2.2 Revision of Magnetism

Magnetism is a property by virtue of which a piece of solid body attracts iron pieces and pieces of some other metals. Such a piece of solid body is called a **natural magnet.** The two ends of a magnet are called its **poles.** When such a magnet is suspended freely by a piece of a silk fiber, it turns and adjusts itself in the direction of North and South of the earth. The end adjusting itself in the direction of North is called **N pole** while other is called **S pole.** When such two magnets are brought near each other, their behaviour is governed by some laws called **laws of magnetism.**

2.2.1 Laws of Magnetism

Law 1 : It states that 'like' magnetic poles repel and 'unlike' poles attract each other.

When the two magnets are brought near each other, such that two like poles i.e. N and N or S and S are facing each other, then the two magnets experience a force of repulsion. As against this, if two unlike poles i.e. N and S or S and N are facing towards each other, then they experience a force of attraction and try to attract each other.

Law 2 : This law is experimentally proved by Scientist Coulomb and hence also known as **Coulomb's Law.**

The force (F) exerted by one pole on the other pole is,

1. Directly proportional to the product of the pole strengths.
2. Inversely proportional to the square of the distance between them and
3. Dependent on the nature of medium surrounding the poles.

Mathematically this law can be expressed as,

$$F \propto \frac{M_1 \, M_2}{d^2}$$

where M_1 and M_2 are the pole strengths of the poles while 'd' is the distance between the two poles.

$$\boxed{F = \frac{K \, M_1 \, M_2}{d^2}}$$

where K is constant which depends on the nature of the surrounding.

2.2.2 Magnetic Field and Flux

The region around a magnet within which the influence of the magnet can be experienced is called its **magnetic field.** The presence of magnetic field is represented by imaginary lines around a magnet. These are called magnetic lines of force.

The total number of lines of force existing in a particular magnetic field is called **magnetic flux,** denoted by a symbol 'ϕ'. It is measured in a unit weber.

$$\boxed{1 \text{ weber} = 10^8 \text{ lines of force.}}$$

> **Key Point** : *The lines of force never intersect each other and are like stretched rubber bands and always try to contract in length.*

These properties of lines of force play an important role in the understanding of the working principle of the d.c. machines.

The lines of flux have a fixed direction. These flux lines start at N-pole and terminate at S-pole, **external** to the magnet. While the direction of flux lines is from S-pole to N-pole, **internal** to the magnet. The distribution and direction of such flux lines for a bar magnet is shown in the Fig. 2.1.

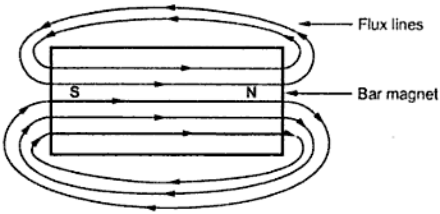


Fig. 2.1 Direction of the flux lines

These lines always form a closed loop and never intersect each other.

2.3 Revision of Electromagnetism

When a conductor carries a current, it creates a magnetic field around it. The direction of such magnetic field depends on the direction of the current passing through the conductor. So electric current and magnetism are very closely related to each other. This relationship plays an important role in the d.c. machines.

Let us see in brief, the rule to determine the direction of the flux produced by a current carrying conductor.

2.3.1 Right Hand Thumb Rule

It states that "Hold the current carrying conductor in the right hand such that the thumb is pointing in the direction of current and parallel to the conductor, then curled fingers point in the direction of the magnetic field or flux around it".

The Fig. 2.2 explains the rule.

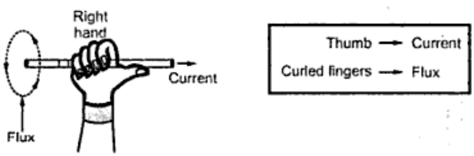
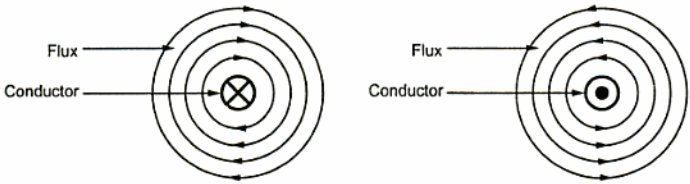


Fig. 2.2 Right hand thumb rule

Conventionally, such conductors are observed, assuming them to be placed perpendicular to the plane of the paper. So current moving away from the observer is denoted by a 'cross' while current coming towards the observer is denoted by a 'dot'. If now right hand is adjusted in such a way, that the thumb is pointing in the direction of current denoted as 'cross' i.e. going into the paper, then curled fingers indicate the direction of flux as clockwise, as shown in the Fig. 2.3 (a). While if thumb of right hand is



(a) Current moving away from observer (b) Current moving towards observer

Fig. 2.3

adjusted in the direction of current shown as 'dot' i.e. coming out of paper, then curled fingers indicate the direction of flux as anticlockwise as shown in the Fig. 2.3 (b).

2.3.2 Magnetic Field due to Circular Conductor

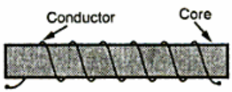


Fig. 2.4 Solenoid

Consider an arrangement in which a long conductor is wound with number of turns on a core, close together to form a coil. This is called a **solenoid** as shown in the Fig. 2.4. When such a conductor carries a current, the magnetic field gets produced around the core.

Identifying the direction of flux and hence identifying the two ends of the core as N pole or S pole is important in understanding the principle of d.c. machine. The right hand thumb rule can be modified for such case as stated below,

The right hand thumb rule : Hold the solenoid in the right hand such that curled fingers point in the direction of the current through the curled conductor, then the outstretched thumb along the axis of the solenoid points to the North pole of the solenoid or points in the direction of flux lines **inside** the core.

This is represented in the Fig. 2.5.

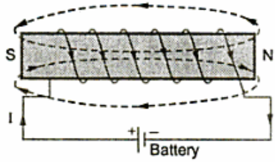


Fig. 2.5 Direction of flux around circular conductor

Key Point : *The direction of flux can be reversed either by changing direction of current through the conductor by reversing the polarities of the supply or by changing the direction of winding of the conductors around the core.*

With this background, let us start the detail study of a d.c. generator.

2.4 Principle of Operation of a D.C. Generator

All the generators work on a principle of dynamically induced e.m.f. This principle is nothing but the Faraday's law of electromagnetic induction . It states that, 'Whenever the number of magnetic lines of force i.e. flux linking with a conductor or a coil changes, an electromotive force is set up in that conductor or coil.' The change in flux associated with the conductor can exist only when there exists a relative motion between a conductor and the flux. The relative motion can be achieved by rotating conductor with respect to flux or by rotating flux with respect to a conductor. So a voltage gets generated in a conductor, as long as there exists a relative motion between conductor and the flux.

Such an induced e.m.f. which is due to physical movement of coil or conductor with respect to flux or movement of flux with respect to coil or conductor is called **dynamically induced e.m.f.**

> **Key Point** : *So a generating action requires following basic components to exist, i) The conductor or a coil ii) The flux iii) The relative motion between conductor and flux.*

In a practical generator, the conductors are rotated to cut the magnetic flux, keeping flux stationary. To have a large voltage as the output, the number of conductors are connected together in a specific manner, to form a winding. This winding is called **armature winding** of a d.c. machine. The part on which this winding is kept is called **armature** of a d.c. machine. To have the rotation of conductors, the conductors placed on the armature are rotated with the help of some external device. Such an external device is called a **prime mover.** The commonly used prime movers are diesel engines, steam engines, steam turbines, water turbines etc. The necessary magnetic flux is produced by current carrying winding which is called **field winding.** The direction of the induced e.m.f. can be obtained by using Fleming's right hand rule.

2.5 Fleming's Right Hand Rule

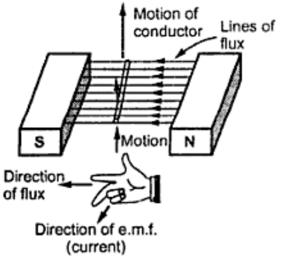
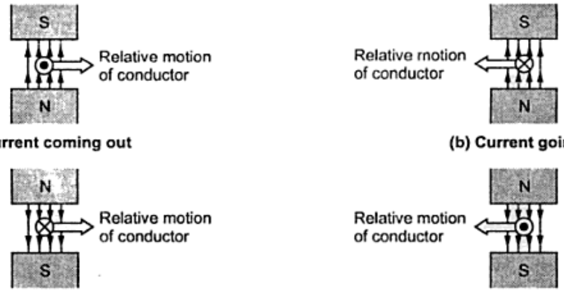


Fig. 2.6 Fleming's right hand rule

If three fingers of a right hand, namely thumb, index finger and middle finger are outstretched so that everyone of them is at right angles with the remaining two, and if in this position index finger is made to point in the direction of lines of flux, thumb in the direction of the relative motion of the conductor with respect to flux then the outstretched middle finger gives the direction of the e.m.f. induced in the conductor. Visually the rule can be represented as shown in the Fig. 2.6.

This rule mainly gives direction of current which induced e.m.f. in conductor will set up when closed path is provided to it.

Verify the direction of the current through conductor in the four cases shown in the Fig. 2.7 by using Fleming's right hand rule.



(a) Current coming out **(b) Current going in**

(c) Current going in **(d) Current coming out**

Fig. 2.7 Fleming's right hand rule

Key Point : *It can be observed from the Fig. 2.7 that if the direction of relative motion of conductor is reversed keeping flux direction same or if flux direction is reversed keeping direction of relative motion of conductor same then the direction of induced e.m.f. and hence direction of current it sets up in an external circuit gets reversed.*

The magnitude of the induced e.m.f. is given by,

$$\boxed{E = B \times l \times v}$$

where l = Active length of conductor in m.

v = Relative velocity component of conductor in m/s in the direction perpendicular to direction of the flux.

The active length means the length of conductor which is under the influence of magnetic field. In all the cases above, direction of motion of conductor is perpendicular to the plane of the flux.

But if it is not perpendicular then the component of velocity which is perpendicular to the plane of the flux, is only responsible for inducing e.m.f. in the conductor. This is shown in the Fig. 2.8 (a). In this Fig. 2.8 (a), though the velocity is v, its component v′ which is perpendicular to the flux lines is only responsible for the induced e.m.f.

If the plane of the rotation of conductor is parallel to the plane of the flux, there will not be any cutting of flux and hence there cannot be any induced e.m.f. in the conductor. This is shown in the Fig. 2.8 (b).

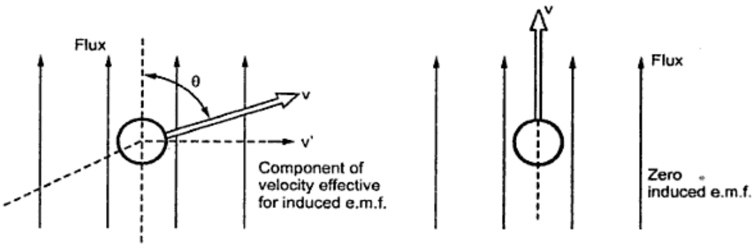


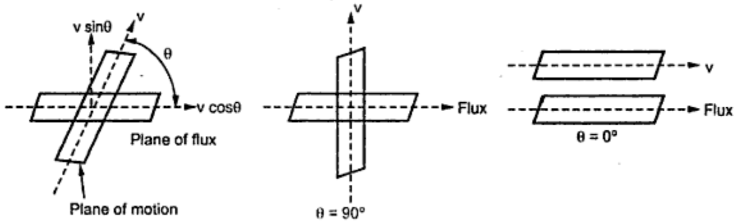
Fig. 2.8

Key Point : *So to have an induced e.m.f. in the conductor not only the relative motion between the conductor and the flux is necessary but plane of rotation and plane of flux should not be parallel to each other.*

If angle between the plane of rotation and the plane of the flux is 'θ' as measured from the axis of the plane of flux then the induced e.m.f. is given by,

$$\boxed{E = B\,l\,(v \sin\theta) \text{ volts}}$$

Where v sinθ is the component of velocity which is perpendicular to the plane of flux and hence responsible for the induced e.m.f. This is shown in the Fig. 2.9.



(a) E ∝ v sinθ (b) Maximum E (c) E = 0

Fig. 2.9

From the equation of the induced e.m.f., it can be seen that the basic nature of the induced e.m.f. in a d.c. generator is purely sinusoidal i.e. alternating. To have d.c. voltage, a device is used in a d.c. generator to convert the alternating e.m.f. to unidirectional e.m.f. This device is called commutator. An alternator is a machine which produces an alternating e.m.f. is without a commutator. So an alternator with a commutator is the basic d.c. generator. Practically there is a difference between the construction of an alternator and a d.c. generator though the basic principle of working is same.

2.6 Construction of a Practical D.C. Machine

As stated earlier, whether a machine is **d.c. generator or a motor** the construction basically remains the same as shown in the Fig. 2.10.

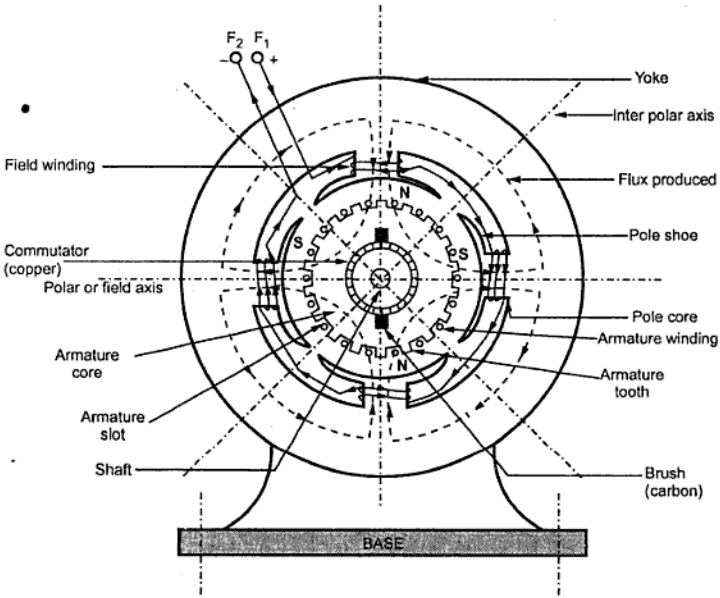


Fig. 2.10 A cross-section of typical d.c. machine

It consists of the following parts :

2.6.1 Yoke

a) Functions :

1. It serves the purpose of outermost cover of the d.c. machine. So that the insulating materials get protected from harmful atmospheric elements like moisture, dust and various gases like SO_2, acidic fumes etc.
2. It provides mechanical support to the poles.
3. It forms a part of the magnetic circuit. It provides a path of low reluctance for magnetic flux. The low reluctance path is important to avoid wastage of power to provide same flux. Large current and hence the power is necessary if the path has high reluctance, to produce the same flux.

b) Choice of material : To provide low reluctance path, it must be made up of some magnetic material. It is prepared by using cast iron because it is cheapest. For large machines rolled steel, cast steel, silicon steel is used which provides high permeability i.e. low reluctance and gives good mechanical strength.

2.6.2 Poles

Each pole is divided into two parts namely, I) Pole core and II) Pole shoe.

This is shown in the Fig. 2.11.

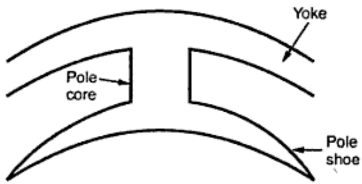


Fig. 2.11 Pole structure

a) Functions of pole core and pole shoe :

1. Pole core basically carries a field winding which is necessary to produce the flux.
2. It directs the flux produced through air gap to armature core, to the next pole.
3. Pole shoe enlarges the area of armature core to come across the flux, which is necessary to produce larger induced e.m.f. To achieve this, pole shoe has been given a particular shape.

b) Choice of material : It is made up of magnetic material like cast iron or cast steel.

As it requires a definite shape and size, laminated construction is used. The laminations of required size and shape are stamped together to get a pole which is then bolted to the yoke.

2.6.3 Field Winding (F1 - F2)

The field winding is wound on the pole core with a definite direction.

a) Functions : To carry current due to which pole core, on which the field winding is placed behaves as an electromagnet, producing necessary flux.

As it helps in producing the magnetic field i.e. exciting the pole as an electromagnet it is called **Field winding or Exciting winding.**

b) Choice of material : It has to carry current hence obviously made up of some conducting material. So aluminium or copper is the choice. But field coils are required to take any type of shape and bend about pole core and copper has good pliability i.e. it can bend easily. So copper is the proper choice.

Key Point : *Field winding is divided into various coils called field coils. These are connected in series with each other and wound in such a direction around pole cores, such that alternate 'N' and 'S' poles are formed.*

By using right hand thumb rule for current carrying circular conductor, it can be easily determined that how a particular core is going to behave as 'N' or 'S' for a particular winding direction around it. The direction of winding and flux can be observed in the Fig. 1.10.

2.6.4 Armature

It is further divided into two parts namely,

I) Armature core and II) Armature winding

I) Armature core : Armature core is cylindrical in shape mounted on the shaft. It consists of slots on its periphery and the air ducts to permit the air flow through armature which serves cooling purpose.

a) Functions :

1. Armature core provides house for armature winding i.e. armature conductors.
2. To provide a path of low reluctance to the magnetic flux produced by the field winding.

b) Choice of material : As it has to provide a low reluctance path to the flux, it is made up of magnetic material like cast iron or cast steel.

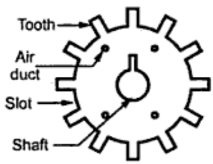


Fig. 2.12 Single circular lamination of armature core

It is made up of laminated construction to keep eddy current loss as low as possible. A single circular lamination used for the construction of the armature core is shown in the Fig. 2.12.

II) Armature winding : Armature winding is nothing but the interconnection of the armature conductors, placed in the slots provided on the armature core periphery. When the armature is rotated, in case of generator, magnetic flux gets cut by armature conductors and e.m.f. gets induced in them.

a) Functions :

1. Generation of e.m.f. takes place in the armature winding in case of generators.
2. To carry the current supplied in case of d.c. motors.
3. To do the useful work in the external circuit.

b) Choice of material : As armature winding carries entire current which depends on external load, it has to be made up of conducting material, which is copper.

Armature winding is generally former wound. The conductors are placed in the armature slots which are lined with tough insulating material.

2.6.5 Commutator

We have seen earlier that the basic nature of e.m.f. induced in the armature conductors is alternating. This needs rectification in case of d.c. generator, which is possible by a device called commutator.

a) Functions :

1. To facilitate the collection of current from the armature conductors.
2. To convert internally developed alternating e.m.f. to unidirectional (d.c.) e.m.f.
3. To produce unidirectional torque in case of motors.

b) Choice of material : As it collects current from armature, it is also made up of copper segments.

It is cylindrical in shape and is made up of wedge shaped segments of hard drawn, high conductivity copper. These segments are insulated from each other by thin layer of mica. Each commutator segment is connected to the armature conductor by means of copper lug or strip. This connection is shown in the Fig. 2.13.

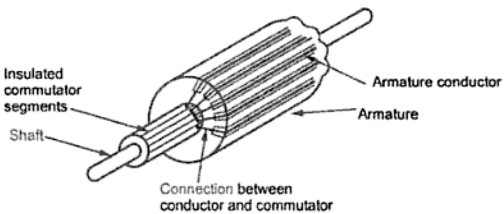


Fig. 2.13 Commutator

2.6.6 Brushes and Brush Gear

Brushes are stationary and resting on the surface of the commutator.

a) Function : To collect current from commutator and make it available to the stationary external circuit.

b) Choice of material : Brushes are normally made up of soft material like carbon.

Brushes are rectangular in shape. They are housed in brush holders, which are usually of box type. The brushes are made to press on the commutator surface by means of a spring, whose tension can be adjusted with the help of lever. A flexible copper conductor called **pig tail** is used to connect the brush to the external circuit. To avoid wear and tear of commutator, the brushes are made up of soft material like carbon.

2.6.7 Bearings

Ball-bearings are usually used as they are more reliable. For heavy duty machines, roller bearings are preferred.

2.7 Types of Armature Winding

We have seen that there are number of armature conductors, which are connected in specific manner as per the requirement, which is called **armature winding.** According to the way of connecting the conductors, armature winding has basically two types namely,

a) Lap winding b) Wave winding

2.7.1 Lap Winding

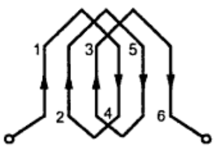


Fig. 2.14 Lap winding

In this case, if connection is started from conductor in slot 1 then connections overlap each other as winding proceeds, till starting point is reached again.

Developed view of part of the armature winding in lap fashion is shown in the Fig. 2.14.

As seen from the Fig. 2.14, there is overlapping of coils while proceeding.

Key Point : *Due to such connection, the* ***total number of conductors get divided into 'P' number of parallel paths, where P = number of poles in the machine.***

Large number of parallel paths indicate high current capacity of machine hence lap winding is preferred for high current rating generators.

2.7.2 Wave Winding

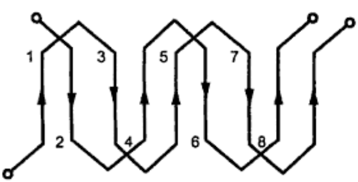


Fig. 2.15 Wave winding

In this type of connection, winding always travels ahead avoiding overlapping. It travels like a progressive wave hence called wave winding. To get an idea of wave winding a part of armature winding in wave fashion is shown in the Fig. 2.15.

Both coils starting from slot 1 and slot 2 are progressing in wave fashion.

Key Point : *Due to this type of connection, the* ***total number of conductors get divided into two number of parallel paths always, irrespective of number of poles of the machine.*** *As number of parallel paths are less, it is preferrable for low current, high voltage capacity generators.*

The number of parallel paths in which armature conductors are divided due to lap or wave fashion of connection is denoted as A. So **A = P** for **lap** connection and **A = 2** for **wave** connection.

2.7.3 Comparison of Lap and Wave Type Winding

Sr. No	Lap winding	Wave winding
1.	Number of parallel paths (A) = poles (P)	Number of parallel paths (A) = 2 (always)
2.	Number of brush sets required is equal to number of poles.	Number of brush sets required is always equal to two.
3.	Preferable for high current, low voltage capacity generators.	Preferable for high voltage, low current capacity generators.
4.	Normally used for generators of capacity more than 500 A.	Preferred for generators of capacity less than 500 A.

2.8 E.M.F. Equation of D.C. Generator

Let P = Number of poles of the generator

ϕ = Flux produced by each pole in webers (Wb)

N = Speed of armature in r.p.m.

Z = Total number of armature conductors

A = Number of parallel paths in which the 'Z' number of conductors are divided

So $A = P$ for lap type of winding

$A = 2$ for wave type of winding

Now e.m.f. gets induced in the conductor according to Faraday's law of electromagnetic induction. Hence average value of e.m.f. induced in each armature conductor is,

$$e = \text{Rate of cutting the flux} = \frac{d\phi}{dt}$$

Now consider one revolution of conductor. In one revolution, conductor will cut total flux produced by all the poles i.e. $\phi \times P$. While time required to complete one revolution is $\frac{60}{N}$ seconds as speed is N r.p.m.

$$\therefore \qquad e = \frac{\phi P}{\frac{60}{N}} = \phi P \frac{N}{60}$$

This is the e.m.f. induced in one conductor. Now the conductors in one parallel path are always in series. There are total Z conductors with A parallel paths, hence $\frac{Z}{A}$ number of conductors are always in series and e.m.f. remains same across all the parallel paths.

$\therefore$ Total e.m.f. can be expressed as,

$$E = \phi P \frac{N}{60} \times \frac{Z}{A} \quad \text{volts}$$

This is nothing but the e.m.f. equation of a d.c. generator.

So $$\boxed{E = \frac{\phi PNZ}{60\,A} \quad \text{e.m.f. equation}}$$

$$E = \frac{\phi NZ}{60} \qquad \text{for lap type as } A = P$$

$$E = \frac{\phi PNZ}{120} \qquad \text{for wave type as } A = 2$$

Example 2.1 : *A 4 pole, lap wound, d.c. generator has a useful flux of 0.07 Wb per pole. Calculate the generated e.m.f. when it is rotated at a speed of 900 r.p.m. with the help of prime mover. Armature consists of 440 number of conductors. Also calculate the generated e.m.f. if lap wound armature is replaced by wave wound armature.*

Solution : P = 4 Z = 440 ϕ = 0.07 Wb and N = 900 r.p.m.

$$E = \frac{\phi PNZ}{60A}$$

i) For lap wound, $A = P = 4$

$$\therefore \qquad E = \frac{\phi NZ}{60} = \frac{0.07 \times 900 \times 440}{60} = \mathbf{462\ V}$$

ii) For wave wound $A = 2$

$$\therefore \qquad E = \frac{\phi PNZ}{120} = \frac{0.07 \times 900 \times 4 \times 440}{120} = \mathbf{924\ V}$$

2.9 Winding Terminologies

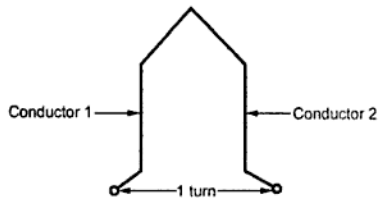


Fig. 2.16 Single turn

a) Conductor : It is the actual armature conductor which is under the influence of the magnetic field, placed in the armature slot.

b) Turn : The two conductors placed in different slots when connected together, forms a turn. While describing armature winding the number of turns may be specified from which, the number of conductors can be decided.

$$Z = 2 \times \text{Number of turns.}$$

c) Coil : For simplicity of connections, the turns are grouped together to form a coil. If coil contains only one turn it is called single turn coil while coil more than one turn is called multiturn coil.

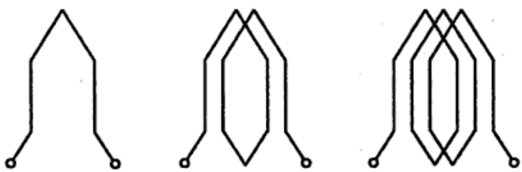


Fig. 2.17 Armature coils

Hence if number of coils, along with number of turns per coil are specified, it is possible to determine the total number of turns and hence total number of armature conductors 'Z' required to calculate generated e.m.f.

Example 2.2 : *A 4 pole, lap wound, d.c. generators has 42 coils with 8 turns per coils. It is driven at 1120 r.p.m. If useful flux per pole is 21 mWb, calculate the generated e.m.f. Find the speed at which it is to be driven to generate the same e.m.f. as calculated above, with wave wound armature.*

Solution : P = 4 $\phi = 21 \text{ mWb} = 21 \times 10^{-3} \text{ Wb}$ N = 1120 r.p.m.

$$\text{Coils} = 42 \text{ and turns / coil} = 8$$

$$\text{Total turns} = \text{coils} \times \text{turns / coil} = 42 \times 8 = 336$$

$$Z = 2 \times \text{total turns} = 2 \times 336 = 672$$

i) For lap wound, A = P

$$\therefore \quad E = \frac{\phi N Z}{60} = \frac{21\times10^{-3}\times1120\times672}{60} = \mathbf{263.424\ V}$$

ii) For wave wound, $A = 2$

and $E = 263.424$ V

$$\therefore \quad E = \frac{\phi P N Z}{120}$$

$$\therefore \quad 263.424 = \frac{21\times10^{-3}\times4\times N\times672}{120}$$

$$N = \mathbf{560\ r.p.m.}$$

2.10 Single Layer and Double Layer Winding

Basically there are two physical types of the windings. These are i) Single layer winding ii) Double layer winding. The sequential arrangement of coils around the armature is different for both these types of windings.

2.10.1 Single Layer Winding

In this type of winding, the complete slot is containing only one coil side of a coil. This type of winding is not normally used for machines having commutators. It is shown in the Fig. 2.18.

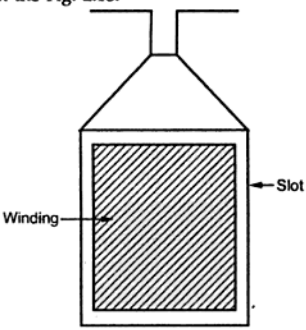


Fig. 2.18 Single layer winding

In single layer windings permit the use of semi enclosed and closed types of slots. Also the coils can be pushed through the slots from one end of the core and are connected during the process of windings at the other end. Here the insulation can be properly applied and consolidated which is advantageous in large output machines with high voltage.

The single layer windings used in high voltage machines use small groups of concentrically placed coils. The interlinking between these coils is in such a way so as to minimize the space taken up outside the slot and in the overhang connections.

2.10.2 Double Layer Winding

It is shown in the Fig. 2.19. It consists of identical coils with one coilside of each coil in top half of the slot and the other coilside in bottom half of another slot which is nearly one pole pitch away.

In the Fig. 2.19 (a) there are two coilsides per slot while in (b) there are eight coilsides per slot. Each layer may contain more than one coil side if large number of coils are required. For placing double layer windings, usually open slots are used.

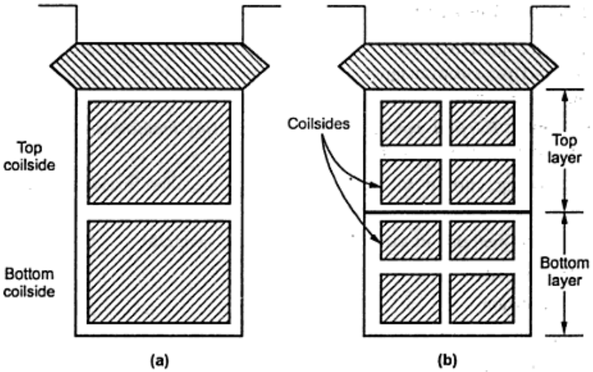


Fig. 2.19 Double layer winding

Advantages of double layer winding

The double layer winding has following advantages,

1) It provides neat arrangement as all coils are identical.
2) Greater flexibility can be achieved with double layer winding as coil span can be easily selected.

2.11 Symbolic Representation of D.C. Generator

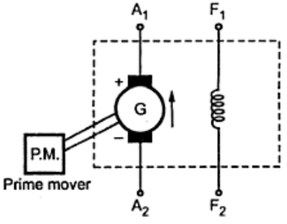


Fig. 2.20 Symbolic representation of D.C. generator

The armature is denoted by a circle with two brushes. Mechanically it is connected to another device called prime mover. The two ends of armature are denoted as A_1 - A_2. The field winding is shown near armature and the two ends are denoted as F_1 - F_2. The representation of field vary little bit, depending on the type of generator.

The symbolic representation is shown in the Fig. 2.20. Many times an arrow (↑) is indicated near armature. This arrow denotes the direction of current which induced e.m.f. will set up, when connected to an external load.

Key Point : *Every practical generator needs a* ***prime mover*** *to rotate its armature. Hence to avoid complexity of the diagram, prime mover need not be included in the symbolic representation of generator.*

2.12 Types of Generators

The magnetic field required for the operation of a d.c. generator is produced by an electromagnet. This electromagnet carries a field winding which produces required magnetic flux when current is passed through it.

Key Point : *The field winding is also called* ***exciting winding*** *and current carried by the field winding is called an* ***exciting current.***

Thus supplying current to the field winding is called excitation and the way of supplying the exciting current is called **method of excitation.**

There are two methods of excitation used for d.c. generators,

1. Separate excitation
2. Self excitation

Depending on the method of excitation used, the d.c. generators are classified as,

1. Separately excited generator
2. Self excited generator

In **separately excited generator,** a separate external d.c. supply is used to provide exciting current through the field winding.

The d.c. generator produces d.c. voltage. If this generated voltage itself is used to excite the field winding of the same d.c. generator, it is called **self excited generator.** The d.c. voltage is produced in the armature winding of a d.c. generator, which is used to excite the field winding of the same generator. Depending on how electrically the armature winding is connected to the field winding, the self excited d.c. generators are classified as,

a) Shunt generators b) Series generators c) Compound generators.

In shunt the two windings, field and armature are in parallel while in series type the two windings are in series. In compound type the part of the field winding is in parallel while other part in series with the armature winding.

The compound generators are further classified as long shunt and short shunt compound generators. The overall classification of d.c. generators is shown in the Fig. 2.21.

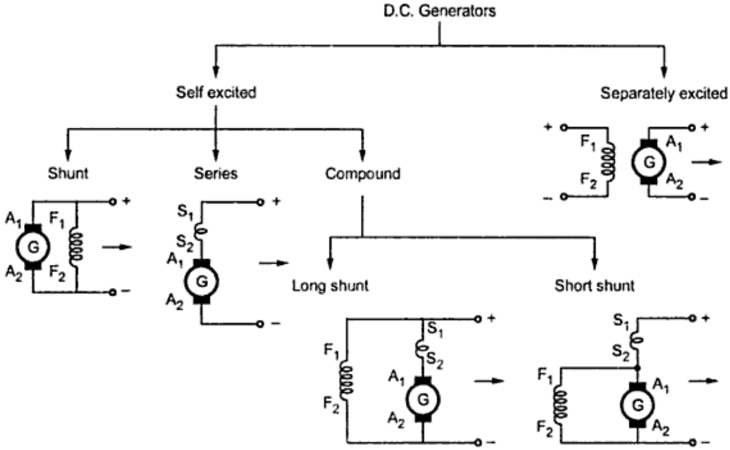


Fig. 2.21 Types of d.c. generators

2.13 Separately Excited Generator

When the field winding is supplied from external, separate d.c. supply i.e. excitation of field winding is separate then the generator is called separately excited generator. Schematic representation of this type is shown in the Fig. 2.22.

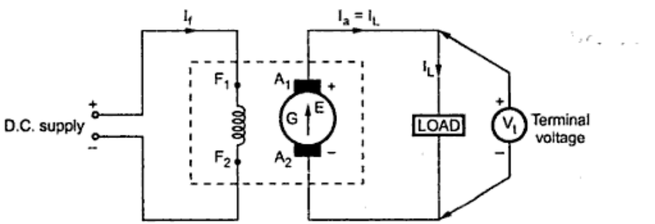


Fig. 2.22 Separately excited generator

The field winding of this type of generator has large number of turns of thin wire. So length of such winding is more with less cross-sectional area. So resistance of this field winding is high in order to limit the field current.

2.13.1 Voltage and Current Relations

The field winding is excited separately, so the field current depends on supply voltage and resistance of the field winding.

For armature side, we can see that it is supplying a load, demanding a load current of I_L at a voltage of V_t which is called **terminal voltage.**

Now $$I_a = I_L$$

The internally induced e.m.f. E is supplying the voltage of the load hence terminal voltage V_t is a part of E. But E is not equal to V_t while supplying a load. This is because when armature current I_a flows through armature winding, due to armature winding resistance R_a ohms, there is a voltage drop across armature winding equal to $I_a R_a$ volts. The induced e.m.f. has to supply this drop, along with the terminal voltage V_t. To keep $I_a R_a$ drop to minimum, the resistance R_a is designed to be very very small. In addition to this drop, there is some voltage drop at the contacts of the brush called brush contact drop. But this drop is negligible and hence generally neglected. So in all, induced e.m.f. E has three components namely,

i) Terminal voltage V_t ii) Armature resistance drop $I_a R_a$ iii) Brush contact drop V_{brush}

So voltage equation for separately excited generator can be written as,

$$\boxed{E = V_t + I_a R_a + V_{brush}}$$

Where $$E = \frac{\phi P N Z}{60 A}$$

Generally V_{brush} is neglected as is negligible compared to other voltages.

2.14 Self Excited Generator

When the field winding is supplied from the armature of the generator itself then it is said to be self excited generator. Now without generated e.m.f., field cannot be excited in such generator and without excitation there cannot be generated e.m.f. So one may obviously wonder, how this type of generator works. The answer to this is residual magnetism possessed by the field poles, under normal condition.

Practically though the generator is not working, without any current through field winding, the field poles possess some magnetic flux. This is called **residual flux** and the property is called **residual magnetism.** Thus when the generator is started, due to such residual flux, it develops a small e.m.f. which now drives a small current through the field winding. This tends to increase the flux produced. This inturn increases the induced e.m.f. This further increases the field current and the flux. The process is cumulative and continues till the generator develops rated voltage across its armature. This is **voltage building process** in self excited generators.

Based on how field winding is connected to the armature to derive its excitation, this type is further divided into following three types :

i) Shunt generator ii) Series generator iii) Compound generator

Let us see the connection diagrams and voltage, current relations for these types of generators.

2.15 Shunt Generator

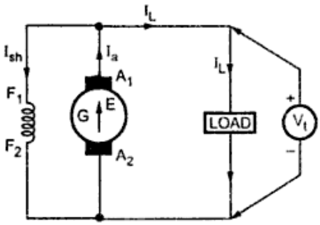


Fig. 2.23 Shunt generator

When the field winding is connected in parallel with the armature and the combination across the load then the generator is called **shunt generator.**

The field winding has large number of turns of thin wire so it has high resistance. Let R_{sh} be the resistance of the field winding.

2.15.1 Voltage and Current Relations

From the Fig. 2.23, we can write

$$\boxed{I_a = I_L + I_{sh}}$$

Now voltage across load is V_t which is same across field winding as both are in parallel with each other.

$$\therefore \qquad \boxed{I_{sh} = \frac{V_t}{R_{sh}}}$$

While induced e.m.f. E, still requires to supply voltage drop $I_a R_a$ and brush contact drop.

$$\therefore \qquad \boxed{E = V_t + I_a R_a + V_{brush}}$$

where $$E = \frac{\phi P N Z}{60 A}$$

In practice, brush contact drop can be neglected.

2.16 Series Generator

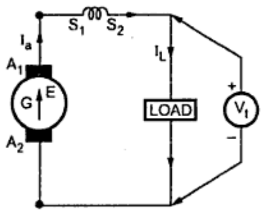


Fig. 2.24 Series generator

When the field winding is connected in series with the armature winding while supplying the load then the generator is called **series generator.** It is shown in the Fig. 2.24.

Field winding, in this case is denoted as S_1 and S_2. The resistance of series field winding is very small and hence naturally it has less number of turns of thick cross-section wire as shown in the Fig. 2.24.

Let R_{se} be the resistance of the series field winding.

2.16.1 Voltage and Current Relations

As all armature, field and load are in series they carry the same current.

$$\therefore \quad \boxed{I_a = I_{se} = I_L}$$

where I_{se} = Current through series field winding.

Now in addition to drop I_aR_a, induced e.m.f. has to supply voltage drop across series field winding too. This is $I_{se} R_{se}$ i.e. I_aR_{se} as $I_a = I_{se}$. So voltage equation can be written as,

$$\therefore \quad \boxed{E = V_t + I_a R_a + I_a R_{se} + V_{brush}}$$

$$\therefore \quad E = V_t + I_a (R_a + R_{se}) + V_{brush}$$

Where $E = \dfrac{\phi P N Z}{60 A}$

2.17 Compound Generator

In this type, the part of the field winding is connected in parallel with armature and part in series with the armature. Both series and shunt field windings are mounted on the same poles. Depending upon the connection of shunt and series field winding, compound generator is further classified as : i) Long shunt compound generator, ii) Short shunt compound generator.

2.17.1 Long Shunt Compound Generator

In this type, shunt field winding is connected across the series combination of armature and series field winding as shown in the Fig. 2.25.

Voltage and current relations are as follows :

From the Fig. 2.25, $I_a = I_{se}$

and $$\boxed{I_a = I_{sh} + I_L}$$

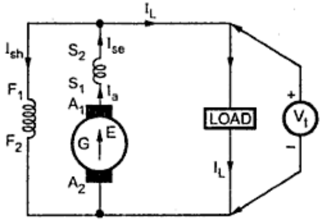


Fig. 2.25 Long shunt compound generator

Voltage across shunt field winding is V_t.

$$\therefore \qquad \boxed{I_{sh} = \frac{V_t}{R_{sh}}}$$

Where R_{sh} = Resistance of shunt field winding

And voltage equation is,

$$\boxed{E = V_t + I_a R_a + I_a R_{se} + V_{brush}}$$

Where R_{se} = Resistance of series field winding

2.17.2 Short Shunt Compound Generator

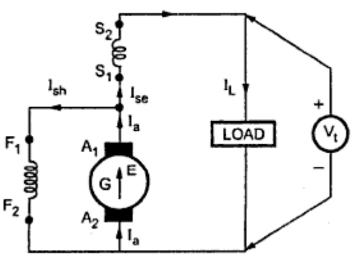


Fig. 2.26 Short shunt compound generator

In this type, shunt field winding is connected, only across the armature, excluding series field winding as shown in the Fig. 2.26.

Voltage and current relations are as follows :

For the Fig. 2.26, $I_a = I_{se} + I_{sh}$

and $I_{se} = I_L$

$$\therefore \qquad \boxed{I_a = I_L + I_{sh}}$$

The drop across shunt field winding is drop across the armature only and not the total V_t, in this case. So drop across shunt field winding is $E - I_a R_a$.

$$\therefore \qquad \boxed{I_{sh} = \frac{E - I_a R_a}{R_{sh}}}$$

Now the voltage equation is $E = V_t + I_a R_a + I_{se} R_{se} + V_{brush}$

Now $\quad I_{se} = I_L$

$$\therefore \qquad E = V_t + I_a R_a + I_L R_{se} + V_{brush}$$

Neglecting V_{brush}, we can write,

$$E = V_t + I_a R_a + I_L R_{se}$$

$$\therefore \qquad E - I_a R_a = V_t + I_L R_{se}$$

$$\therefore \qquad \boxed{I_{sh} = \frac{V_t + I_L R_{se}}{R_{sh}}}$$

Any of the two above expressions of I_{sh} can be used, depending on the quantities known while solving the problems.

2.17.3 Cumulative and Differential Compound Generator

It is mentioned earlier that the two windings, shunt and series field are wound on the same poles. Depending on the direction of winding on the pole, two fluxes produced by shunt and series field may help or may oppose each other. This fact decides whether generator is cumulative or differential compound. If the two fluxes help each other as shown in Fig. 2.27 the generator is called **cumulative compound generator**.

$$\boxed{\phi_T = \phi_{sh} + \phi_{se}}$$

Where $\quad \phi_{sh}$ = Flux produced by shunt.

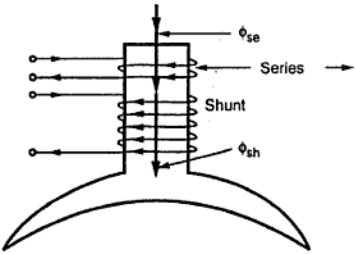
ϕ_{se} = Flux produced by series, field windings.

If the two windings are wound in such a direction that the fluxes produced by them oppose each other then the generator is called **differential compound generator**. This is shown in the Fig. 2.28.

$$\boxed{\phi_T = \phi_{sh} + \phi_{se}}$$

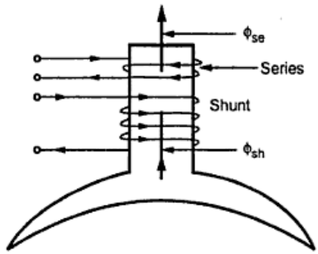
Where $\quad \phi_{sh}$ = Flux produced by shunt field winding.

ϕ_{se} = Flux produced by series field winding.



Cumulative compound generator

Fig. 2.27



Differential compound generator

Fig. 2.28

Example 2.3 : *A d.c. shunt generator has shunt field winding resistance of 100 Ω. It is supplying a load of 5 kW at a voltage of 250 V. If its armature resistance is 0.22 Ω, calculate the induced e.m.f. of generator.*

Solution : Consider shunt generator as shown in the Fig. 2.29.

$$I_a = I_L + I_{sh}$$

$$I_{sh} = \frac{V_t}{R_{sh}}$$

Now $V_t = 250$ V

and $R_{sh} = 100\ \Omega$

$\therefore$ $I_{sh} = \frac{250}{100} = 2.5$ A

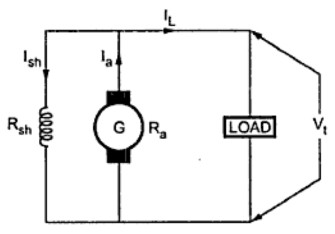


Fig. 2.29

Load power = 5 kW.

$\therefore$ $P = V_t \times I_L$

$\therefore$ $I_L = \frac{P}{V_t} = \frac{5 \times 10^3}{250} = 20$ A

$\therefore$ $I_a = I_L + I_{sh} = 20 + 2.5 = 22.5$ A

Now $E = V_t + I_a R_a$ (neglect V_{brush})

$\therefore$ $E = 250 + 22.5 \times 0.22 =$ **254.95 V**

This is the induced e.m.f. to supply the given load.

Example 2.4 : *A 250 V, 10 kW, separately excited generator has an induced e.m.f. of 255 V at full load. If the brush drop is 2 V per brush, calculate the armature resistance of the generator.*

Solution : Consider separately excited generator as shown in the Fig. 2.30.

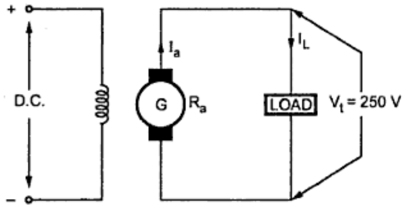


Fig. 2.30

$$I_a = I_L$$

Note that 250 V, 10 kW generator means the full load capacity of generator is to supply 10 kW load at a terminal voltage V_t = 250 V.

$\therefore \quad V_t = 250$ V and P = 10 kW

and $\quad P = V_t \times I_L$

$\therefore \quad I_L = \dfrac{10 \times 10^3}{250} = 40$ A

$\therefore \quad I_a = I_L = 40$ A ... As separately excited

Now $\quad E = V_t + I_a R_a + V_{brush}$

Now there are two brushes and brush drop is 2 V/brush.

$\therefore \quad V_{brush} = 2 \times 2 = 4$ V

$\therefore \quad E = 250 + 40 \times R_a + 4$

But $\quad E = 255$ V on full load

$\therefore \quad 255 = 250 + 40 R_a + 4$

$\therefore \quad \mathbf{R_a = 0.025\ \Omega}$

Example 2.5 : *A d.c. series generator has armature resistance of 0.5 Ω and series field resistance of 0.03 Ω. It drives a load of 50 A. If it has 6 turns/coil and total 540 coils on the armature and is driven at 1500 r.p.m., calculate the terminal voltage at the load. Assume 4 poles, lap type winding, flux per pole as 2 mWb and total brush drop as 2 V.*

Solution : Consider the series generator as shown in Fig. 2.31.

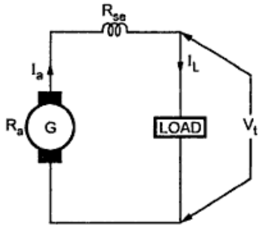


Fig. 2.31

$$R_a = 0.5\ \Omega,\ R_{se} = 0.03\ \Omega$$

$$V_{brush} = 2\ V$$

$$N = 1500 \text{ r.p.m.}$$

Total coils are 540 with 6 turns/coil.

$\therefore$ Total turns $= 540 \times 6 = 3240$

$\therefore$ Total conductors $Z = 2 \times$ turns

$$= 2 \times 3240 = 6480$$

$$\therefore \quad E = \frac{\phi P N Z}{60\ A}$$

For lap type, $A = P$

and $\phi = 2\ mWb = 2 \times 10^{-3}\ Wb$

$$\therefore \quad E = \frac{2 \times 10^{-3} \times 1500 \times 6480}{60}$$

$$= 324\ V$$

$$E = V_t + I_a (R_a + R_{se}) + V_{brush} \qquad \text{... Total } V_{brush} \text{ given}$$

Where $I_a = I_L = 50\ A$

$$\therefore \quad 324 = V_t + 50\ (0.5 + 0.03) + 2$$

$$\therefore \quad V_t = \mathbf{295.5\ V}$$

➠ **Example 2.6 :** *A short shunt compound d.c. generator supplies a current of 75 A at a voltage of 225 V. Calculate the generated voltage if the resistance of armature, shunt field and series field windings are 0.04 Ω , 90 Ω and 0.02 Ω respectively.*

Solution : Consider a short shunt generator as shown in the Fig. 2.32.

$R_a = 0.04\ \Omega,\quad R_{sh} = 90\ \Omega,\quad R_{se} = 0.02\ \Omega$

$$V_t = 225\ \text{V}$$

$$I_L = 75\ \text{A}$$

$$I_a = I_L + I_{sh}$$

Now $\quad E = V_t + I_a R_a + I_L R_{se}$

and drop across armature terminals is,

$$E - I_a R_a = V_t + I_L R_{se}$$

$$= 225 + 75 \times 0.02 = 226.5\ \text{V}$$

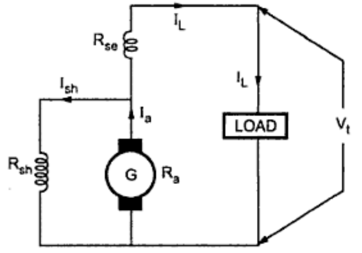


Fig. 2.32

$$\therefore \qquad I_{sh} = \frac{E - I_a R_a}{R_{sh}} = \frac{V_t + I_L R_{se}}{R_{sh}}$$

$$= \frac{226.5}{90} = 2.5167\ \text{A}$$

$$\therefore \qquad I_a = I_L + I_{sh} = 75 + 2.5167 = 77.5167\ \text{A}$$

$$\therefore \qquad E = V_t + I_a R_a + I_L R_{se}$$

$$= 225 + 77.5167 \times 0.04 + 75 \times 0.02 = \mathbf{229.6\ V}$$

2.18 Applications of Various Types of Generators

Separately excited generators :

As a separate supply is required to excite field, the use is restricted to some special applications like electro-plating, electro-refining of materials etc.

Shunt generators :

Commonly used in battery charging and ordinary lighting purposes.

Series generators :

Commonly used as boosters on d.c. feeders, as a constant current generators for welding generator and arc lamps.

Cumulatively compound generators :

These are used for domestic lighting purposes and to transmit energy over long distance.

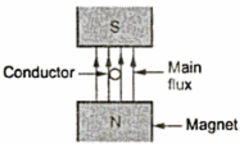
Differential compound generators :

The use of this type of generators is very rare and it is used for special application like electric arc welding.

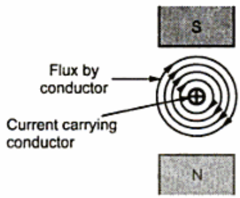
2.19 Principle of Operation of a D.C. Motor

The principle of operation of a d.c. motor can be stated in a single statement as 'when a current carrying conductor is placed in a magnetic field; it experiences a mechanical force'. In a practical d.c. motor, field winding produces a required magnetic field while armature conductors play a role of a current carrying conductors and hence armature conductors experience a force. As conductors are placed in the slots which are on the periphery, the individual force experienced by the conductors acts as a twisting or turning force on the armature which is called a **torque.** The torque is the product of force and the radius at which this force acts. So overall armature experiences a torque and starts rotating. Let us study this motoring action in detail.

Consider a single conductor placed in a magnetic field as shown in the Fig. 2.33 (a). The magnetic field is produced by a permanent magnet but in a practical d.c. motor it is produced by the field winding when it carries a current.



(a) Conductor in a magnetic field



(b) Flux produced by current carrying conductor

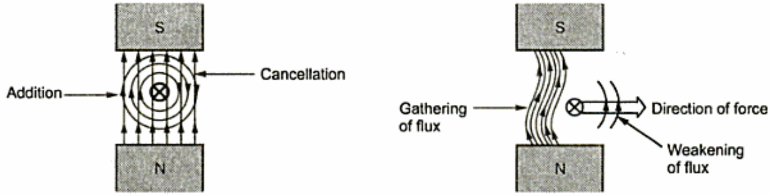
Fig. 2.33

Now this conductor is excited by a separate supply so that it carries a current in a particular direction. Consider that it carries a current away from an observer as shown in the Fig. 2.33 (b). Any current carrying conductor produces its own magnetic field around it, hence this conductor also produces its own flux, around. The direction of this flux can be determined by right hand thumb rule. For direction of current considered, the direction of flux around a conductor is clockwise. For simplicity of understanding, the main flux produced by the permanent magnet is not shown in the Fig. 2.33 (b).

Now there are two fluxes present,

1. The flux produced by the permanent magnet called main flux.
2. The flux produced by the current carrying conductor.

These are shown in the Fig. 2.34 (a). From this, it is clear that on one side of the conductor, both the fluxes are in the same direction. In this case, on the left of the conductor there is gathering of the flux lines as two fluxes help each other. As against this, on the right of the conductor, the two fluxes are in opposite direction and hence try to cancel each other. Due to this, the density of the flux lines in this area gets weakened. So on the left, there exists high flux density area while on the right of the conductor there exists low flux density area as shown in the Fig. 2.34 (b).



(a) Interaction of two fluxes (b) Force experienced by the conductor

Fig. 2.34

This flux distribution around the conductor acts like a stretched rubber band under tension. This exerts a mechanical force on the conductor which acts from high flux density area towards low flux density area, i.e. from left to right for the case considered as shown in the Fig. 2.34 (b).

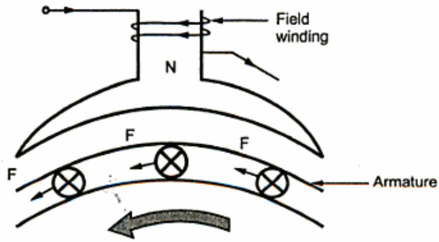


Fig. 2.35 Torque exerted on armature

Key Point : *In the practical d.c. motor, the permanent magnet is replaced by a field winding which produces the required flux called main flux and all the armature conductors, mounted on the periphery of the armature drum, get subjected to the mechanical force. Due to this, overall armature experiences a twisting force called torque and armature of the motor starts rotating.*

2.20 Direction of Rotation of Motor

The magnitude of the force experienced by the conductor in a motor is given by,

$$\boxed{F = B\,l\,I \quad \text{Newtons (N)}}$$

B = Flux density due to the flux produced by the field winding.

l = Active length of the conductor.

I = Magnitude of the current passing through the conductor.

The direction of such force i.e. the direction of rotation of a motor can be determined by Fleming's left hand rule. So **Fleming's right hand rule** is to determine direction of induced e.m.f. i.e. for **generating action** while **Fleming's left hand rule** is to determine direction of force experienced i.e. for **motoring action.**

2.20.1 Fleming's Left Hand Rule

The rule states that, 'Outstretch the three fingers of the left hand namely the first finger, middle finger and thumb such that they are mutually perpendicular to each other. Now point the first finger in the direction of magnetic field and the middle finger in the direction of the current then the thumb gives the direction of the force experienced by the conductor'.

The Fleming's left hand rule can be diagramatically shown as in the Fig. 2.36.

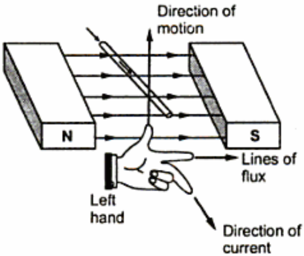


Fig. 2.36 Fleming's left hand rule

Apply the rule to crosscheck the direction of force experienced by a single conductor, placed in the magnetic field, shown in the Fig. 2.37 (a), (b), (c) and (d).

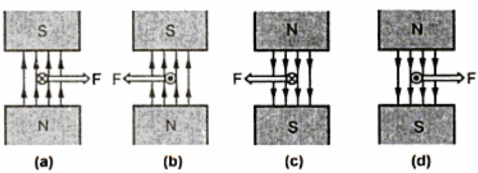


Fig. 2.37 Direction of force experienced by conductor

It can be seen from the Fig. 2.37 that if the direction of the main field in which current carrying conductor is placed, is reversed, force experienced by the conductor reverses its direction. Similarly keeping main flux direction unchanged, the direction of current passing through the conductor is reversed, the force experienced by the conductor reverses its direction. However if both the directions are reversed, the direction of the force experienced remains the same.

Key Point: ***So in a practical motor, to reverse its direction of rotation, either direction of main field produced by the field winding is reversed or direction of the current passing through the armature is reversed.***

The direction of the main field can be reversed by changing the direction of current passing through the field winding, which is possible by interchanging the polarities of supply which is given to the field winding. In short, to have a motoring action two fluxes must exist, the interaction of which produces a torque.

2.21 Significance of Back E.M.F.

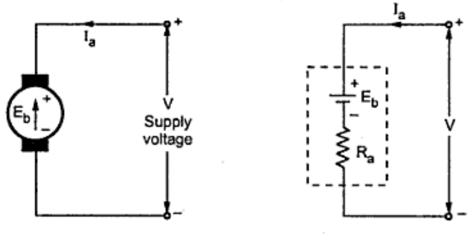
It is seen in the generating action, that when a conductor cuts the lines of flux, e.m.f. gets induced in the conductor. The question is obvious that in a d.c. motor, after a motoring action, armature starts rotating and armature conductors cut the main flux. So is there a generating action existing in a motor ? The answer to this question is 'Yes'.

After a motoring action, there exists a generating action. There is an induced e.m.f. in the rotating armature conductors according to Faraday's law of electromagnetic induction. This induced e.m.f. in the armature always acts in the opposite direction of the supply voltage. This is according to the **Lenz's law** which states that the direction of the induced e.m.f. is always so as to oppose the cause producing it. In a d.c. motor, electrical input i.e. the supply voltage is the cause and hence this induced e.m.f. opposes the supply voltage. This e.m.f. tries to set up a current through the armature which is in the opposite direction to that, which supply voltage is forcing through the conductor.

So as this e.m.f. always opposes the supply voltage, it is called **back e.m.f.** and denoted as E_b. Though it is denoted as E_b, basically it gets generated by the generating action which we have seen earlier in case of generators. So its magnitude can be determined by the e.m.f. equation which is derived earlier. So,

$$\boxed{E_b = \frac{\phi P N Z}{60 A} \text{ volts}}$$

where all symbols carry the same meaning as seen earlier in case of generators.



(a) Back e.m.f. in a d.c. motor (b) Equivalent circuit

Fig. 2.38

This e.m.f. is shown schematically in the Fig. 2.38 (a). So if V is supply voltage in volts and R_a is the value of the armature resistance, the equivalent electric circuit can be shown as in the Fig. 2.38 (b).

2.21.1 Voltage Equation of a D.C. Motor

In case of a generator, generated e.m.f. has to supply armature resistance drop and remaining part is available across the load as a terminal voltage. But in case of d.c. motor, supply voltage V has to overcome back e.m.f. E_b which is opposing V and also various drops as armature resistance drop $I_a R_a$, brush drop etc. Infact the electrical work done in overcoming the back e.m.f. gets converted into the mechanical energy developed in the armature. Hence the voltage equation of a d.c. motor can be written as,

$$\boxed{V = E_b + I_a R_a + \text{brush drop}}$$

Neglecting the brush drop, the generalised voltage equation is,

$$V = E_b + I_a R_a$$

The back e.m.f. is always less than supply voltage ($E_b < V$). But R_a is very small hence under normal running conditions, the difference between back e.m.f. and supply voltage is very small. The net voltage across the armature is the difference between the supply voltage and back e.m.f. which decides the armature current. Hence from the voltage equation we can write,

$$\boxed{I_a = \frac{V - E_b}{R_a}}$$

Key Point : *Voltage equation gets changed a little bit depending upon the type of the motor, which is discussed later.*

➠ **Example 2.7 :** *A 220 V, d.c. motor has an armature resistance of 0.75 Ω. It is drawing an armature current of 30 A, driving a certain load. Calculate the induced e.m.f. in the motor under this condition.*

Solution : V = 200 V, I_a = 30 A, R_a = 0.75 Ω are the given values.

For a motor, $V = E_b + I_a R_a$

$\therefore \quad 220 = E_b + 30 \times 0.75$

$\therefore \quad E_b = \mathbf{197.5\ volts}$

This is the induced e.m.f. called back e.m.f. in a motor.

➠ **Example 2.8 :** *A 4 pole, d.c. motor has lap connected armature winding. The flux per pole is 30 mWb. The number of armature conductors is 250. When connected to 230 V d.c. supply it draws an armature current of 40 A. Calculate the back e.m.f. and the speed with which motor is running. Assume armature resistance is 0.6 Ω.*

Solution : P = 4, A = P = 4 as lap, V = 230 V, Z = 250

$$\phi = 30 \text{ mWb} = 30 \times 10^{-3} \text{ Wb}$$

$$I_a = 40 \text{ A}$$

From voltage equation,
$$V = E_b + I_a R_a$$

$$\therefore \quad 230 = E_b + 40 \times 0.6$$

$$\therefore \quad E_b = 206 \text{ V}$$

And
$$E_b = \frac{\phi P N Z}{60 A}$$

$$\therefore \quad 206 = \frac{30 \times 10^{-3} \times 4 \times N \times 250}{60 \times 4}$$

$$\therefore \quad N = \mathbf{1648 \text{ r.p.m.}}$$

2.21.2 Back E.M.F. as a Regulating Mechanism

Due to the presence of back e.mf. the d.c. motor becomes a regulating machine i.e. motor adjusts itself to draw the armature current just enough to satisfy the load demand. The basic principle of this fact is that the back e.m.f. is proportional to speed, $E_b \propto N$.

When load is suddenly put on to the motor, motor tries to slow down. So speed of the motor reduces due to which back e.m.f. also decreases. So the net voltage across the armature ($V - E_b$) increases and motor draws more armature current. As $F = B\, l\, I$, due to increased current, force experienced by the conductors and hence the torque on the armature increases. The increase in the torque is just sufficient to satisfy increased load demand. The motor speed stops decreasing when the armature current is just enough to produce torque demanded by the new load.

When load on the motor is decreased, the speed of the motor tries to increase. Hence back e.m.f. increases. This causes ($V - E_b$) to reduce which eventually reduces the current drawn by the armature. The motor speed stops increasing when the armature current is just enough to produce the less torque required by the new load.

> **Key Point** : *So back e.m.f. regulates the flow of armature current and it automatically alters the armature current to meet the load requirement. This is the practical significance of the back e.m.f.*

2.22 Power Equation of a D.C. Motor

The voltage equation of a d.c. motor is given by,

$$V = E_b + I_a R_a$$

Multiplying both sides of the above equation by I_a we get,

$$\boxed{VI_a = E_b I_a + I_a^2 R_a}$$

This equation is called **power equation** of a d.c. motor.

VI_a = Net electrical power input to the armature measured in watts.

$I_a^2 R_a$ = Power loss due to the resistance of the armature called **armature copper loss.**

So difference between VI_a and $I_a^2 R_a$ i.e. input – losses gives the output of the armature.

So $E_b I_a$ is called **electrical equivalent of gross mechanical power developed by** the armature. This is denoted as P_m.

∴ Power input to the armature – Armature copper loss = Gross mechanical power developed in the armature.

2.22.1 Condition for Maximum Power

For a motor from power equation it is known that,

$$P_m = \text{Gross mechanical power developed} = E_b I_a$$

$$= VI_a - I_a^2 R_a$$

For maximum P_m, $\frac{dP_m}{dI_a} = 0$

$$\therefore \quad 0 = V - 2 I_a R_a$$

$$\therefore \quad \boxed{I_a = \frac{V}{2 R_a} \quad \text{i.e.} \quad I_a R_a = \frac{V}{2}}$$

Substituting in voltage equation,

$$V = E_b + I_a R_a = E_b + \frac{V}{2}$$

$$\therefore \quad \boxed{E_b = \frac{V}{2}} \quad \text{.... Condition for maximum power}$$

Key Point : *This is practically impossible to achieve as for this E_b, current required is much more than its normal rated value. Large heat will be produced and efficiency of motor will be less than 50 %.*

2.23 Torque Equation of a D.C. Motor

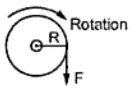


Fig. 2.39

It is seen that the turning or twisting force about an axis is called torque. Consider a wheel of radius R meters acted upon by a circumferential force F newtons as shown in the Fig. 2.39.

The wheel is rotating at a speed of N r.p.m.

Then angular speed of the wheel is,

$$\omega = \frac{2\pi N}{60} \quad \text{rad/sec}$$

So workdone in one revolution is,

$$W = F \times \text{distance travelled in one revolution}$$

$$= F \times 2\pi R \quad \text{joules}$$

And $$P = \text{Power developed} = \frac{\text{Workdone}}{\text{Time}}$$

$$= \frac{F \times 2\pi R}{\text{Time for 1 rev}} = \frac{F \times 2\pi R}{\left(\frac{60}{N}\right)} = (F \times R) \times \left(\frac{2\pi N}{60}\right)$$

$\therefore$ $$P = T \times \omega \quad \text{watts}$$

Where $$T = \text{Torque in N - m}$$

$$\omega = \text{Angular speed in rad/sec.}$$

Let T_a be the gross torque developed by the armature of the motor. It is also called **armature torque**. The gross mechanical power developed in the armature is $E_b\ I_a$, as seen from the power equation. So if speed of the motor is N r.p.m. then,

Power in armature = Armature torque $\times \omega$

$\therefore$ $$E_b\ I_a = T_a \times \frac{2\pi N}{60}$$

but E_b in a motor is given by,

$$E_b = \frac{\phi P N Z}{60\ A}$$

$\therefore$ $$\frac{\phi P N Z}{60\ A} \times I_a = T_a \times \frac{2\pi N}{60}$$

$\therefore$ $$\boxed{\begin{aligned} T_a &= \frac{1}{2\pi}\ \phi\ I_a \times \frac{PZ}{A} \\ T_a &= 0.159\ \phi\ I_a \cdot \frac{PZ}{A} \quad \text{N- m} \end{aligned}}$$

This is the **torque equation** of a d.c. motor.

Example 2.9 : *A 4 pole d.c. motor takes a 50 A armature current. The armature has lap connected 480 conductors. The flux per pole is 20 mWb. Calculate the gross torque developed by the armature of the motor.*

Solution : P = 4, A = P = 4, Z = 480

$$\phi = 20 \text{ mWb} = 20 \times 10^{-3} \text{ Wb}, I_a = 50 \text{ A}$$

Now
$$T_a = 0.159 \times \phi I_a \cdot \frac{PZ}{A} = 0.159 \times 20 \times 10^{-3} \times 50 \times \frac{4 \times 480}{4}$$

$$= \mathbf{76.394 \text{ N-m}}$$

2.23.1 Types of Torque in the Motor

Basically the torque is developed in the armature and hence gross torque produced is denoted as T_a.

The mechanical power developed in the armature is transmitted to the load through the shaft of the motor. It is impossible to transmit the entire power developed by the armature to the load. This is because while transmitting the power through the shaft, there is a power loss due to the friction, windage and the iron loss. The torque required to overcome these losses is called **lost torque**, denoted as T_f. These losses are also called **stray losses.**

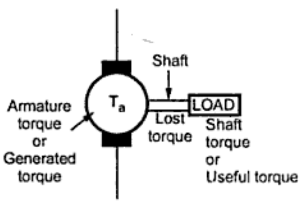


Fig. 2.40 Types of torque

The torque which is available at the shaft for doing the useful work is known as **load torque** or **shaft torque** denoted as T_{sh}.

$$\therefore \qquad \boxed{T_a = T_f + T_{sh}}$$

The shaft torque magnitude is always less than the armature torque, ($T_{sh} < T_a$).

The speed of the motor remains same all along the shaft say N r.p.m. Then the product of shaft torque T_{sh} and the angular speed ω rad/sec is called power available at the shaft i.e. net output of the motor. The maximum power a motor can deliver to the load safely is called **output rating** of a motor. Generally it is expressed in H.P. It is called **H.P. rating** of a motor.

$$\boxed{\text{Net output of motor} = P_{out} = T_{sh} \times \omega}$$

2.23.2 No Load Condition of a Motor

On no load, the load requirement is absent. So $T_{sh} = 0$. This does not mean that motor is at hault. The motor can rotate at a speed say N_0 r.p.m. on no load. The motor draws an armature current of I_{a0}.

$$\boxed{I_{a0} = \frac{V - E_{b0}}{R_a}}$$

where E_{b0} is back e.m.f. on no load, proportional to speed N_0.

Now armature torque T_a for a motor is,

$$T_a \propto \phi I_a$$

As flux is present and armature current is present, hence T_{a0} i.e. armature torque exists on no load.

Now $\quad T_a = T_f + T_{sh}$

but on no load, $T_{sh} = 0$

$$\therefore \quad \boxed{T_{a0} = T_f}$$

So on no load, motor keeps on rotating at a speed of N_0 r.p.m. drawing an armature current of I_{a0}. This is just enough to produce a torque T_{a0} which satisfies the friction, windage and iron losses of the motor. On no load, speed of the motor is large hence E_{b0} is also large hence $(V - E_{b0})$ is very small hence armature current I_{a0} is also small. So **motor draws less current on no load and takes more and more current as motor load increases.**

So on no load,

Torque developed = Torque required to overcome friction, windage, iron losses.

$$\therefore \quad \boxed{\text{Power developed } (E_{b0} \times I_{a0}) = \text{Friction, windage and, iron losses}}$$

where $\quad E_{b0}$ = Back e.m.f. on no load.

and $\quad I_{a0}$ = Armature current drawn on no load.

This component of stray losses i.e. $E_{b0}\, I_{a0}$ is practically assumed to be constant though the load on the motor is changed from zero to the full capacity of the motor. So T_f is practically assumed constant for all load conditions.

Example 2.10 : *A 4 pole, lap wound d.c. motor has 540 conductors. Its speed is found to be 1000 r.p.m. when it is made to run light. The flux per pole is 25 mWb. It is connected to 230 V d.c. supply. The armature resistance is 0.8 Ω. Calculate.*
i) Induced e.m.f. ii) Armature current iii) Stray losses iv) Lost torque.

Solution : P = 4, A = P = 4

Running light means it is on no load.

$$\therefore \quad N_0 = 1000 \text{ r.p.m.}$$

$$Z = 540 \text{ and } \phi = 25 \times 10^{-3} \text{ Wb}$$

$$\therefore \quad E_{b0} = \frac{\phi P N_0 Z}{60 A} = \frac{25\times10^{-3}\times4\times1000\times540}{60\times4} = 225 \text{ V}$$

i) Induced e.m.f., E_{b0} = **225 V**

ii) From voltage equation, $V = E_b + I_a R_a$

$$\therefore \quad V = E_{b0} + I_{a0} R_a$$

$$\therefore \quad 230 = 225 + I_{a0} \times 0.8$$

$$\therefore \quad I_{a0} = \mathbf{6.25 \text{ A}}$$

iii) On no load, power developed is fully the power required to overcome stray losses.

$$\therefore \quad \text{Stray losses} = E_{b0} I_{a0} = 225 \times 6.25 = \mathbf{1406.25 \text{ W}}$$

iv) Lost torque $$T_f = \frac{E_{b0} I_{a0}}{\omega_0} = \frac{1406.25}{\frac{2\pi N_0}{60}} = \frac{1406.25\times60}{2\pi\times1000} = \mathbf{13.428 \text{ N-m.}}$$

2.24 Types of D.C. Motors

Similar to the d.c. generators, the d.c. motors are classified depending upon the way of connecting the field winding with the armature winding. The different types of d.c. motors are shunt motors, series motors and compound motors. The compound motors are further classified as short shunt compound and long shunt compound motors. Let us see the connection diagrams and different voltage and current relations of these types of motors.

2.25 D.C. Shunt Motor

In this type, the field winding is connected across the armature winding and the combination is connected across the supply, as shown in the Fig. 2.41.

Let R_{sh} be the resistance of shunt field winding.

R_a be the resistance of armature winding.

The value of R_a is very small while R_{sh} is quite large. Hence shunt field winding has more number of turns with less cross-sectional area.

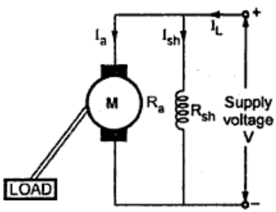


Fig. 2.41 D.C. shunt motor

2.25.1 Voltage and Current Relationship

The voltage across armature and field winding is same equal to the supply voltage V.

The total current drawn from the supply is denoted as line current I_L.

$$I_L = I_a + I_{sh}$$

$$I_{sh} = \frac{V}{R_{sh}}$$

and $$V = E_b + I_a R_a + V_{brush}$$

V_{brush} is generally neglected.

Now flux produced by the field winding is proportional to the current passing through it i.e. I_{sh}.

$$\boxed{\phi \propto I_{sh}}$$

Key Point : *As long as supply voltage is constant, which is generally so in practice, the flux produced is constant. Hence **d.c. shunt motor is called constant flux motor.***

2.26 D.C. Series Motor

In this type of motor, the series field winding is connected in series with the armature and the supply, as shown in the Fig. 2.42.

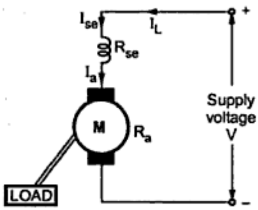


Fig. 2.42 D.C. series motor

Let R_{se} be the resistance of the series field winding. The value of R_{se} is very small and it is made of small number of turns having large cross-sectional area.

2.26.1 Voltage and Current Relationship

Let I_L be the total current drawn from the supply.

So $$I_L = I_{se} = I_a$$

and $$V = E_b + I_a R_a + I_{se} R_{se} + V_{brush}$$

$$V = E_b + I_a (R_a + R_{se}) + V_{brush}$$

Supply voltage has to overcome the drop across series field winding in addition to E_b and drop across armature winding.

Key Point : *In series motor, entire armature current is passing through the series field winding. So flux produced is proportional to the armature current.*

$$\boxed{\phi \propto I_{se} \propto I_a \quad \text{for series motor}}$$

2.27 D.C. Compound Motor

The compound motor consists of part of the field winding connected in series and part of the field winding connected in parallel with armature. It is further classified as long shunt compound and short shunt compound motor.

2.27.1 Long Shunt Compound Motor

In this type, the shunt field winding is connected across the combination of armature and the series field winding as shown in the Fig. 2.43.

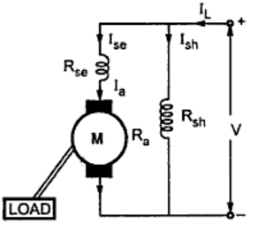


Fig. 2.43 Long shunt compound motor

Let R_{se} be the resistance of series field and R_{sh} be the resistance of shunt field winding. The total current drawn from supply is I_L.

So $\quad I_L = I_{se} + I_{sh}$

But $\quad I_{se} = I_a$

$\therefore \quad I_L = I_a + I_{sh}$

And $\quad I_{sh} = \dfrac{V}{R_{sh}}$

And $\quad V = E_b + I_a R_a + I_{se} R_{se} + V_{brush}$

But as $\quad I_{se} = I_a,$

$\therefore \quad V = E_b + I_a (R_a + R_{se}) + V_{brush}$

2.27.2 Short Shunt Compound Motor

In this type, the shunt field is connected purely in parallel with armature and the series field is connected in series with this combination shown in the Fig. 2.44.

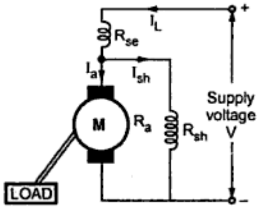


Fig. 2.44 Short shunt compound motor

$$I_L = I_{se}$$

The entire line current is passing through the series field winding.

and $$I_L = I_a + I_{sh}$$

Now the drop across the shunt field winding is to be calculated from the voltage equation.

So $$V = E_b + I_{se} R_{se} + I_a R_a + V_{brush}$$

but $$I_{se} = I_L$$

$\therefore$ $$V = E_b + I_L R_{se} + I_a R_a + V_{brush}$$

$\therefore$ Drop across shunt field winding is,

$$= V - I_L R_{se} = E_b + I_a R_a + V_{brush}$$

$\therefore$ $$\boxed{I_{sh} = \frac{V - I_L R_{se}}{R_{sh}} = \frac{E_b + I_a R_a + V_{brush}}{R_{sh}}}$$

Apart from these two, compound motor can be classified into two more types,

i) Cumulatively compound motors and ii) Differential compound motors.

Key Point : *If the two field windings are wound in such a manner that the fluxes produced by the two always help each other, the motor is called* ***cumulatively compound.*** *If the fluxes produced by the two field windings are trying to cancel each other i.e. they are in opposite direction, the motor is called* ***differential compound.***

A long shunt compound motor can be of cumulative or differential type. Similarly short shunt compound motor can be cumulative or differential type.

2.28 Torque and Speed Equations

Before analyzing the various characteristics of motors, let us revise the torque and speed equations as applied to various types of motors.

$$\therefore \quad \boxed{T \propto \phi I_a} \quad \text{from torque equation.}$$

This is because, $0.159 \frac{PZ}{A}$ is a constant for a given motor.

Now ϕ is the flux produced by the field winding and is proportional to the current passing through the field winding.

$$\phi \propto I_{field}$$

But for various types of motors, current through the field winding is different. Accordingly torque equation must be modified.

For a d.c. shunt motor, I_{sh} is constant as long as supply voltage is constant. Hence ϕ flux is also constant.

$$\therefore \quad T \propto I_a \quad \text{for shunt motors}$$

For a d.c. series motor, I_{se} is same as I_a. Hence flux ϕ is proportional to the armature current I_a.

$$\therefore \quad T \propto I_a \phi \propto I_a^2 \quad \text{for series motors.}$$

Similarly as $E_b = \frac{\phi PNZ}{60 A}$, we can write the speed equation as,

$$E_b \propto \phi N$$

$$\therefore \quad \boxed{N \propto \frac{E_b}{\phi}}$$

But $\quad V = E_b + I_a R_a \quad$ neglecting brush drop.

$$\therefore \quad E_b = V - I_a R_a$$

$\therefore$ Speed equation becomes,

$$N \propto \frac{V - I_a R_a}{\phi}$$

So for shunt motor as flux ϕ is constant,

$$\therefore \quad N \propto V - I_a R_a$$

While for series motor, flux ϕ is proportional to I_a

$$\therefore \quad \boxed{N \propto \frac{V - I_a R_a - I_a R_{se}}{I_a}}$$

These relations play an important role in understanding the various characteristics of different types of motors.

2.28.1 Speed Regulation

The speed regulation for a d.c. motor is defined as the ratio of change in speed corresponding to no load and full load condition to speed corresponding to full load.

Mathematically it is expressed as,

$$\boxed{\% \text{ speed regulation} = \frac{N_{\text{no load}} - N_{\text{full load}}}{N_{\text{full load}}} \times 100}$$

2.29 D.C. Motor Characteristics

The performance of a d.c. motor under various conditions can be judged by the following characteristics.

i) Torque - armature current characteristics (T Vs I_a) :

The graph showing the relationship between the torque and the armature current is called a torque-armature current characteristic. These are also called electrical characteristics.

ii) Speed - armature current characteristics (N Vs I_a) :

The graph showing the relationship between the speed and armature current characteristics.

iii) Speed - torque characteristics (N Vs T) :

The graph showing the relationship between the speed and the torque of the motor is called speed-torque characteristics of the motor. These are also called mechanical characteristics.

The nature of these characteristics can easily be obtained by using speed and torque equations derived in section 2.28. These characteristics play a very important role in selecting a type of motor for a particular application.

2.30 Characteristics of D.C. Shunt Motor

i) Torque - armature current characteristics

For a d.c. motor $T \propto \phi I_a$

For a constant values of R_{sh} and supply voltage V, I_{sh} is also constant and hence flux is also constant.

$$\therefore \qquad \boxed{T_a \propto I_a}$$

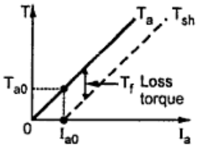


Fig. 2.45 T Vs I_a for shunt motor

The equation represents a straight line, passing through the origin, as shown in the Fig. 2.45. Torque increases linearily with armature current. It is seen earlier that armature current is decided by the load. So as load increases, armature current increases, increasing the torque developed linearly.

Now if shaft torque is plotted against armature current, it is known that shaft torque is less than the armature torque and the difference between the two is loss torque T_f as shown. On no load $T_{sh} = 0$ but armature torque is present which is just enough to overcome stray losses shown as T_{a0}. The current required is I_{a0} on no load to produce T_{a0} and hence T_{sh} graph has an intercept of I_{a0} on the current axis.

To generate high starting torque, this type of motor requires a large value of armature current at start. This may damage the motor hence d.c. shunt motors can develop moderate starting torque and hence suitable for such applications where starting torque requirement is moderate.

ii) Speed - armature current characteristics

From the speed equation we get,

$$N \propto \frac{V - I_a R_a}{\phi}$$

$$\propto V - I_a R_a \qquad \text{as } \phi \text{ is constant.}$$

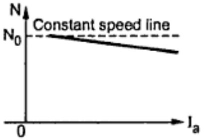


Fig. 2.46 N Vs I_a for shunt motor

So as load increases, the armature current increases and hence drop $I_a R_a$ also increases.

Hence for constant supply voltage, $V - I_a R_a$ decreases and hence speed reduces. But as R_a is very small, for change in I_a from no load to full load, drop $I_a R_a$ is very small and hence drop in speed is also not significant from no load to full load.

So the characteristics is slightly droping as shown in the Fig. 2.46.

But for all practical purposes these type of motors are considered to be a constant speed motors.

iii) Speed - torque characteristics

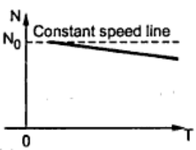


Fig. 2.47 N Vs T for shunt motor

These characteristics can be derived from the above two characteristics. This graph is similar to speed-armature current characteristics as torque is proportional to the armature current. This curve shows that the speed almost remains constant though torque changes from no load to full load conditions. This is shown in the Fig. 2.47.

2.31 Characteristics of D.C. Series Motor

i) Torque - armature current characteristics

In case of series motor the series field winding is carrying the entire armature current. So flux produced is proportional to the armature current.

$$\therefore \qquad \phi \propto I_a$$

$$\text{Hence} \qquad \boxed{T_a \propto \phi I_a \propto I_a^2}$$

Thus torque in case of series motor is proportional to the square of the armature current. This relation is parabolic in nature as shown in the Fig. 2.48.

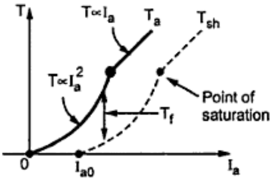


Fig. 2.48 T Vs I_a for series motor

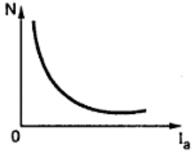


Fig. 2.49 N Vs I_a for series motor

As load increases, armature current increases and torque produced increases proportional to the square of the armature current upto a certain limit.

As the entire I_a passes through the series field, there is a property of an electromagnet called **saturation**, may occur. Saturation means though the current through the winding increases, the flux produced remains constant. Hence after saturation the characteristics take the shape of straight line as flux becomes constant, as shown. The difference between T_a and T_{sh} is loss torque T_f which is also shown in the Fig. 2.48.

At start as $T \propto I_a^2$, these types of motors can produce high torque for small amount of armature current hence the series motors are suitable for the applications which demand high starting torque.

ii) Speed - armature current characteristics

From the speed equation we get,

$$N \propto \frac{E_b}{\phi}$$

$$\propto \frac{V - I_a R_a - I_a R_{se}}{I_a} \qquad \text{as } \phi \propto I_a \text{ in case of series motor}$$

Now the values of R_a and R_{se} are so small that the effect of change in I_a on speed overrides the effect of change in $V - I_a R_a - I_a R_{se}$ on the speed.

Hence in the speed equation, $E_b \cong V$ and can be assumed constant. So speed equation reduces to,

$$\boxed{N \propto \frac{1}{I_a}}$$

So speed-armature current characteristics is rectangular hyperbola type as shown in the Fig. 2.49.

iii) Speed - torque characteristics

In case of series motors, $T \propto I_a^2$ and $N \propto \frac{1}{I_a}$

Hence we can write, $$\boxed{N \propto \frac{1}{\sqrt{T}}}$$

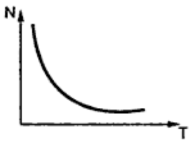


Fig. 2.50 N Vs T for series motor

Thus as torque increases when load increases, the speed decreases. On no load, torque is very less and hence speed increases to dangerously high value. Thus the nature of the speed-torque characteristics is similar to the nature of the speed-armature current characteristics.

The speed-torque characteristics of a series motor is shown in the Fig. 2.50.

2.32 Why Series Motor is Never Started on No Load ?

It is seen earlier that motor armature current is decided by the load. On light load or no load, the armature current drawn by the motor is very small.

In case of a d.c. series motor, $\phi \propto I_a$ and

on no load as I_a is small hence flux produced is also very small.

According to speed equation,

$$N \propto \frac{1}{\phi} \quad \text{as } E_b \text{ is almost constant.}$$

So on very light load or no load as flux is very small, the motor tries to run at dangerously high speed which may damage the motor mechanically. This can be seen from the speed-armature current and the speed-torque characteristics that on low armature current and low torque condition motor shows a tendency to rotate with dangerously high speed.

This is the reason why series motor should never be started on light loads or no load conditions. For this reason it is not selected for belt drives as breaking or slipping of belt causes to throw the entire load off on the motor and made to run motor with no load which is dangerous.

2.33 Characteristics of D.C. Compound Motor

Compound motor characteristics basically depends on the fact whether the motor is cumulatively compound or differential compound. All the characteristics of the compound motor are the combination of the shunt and series characteristic.

Cumulative compound motor is capable of developing large amount of torque at low speeds just like series motor. However it is not having a disadvantage of series motor even at light or no load. The shunt field winding produces the definite flux and series flux helps the shunt field flux to increase the total flux level.

So cumulative compound motor can run at a reasonable speed and will not run with dangerously high speed like series motor, on light or no load condition.

In differential compound motor, as two fluxes oppose each other, the resultant flux decreases as load increases, thus the machine runs at a higher speed with increase in the load. This property is dangerous as on full load, the motor may try to run with dangerously high speed. So differential compound motor is generally not used in practice.

The various characteristics of both the types of compound motors cumulative and the differential are shown in the Fig. 2.51 (a), (b) and (c).

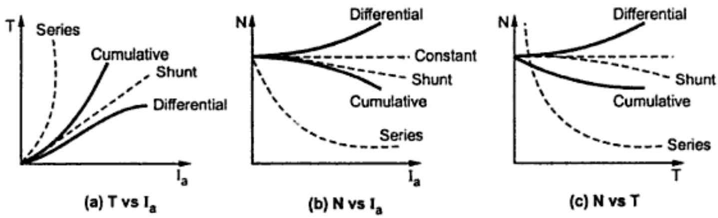


Fig. 2.51 Characteristics of d.c. compound motor

The exact shape of these characteristics depends on the relative contribution of series and shunt field windings. If the shunt field winding is more dominant then the characteristics take the shape of the shunt motor characteristics. While if the series field winding is more dominant then the characteristics take the shape of the series characteristics.

2.34 Applications of D.C. Motors

Instead of just stating the applications, the behaviour of the various characteristics like speed, starting torque etc., which makes the motor more suitable for the applications, is also stated in the Table 2.1.

Type of motor	Characteristics	Applications
Shunt	Speed is fairly constant and medium starting torque.	1) Blowers and fans 2) Centrifugal and reciprocating pumps 3) Lathe machines 4) Machine tools 5) Milling machines 6) Drilling machines.
Series	High starting torque. No load condition is dangerous. Variable speed.	1) Cranes 2) Hoists, Elevators 3) Trolleys 4) Conveyors 5) Electric locomotives.
Cumulative compound	High starting torque. No load condition is allowed.	1) Rolling mills 2) Punches 3) Shears 4) Heavy planers 5) Elevators.
Differential compound	Speed increases as load increases.	Not suitable for any practical application.

Table 2.1

➠ **Example 2.11 :** *A 4 pole, 250 V, d.c. series motor has a wave connected armature with 200 conductors. The flux per pole is 25 mWb when motor is drawing 60 A from the supply. Armature resistance is 0.15 Ω while series field winding resistance is 0.2 Ω. Calculate the speed under this condition.*

Solution :

$$P = 4,\ Z = 200$$

$$A = 2,\ \phi = 25 \times 10^{-3}\,\text{Wb}$$

$$I_a = I_L = 60\ \text{A}$$

$$R_a = 0.15\ \Omega$$

$$R_{se} = 0.2\ \Omega$$

$$V = E_b + I_a R_a + I_a R_{se}$$

$$\therefore \quad 250 = E_b + 60\,(0.15 + 0.2)$$

$$\therefore \quad E_b = 229\ \text{V}$$

Now $$E_b = \frac{\phi P N Z}{60\ A}$$

$$\therefore \quad 229 = \frac{25 \times 10^{-3} \times 4\,N \times 200}{60 \times 2}$$

$$\therefore \quad N = \mathbf{1374\ r.p.m.}$$

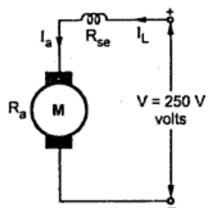


Fig. 2.52

Example 2.12 : *A 250 V, d.c. shunt motor takes a line current of 20 A. Resistance of shunt field winding is 200 Ω and resistance of the armature is 0.3 Ω. Find the armature current and the back e.m.f.*

Solution :

$$V = 250\text{ V},\ I_L = 20\text{ A}$$

$$R_a = 0.3\ \Omega,\quad R_{sh} = 200\ \Omega$$

$$I_L = I_a + I_{sh}$$

$$I_{sh} = \frac{V}{R_{sh}} = \frac{250}{200} = 1.25\text{ A}$$

$$\therefore \quad I_a = I_L - I_{sh} = 20 - 1.25 = \mathbf{18.75\ A}$$

Now $$V = E_b + I_a R_a$$

$$\therefore \quad E_b = V - I_a R_a = 250 - 18.75 \times 0.3 = \mathbf{244.375\ V}$$

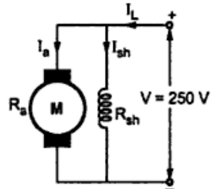


Fig. 2.53

Example 2.13 : *A d.c. shunt motor runs at a speed of 1000 r.p.m. on no load taking a current of 6 A from the supply, when connected to 220 V d.c. supply. Its full load current is 50 A. Calculate its speed on full load. Assume R_a = 0.3 Ω and R_{sh} = 110 Ω.*

Solution : Let no load, speed be N_0 = 1000 r.p.m.

$$I_{L0} = \text{Line current on no load} = 6\text{ A}$$

$$I_{L0} = I_{a0} + I_{sh}$$

$$I_{sh} = \frac{V}{R_{sh}} = \frac{220}{110} = 2\text{ A}$$

$$\therefore \quad I_{a0} = I_{L0} - I_{sh} = 6 - 2 = 4\text{ A}$$

∴ Back e.m.f. on no load E_{b0} can be determined from the voltage equation.

$$V = E_{b0} + I_{a0} R_a$$

$$\therefore \quad 220 = E_{b0} + 4 \times 0.3$$

$$\therefore \quad E_{b0} = 218.8\text{ V}$$

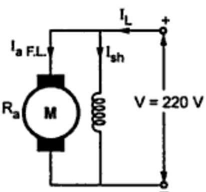


Fig. 2.54

On full load condition, supply voltage is constant and hence,

$$I_{sh} = \frac{V}{R_{sh}} = \frac{220}{110} = 2 \text{ A} \quad \text{(remains same)}$$

Now $\qquad I_L = I_{a\,F.L.} + I_{sh}$

$\therefore \qquad 50 = I_{a\,F.L.} + 2$

$\therefore \qquad I_{a\,F.L.} = 48 \text{ A}$

and $\qquad V = E_{b\,F.L.} + I_{a\,F.L.} R_a$

$\therefore \qquad 220 = E_{b\,F.L.} + 48 \times 0.3$

$\therefore \qquad E_{b\,F.L.} = 205.6 \text{ V}$

From the speed equation,

$$N \propto \frac{E_b}{\phi}$$

But ϕ is constant as I_{sh} is constant for both the load conditions.

$$\therefore \qquad \frac{N_0}{N_{F.L.}} = \frac{E_{b0}}{E_{b\,F.L.}}$$

$$\therefore \qquad N_{F.L.} = N_0 \frac{E_{b\,F.L.}}{E_{b0}} = 1000 \times \frac{205.6}{218.8} = \mathbf{939.67 \text{ r.p.m.}}$$

Example 2.14 : *A d.c. series motor is running with a speed of 800 r.p.m. while taking a current of 20 A from the supply. If the load is changed such that the current drawn by the motor is increased to 50 A, calculate the speed of the motor on new load. The armature and series field winding resistances are 0.2 Ω and 0.3 Ω respectively. Assume the flux produced is proportional to the current. Assume supply voltage as 250 V.*

Solution : For load 1, $N_1 = 800$ r.p.m., $I_1 = I_{a1} = 20$ A

For load 2, $\qquad I_2 = I_{a2} = 50 \text{ A}$

$\qquad R_a = 0.2\ \Omega,\ R_{se} = 0.3\ \Omega,$

From voltage equation $V = E_{b1} + I_{a1} R_a + I_{se1} R_{se}$

but $\qquad I_1 = I_{a1} = I_{se1} = 20\text{A}$

$\therefore \qquad 250 = E_{b1} + 20\,(0.2 + 0.3)$

$\therefore \qquad E_{b1} = 240 \text{ V}$

and $\qquad V = E_{b2} + I_{a2} R_a + I_{se2} R_{se}$

$\therefore \quad 250 = E_{b2} + 50\ (0.2 + 0.3)$

$\therefore \quad E_{b2} = 225\ \text{V}$

From the speed equation,

$$N \propto \frac{E_b}{\phi}$$

Now $\quad \phi \propto I_{se} \propto I_a$

$$\therefore \quad \frac{N_1}{N_2} = \frac{E_{b1}}{E_{b2}} \times \frac{\phi_2}{\phi_1}$$

$$\therefore \quad \frac{N_1}{N_2} = \frac{E_{b1}}{E_{b2}} \times \frac{I_{a2}}{I_{a1}}$$

$$\therefore \quad N_2 = N_1 \times \frac{E_{b2}}{E_{b1}} \times \frac{I_{a1}}{I_{a2}} = 800 \times \frac{225}{240} \times \frac{20}{50} = \mathbf{300\ r.p.m.}$$

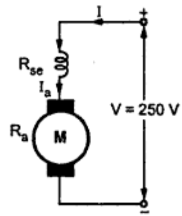


Fig. 2.55

2.35 Necessity of Starter

All the d.c. motors are basically self starting motors. Whenever the armature and the field winding of a d.c. motor receives supply, motoring action takes place. So d.c. motors do not require any additional device to start it. The device to be used as a starter conveys a wrong meaning.

Key Point : ***So starter is not required to start a d.c. motor but it enables us to start the motor in a desired, safe way.***

Now at the starting instant the speed of the motor is zero, (N = 0). As speed is zero, there cannot be any back e.m.f. as $E_b \propto N$ and N is zero at start.

$\therefore \quad E_b$ at start $= 0$

The voltage equation of a d.c. motor is,

$$V = E_b + I_a R_a$$

So at start, $\quad V = I_a R_a \qquad$ as $E_b = 0$

$$\therefore \quad \boxed{I_a = \frac{V}{R_a}} \qquad \text{... At start}$$

Key Point : *Generally motor is switched on with normal voltage and as armature resistance is very small, the armature current at start is very high.*

Consider a motor having full load input power as 8000 watts. The motor rated voltage be 250 V and armature resistance is 0.5 Ω.

Then at start, $E_b = 0$ and motor is operated at 250 V supply, so

$$I_a = \frac{V}{R_a} = \frac{250}{0.5} = 500 \text{ A}$$

While its full load current can be calculated as,

$$I_{Full\ load} = \frac{\text{Power input on full load}}{\text{Supply voltage}} = \frac{8000}{250} = 32 \text{ A}$$

So at start, motor is showing a tendency to draw an armature current which is 15 to 20 times more than the full load current.

Such high current drawn by the armature at start is highly objectionable for the following reasons :

1. In a constant voltage system, such high inrush of current may cause tremendous line voltage fluctuations. This may affect the performance of the other equipments connected to the same line.
2. Such excessively high armature current, blows out the fuses.
3. If motor fails to start due to some problems with the field winding, then a large armature current flowing for a longer time may burn the insulation of the armature winding.
4. As the starting armature current is 10 to 15 times more than the full load current, the torque developed which is proportional to the I_a will also be 10 to 15 times, assuming shunt motor operation. So due to such high torque, the shaft and other accessories are thus be subjected to large mechanical stresses. These stresses may cause permanent mechanical damage to the motor.

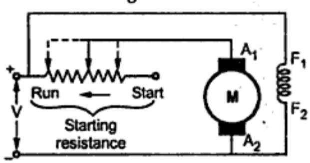


Fig. 2.56 Basic arrangement of a starter

To restrict this high starting armature current, a variable resistance is connected in series with the armature at start. This resistance is called **starter or a starting resistance.** So starter is basically a current limiting device. In the beginning the entire resistance is in the series with the armature and then gradually cut-off as motor gathers speed, producing the back e.m.f. The basic arrangement is shown in the Fig. 2.56.

In addition to the starting resistance, there are some protective devices provided in a starter. There are two types of starters used for d.c. shunt motors.

a) Three point starter

b) Four point starter

Let us see the details of three point starter.

2.36 Three Point Starter

The Fig. 2.57 shows this type of starter.

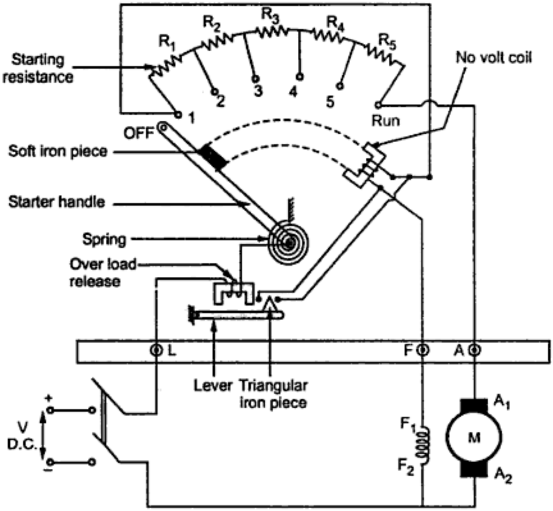


Fig. 2.57 Three point starter

The starter is basically a variable resistance, divided into number of sections. The contact points of these sections are called **studs** and brought out separately shown as OFF, 1, 2, ... upto RUN. There are three main points of this starter :

1. 'L' → Line terminal to be connected to positive of supply.
2. 'A' → To be connected to the armature winding.
3. 'F' → To be connected to the field winding.

Point 'L' is further connected to an electromagnet called **Overload Release** (OLR). The second end of 'OLR' is connected to a point where handle of the starter is pivoted. This handle is free to move from its other side against the force of the spring. This spring brings back the handle to the OFF position under the influence of its own force. Another parallel path is derived from the stud '1', given to the another electromagnet called **No Volt Coil** (NVC). The NVC is further connected to terminal 'F'. The starting resistance is entirely in series with the armature. The OLR and NVC are the two protecting devices of the starter.

Operation : Initially the handle is in the OFF position. The d.c. supply to the motor is switched on. Then handle is slowly moved against the spring force to make a contact with stud No. 1. At this point, field winding gets supply through the parallel path provided to starting resistance, through NVC. While entire starting resistance comes in series with the armature and armature current which is high at start, gets limited. As the handle is moved further, it goes on making contact with studs 2, 3, 4 etc., cutting out the starting resistance gradually from the armature circuit. Finally when the starter handle is in 'RUN' position, the entire starting resistance gets removed from the armature circuit and motor starts operating with normal speed. The handle is moved manually, and the obvious question is how handle will remain in the 'RUN' position, as long as motor is running ?

Let us see the action of NVC which will give the answer to this question along with some other functions of NVC.

2.36.1 Functions of No Volt Coil

1. The supply to the field winding is derived through NVC. So when field current flows, it magnetises the NVC. When the handle is in the 'RUN' position, soft iron piece connected to the handle gets attracted by the magnetic force produced by NVC. Design of NVC is such that it holds the handle in 'RUN' position against the force of the spring as long as supply to the motor is proper. Thus NVC holds the handle in the 'RUN' position and hence also called **hold on coil.**
2. Whenever there is supply failure or if field circuit is broken, the current through NVC gets affected. It looses its magnetism and hence not in a position to keep the soft iron piece on the handle, attracted. Under the spring force, handle comes back to OFF position, switching off the motor. So due to the combination of NVC and the spring, the starter handle always comes back to OFF position whenever there is any supply problem. The entire starting resistance comes back in series with the armature when attempt is made to start the motor everytime. This prevents the damage of the motor caused due to accidental starting.
3. NVC performs the similar action under low voltage conditions and protects the motor from such dangerous supply conditions as well.

2.36.2 Action of Over Load Release

The current through the motor is taken through the OLR, an electromagnet. Under overload condition, high current is drawn by the motor from the supply which passes through OLR. Below this magnet, there is an arm which is fixed at its fulcrum and normally resting in horizontal position. Under overloading, high current though OLR produces enough force of attraction to attract the arm upwards. Normally magnet is so designed that up to a full load value of current, the force of attraction produced is just enough to balance the gravitational force of the arm and hence not lifting it up. At the end of this arm, there is a triangular iron piece fitted. When the arm is pulled upwards the triangular piece touches to the two points which are connected to the two ends of NVC. This shorts the NVC and voltage across NVC becomes zero due to which NVC looses its magnetism. So under the spring force, handle comes back to the OFF position, disconnecting the motor from the supply. Thus motor gets saved from the overload conditions.

In this starter, it can be observed that as handle is moved from different studs one by one, the part of the starting resistance which gets removed from the armature circuit, gets added to the field circuit. As the value of starting resistance is very small as compared to the field winding resistance, this hardly affects the field winding performance. But this addition of the resistance in the field circuit can be avoided by providing a brass arc or copper arc connected just below the stud, the end of which is connected to NVC, as shown in the Fig. 2.58.

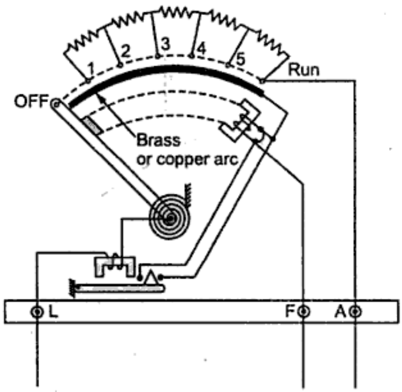


Fig. 2.58 Three point starter with brass arc

The handle moves over this arc, supplying the field current directly bypassing the starting resistance. When such an arc is provided, the connection used earlier to supply field winding, is removed.

2.36.3 Disadvantage

In this starter, the NVC and the field winding are in series. So while controlling the speed of the motor above rated, field current is reduced by adding an extra resistance in series with the field winding. Due to this, the current through NVC also reduces. Due to this, magnetism produced by NVC also reduces. This may release the handle from its RUN position switching off the motor. To avoid the dependency of NVC and the field winding, four point starter is used, in which NVC and the field winding are connected in parallel.

2.37 Four Point Starter

The basic difference between three point and four point starter is the connection of NVC. In three point, NVC is in series with the field winding while in four point starter NVC is connected independently across the supply through the fourth terminal called 'N' in addition to the 'L', 'F' and 'A'.

Hence any change in the field current does not affect the performance of the NVC. Thus it is ensured that NVC always produce a force which is enough to hold the handle in 'RUN' position, against force of the spring, under all the operating conditions. Such a current is adjusted through NVC with the help of fixed resistance R connected in series with the NVC using fourth point 'N' as shown in Fig. 2.59.

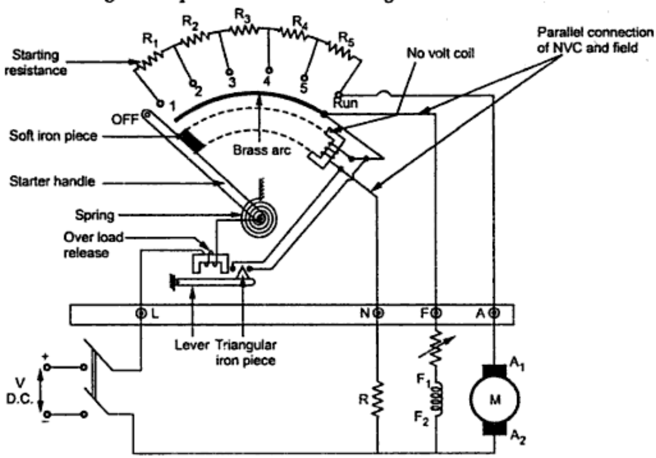


Fig. 2.59 Four point starter

2.37.1 Disadvantage

The only limitation of the four point starter is, it does not provide high speed protection to the motor. If under running condition, field gets opened, the field current reduces to zero. But there is some residual flux present and $N \propto \frac{1}{\phi}$ the motor tries to run with dangerously high speed. This is called **high speeding action** of the motor. In three point starter as NVC is in series with the field, under such field failure, NVC releases handle to the OFF position. But in four point starter NVC is connected directly across the supply and its current is maintained irrespective of the current through the field winding. Hence it always maintains handle in the RUN position, as long as supply is there. And thus it does not protect the motor from field failure conditions which result into the high speeding of the motor.

2.38 Losses in a D.C. Machine

The various losses in a d.c. machine whether it is a motor or a generator are classified into three groups as :

1. Copper losses
2. Iron or core losses
3. Mechanical losses.

2.38.1 Copper Losses

The copper losses are the losses taking place due to the current flowing in a winding. There are basically two windings in a d.c. machine namely armature winding and field winding. The copper losses are proportional to the square of the current flowing through these windings. Thus the various copper losses can be given by,

$$\boxed{\text{Armature copper loss} = I_a^2 R_a}$$

where R_a = Armature winding resistance

and I_a = Armature current

$$\boxed{\text{Shunt field copper loss} = I_{sh}^2 R_{sh}}$$

where R_{sh} = Shunt field winding resistance

and I_{sh} = Shunt field current

$$\boxed{\text{Series field copper loss} = I_{se}^2 R_{se}}$$

where R_{se} = Series field winding resistance

and I_{se} = Series field current

In a compound d.c. machine, both shunt and series field copper losses are present. In addition to the copper losses, there exists brush contact resistance drop. But this drop is usually included in the armature copper loss.

2.38.2 Iron or Core Losses

These losses are also called magnetic losses. These losses include hysteresis loss and eddy current loss.

The hysteresis loss is proportional to the frequency and the maximum flux density B_m in the air gap and is given by,

$$\boxed{\text{Hysteresis loss} = \eta B_m^{1.6} f V \text{ watts}}$$

where η = Steinmetz hystersis coefficient

V = Volume of core in m^3

f = Frequency of magnetic reversals

This loss is basically due to reversal of magnetisation of the armature core.

The eddy current loss exists due to eddy currents. When armature core rotates, it cuts the magnetic flux and e.m.f. gets induced in the core. This induced e.m.f. sets up eddy currents which cause the power loss. This loss is given by,

$$\boxed{\text{Eddy current loss} = K\, B_m^2 f^2\, t^2\, V \text{ watts}}$$

where K = Constant

t = Thickness of each lamination

V = Volume of core

f = Frequency of magnetic reversals

The hysteresis loss is minimised by selecting the core material having low hysteresis coefficient. While eddy current loss is minimised by selecting the laminated construction for the core.

These losses are almost constant for the d.c. machines.

2.28.3 Mechanical Losses

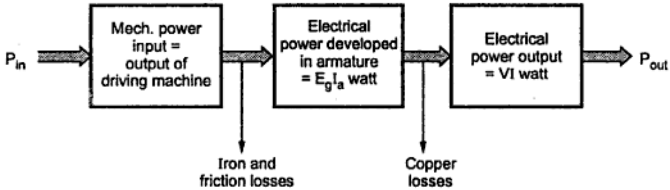
These losses consist of friction and windage losses. Some power is required to overcome mechanical friction and wind resistance at the shaft. This loss is nothing but the friction and windage loss. The mechanical losses are also constant for a d.c. machine.

The magnetic and mechanical losses together are called **stray losses.** For the shunt and compound d.c. machines where field current is constant, field copper losses are also constant. Thus stray losses along with constant field copper losses are called **constant losses.** While the armature current is dependent on the load and thus armature copper losses are called **variable losses.**

Thus for a d.c. machine,

$$\text{Total losses} = \text{Constant losses} + \text{Variable losses}$$

The power flow and energy transformation diagrams at various stages, which takes place in a d.c. machine are represented diagrammatically in Fig. 2.60 (a) and (b).



(a) Generator

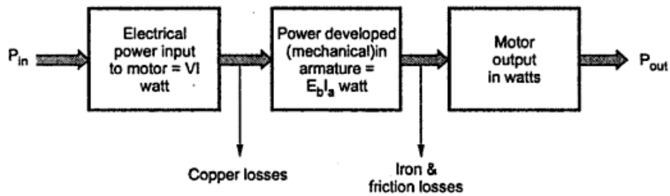


Fig. 2.60 (b) Motor

2.39 Efficiency of a D.C. Machine

For a d.c. machine, its overall efficiency is given by,

$$\% \eta = \frac{\text{Total output}}{\text{Total input}} \times 100$$

Let P_{out} = Total output of a machine

P_{in} = Total input of a machine

P_{cu} = Variable losses

P_i = Constant losses

then $P_{in} = P_{out} + P_{cu} + P_i$

$$\therefore \quad \% \eta = \frac{P_{out}}{P_{in}} \times 100 = \frac{P_{out}}{P_{out} + \text{losses}} \times 100$$

$$\therefore \quad \boxed{\% \eta = \frac{P_{out}}{P_{in} + P_{cu} + P_i} \times 100}$$

2.39.1 Condition for Maximum Efficiency

In case of a d.c. generator the output is given by,

$$P_{out} = VI$$

$$P_{cu} = \text{Variable losses} = I_a^2 R_a = I^2 R_a$$

$$I_a = I \qquad \text{... Neglecting shunt field current}$$

$$\therefore \quad \% \eta = \frac{VI}{VI + I^2 R_a + P_i} \times 100 = \frac{1}{1 + \left(\frac{I R_a}{V} + \frac{P_i}{VI}\right)} \times 100$$

The efficiency is maximum, when the denominator is minimum. According to maxima-minima theorem,

$$\frac{d}{dI}\left[1 + \left(\frac{I R_a}{V} + \frac{P_i}{VI}\right)\right] = 0$$

$$\therefore \quad \frac{R_a}{V} - \frac{P_i}{VI^2} = 0$$

$$\therefore \quad I^2 R_a - P_i = 0$$

$$\therefore \quad I^2 R_a = P_i = P_{cu}$$

Thus for the maximum efficiency, the condition is,

$$\boxed{\text{Variable losses} = \text{Constant losses}}$$

Let us study now the various methods of testing the d.c. motors from the losses and efficiency point of view.

Examples with Solutions

Example 2.15 : *A 4 pole, lap wound 750 r.p.m. d.c. shunt generator has an armature resistance of 0.4 Ω and field resistance of 200 Ω. The armature has 720 conductors and the flux per pole is 30 mWb. If the load resistance is 15 Ω, determine the terminal voltage.*

Solution : Consider the generator as shown in the Fig. 2.61.

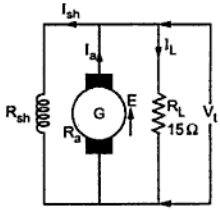


Fig. 2.61

$$P = 4, \quad A = P = 4$$

$$N = 750 \text{ r.p.m.}$$

$$\phi = 30 \text{ mWb} = 30 \times 10^{-3} \text{ Wb}, \; Z = 720$$

$$\therefore \quad E = \frac{\phi P N Z}{60 A} = \frac{30 \times 10^{-3} \times 4 \times 750 \times 720}{60 \times 4} = 270 \text{ V}$$

$$E = V_t + I_a R_a$$

Now $\quad V_t = I_L \times R_L \quad \text{i.e. } I_L = \frac{V_t}{R_L}$

And $\quad I_{sh} = \frac{V_t}{R_{sh}}$

$$I_a = I_L + I_{sh} = \frac{V_t}{R_L} + \frac{V_t}{R_{sh}}$$

Substituting in voltage equation,

$$E = V_t + \left[\frac{V_t}{R_L} + \frac{V_t}{R_{sh}} \right] R_a$$

$$\therefore \quad 270 = V_t + \left[\frac{V_t}{15} + \frac{V_t}{200} \right] 0.4$$

$$\therefore \quad 270 = 1.0286 \, V_t$$

$$\therefore \quad V_t = \mathbf{262.4757 \ V}$$

Example 2.16 : *A 440 V d.c. shunt motor takes a no load current of 2.5 A. The resistance of the shunt field and the armature are 550 Ω and 1.2 Ω respectively. The full load line current is 32 A. Find the full load output and the efficiency of the motor.*

Solution : No load current I = 2.5 A,

$$\text{No load input} = V.\,I = 440 \times 2.5 = 1100 \text{ W}$$

$$I_{sh} = \frac{V}{R_{sh}} = \frac{440}{550} = 0.8 \text{ A}$$

In d.c. shunt motor, $\quad I = I_{sh} + I_a$

$$\therefore \quad I_a = I - I_{sh} = 2.5 - 0.8 = 1.7 \text{ A}$$

$$\text{No load armature copper loss} = I_a^2 R_a = (1.7)^2 \times 1.2$$

$$= 3.468 \text{ watts}$$

Constant losses = No load input − No load armature Cu loss

$$= 1100 - 3.468$$

$$= 1096.532 \text{ W}$$

Now, full load line current i.e. $I = 32$ A

$$I = I_{sh} + I_a$$

$$I_a = I - I_{sh} = 32 - 0.8 = 31.2 \text{ A}$$

Full load armature copper loss $= I_a^2 \cdot R_a = (31.2)^2 \times 1.2 = 1168.128$ W

Total losse = Full load armature Cu loss + Constant losses

$$= 1168.128 + 1096.532 = 2264.66 \text{ W}$$

Full load motor input $= V \cdot I = 440 \times 32 = 14080$ W

Full load motor output = Input – Losses = 14080 – 2264.66 = 11815.34 W

$$\% \text{ efficiency at full load} = \frac{\text{Full load Output}}{\text{Full load Input}} \times 100 = \frac{11815.34}{14080} \times 100$$

$$= 83.91$$

$\therefore$ Efficiency of motor at full load = **83.91 %**

Example 2.17 : *A 250 V d.c. shunt motor has $R_a = 0.08\ \Omega$. When connected to 250 V d.c. supply it develops back e.m.f. of 242 V at 1500 r.p.m. Determine,*

i) Armature current ii) Armature current at start

iii) Back e.m.f. if armature current is changed to 120 A

iv) The speed of the machine if it is operated as a generator in order to deliver an armature current of 87 A at 250 V.

Solution : $R_a = 0.08\ \Omega$, $E_{b1} = 242$ V, V = 250 V

i) $$V = E_{b1} + I_{a1} R_a$$

$\therefore$ $$250 = 242 + I_{a1} \times 0.08$$

$\therefore$ $$I_{a1} = \mathbf{100\ A}$$

ii) At start, $N = 0$ hence $E_b = 0$

$\therefore$ $$I_{a(start)} = \frac{V}{R_a} = \frac{250}{0.08} = \mathbf{3125\ A}$$

iii) If $I_{a2} = 120$ A then

$$E_{b2} = V - I_{a2} R_a = 250 - 120 \times 0.08 = \mathbf{240.4\ V}$$

iv) Machine is running as a generator, shown in the Fig. 2.62.

Let induced e.m.f. as a generator be E_g.

$$E_g = V_t + I_a R_a = 250 + 87 \times 0.08$$
$$= 256.96 \text{ V}$$

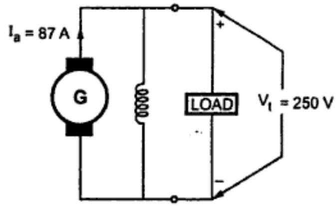


Fig. 2.62

In both cases as a motor or generator $E \propto N \phi$

As flux is constant, $E \propto N$

$$\therefore \quad \frac{E_b}{E_g} = \frac{N_m}{N_g}$$

where N_m = Speed as a motor

N_g = Speed as a generator

$$\therefore \quad \frac{242}{256.96} = \frac{1500}{N_g}$$

$$\therefore \quad N_g = \mathbf{1592.7 \ r.p.m.}$$

Example 2.18 : *A 200 V d.c. series motor drives a load at a certain speed and takes a current of 30 A. The resistance between its terminals is 1.5 Ω. Find the extra resistance to be added in series with the motor circuit to reduced the speed to 60 % of its original value. Assume that the torque produced is proportional to the cube of the speed.*

Solution : $V = 200$ V, $I_{a1} = 30$ A

Resistance across terminals = $R_a + R_{se} = 1.5 \ \Omega$

$$\therefore \quad E_{b1} = V - I_{a1}(R_a + R_{se})$$
$$= 200 - 30 \times 1.5 = 155 \text{ V}$$

$$N_2 = 0.6 \ N_1$$

$$\therefore \quad \frac{N_1}{N_2} = \frac{1}{0.6}$$

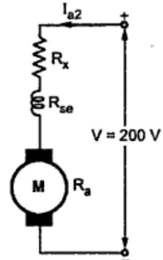


Fig. 2.63

Use torque equation,

$$T \propto \phi I_a \propto I_a^2$$

as ... $\phi \propto I_a$

$$\therefore \quad \frac{T_1}{T_2} = \left(\frac{I_{a1}}{I_{a2}}\right)^2 \qquad \text{... (1)}$$

Also $T \propto N^3$ given, $$\frac{T_1}{T_2} = \left(\frac{N_1}{N_2}\right)^3 = \left(\frac{1}{0.6}\right)^3 \qquad \text{... (2)}$$

Equating equation (1) and (2) , $\left(\frac{1}{0.6}\right)^3 = \left(\frac{30}{I_{a2}}\right)^2$

$$\therefore \quad I_{a2} = 13.9427 \text{ A}$$

$$\therefore \quad E_{b2} = V - I_{a2}(R_a + R_{se} + R_x)$$

$$= 200 - 13.9427\,(1.5 + R_x) \quad \dots (3)$$

Use speed equation, $N \propto \frac{E_b}{\phi} \propto \frac{E_b}{I_a}$ ... $\phi \propto I_a$

$$\therefore \quad \frac{N_1}{N_2} = \frac{E_{b1}}{E_{b2}} \times \frac{I_{a2}}{I_{a1}}$$

$$\therefore \quad \frac{1}{0.6} = \frac{155}{E_{b2}} \times \frac{13.9427}{30}$$

$$E_{b2} = 43.22 \text{ V} \quad \dots (4)$$

Equating equations (3) and (4), $43.22 = 200 - 13.9427\,(1.5 + R_x)$

$$\therefore \quad R_x = \mathbf{9.745\ \Omega}$$

Example 2.19 : *A 250 V d.c. shunt motor takes 4 A when running unloaded. The armature and field resistances are 0.3 Ω and 250 Ω respectively. Calculate the efficiency of the motor when on full load it takes a current of 60 A.*

Solution : No load current $= I_{L0} = 4$ A

$$I_{sh} = \frac{V}{R_{sh}} = \frac{250}{250} = 1 \text{ A}$$

$$\therefore \quad I_{a0} = I_{L0} - I_{sh} = 4 - 1 = 3 \text{ A}$$

$\therefore$ No. load armature copper loss $= I_{a0}^2 R_a = 3^2 \times 0.3 = 2.7$ W

No load input $= V I_{L0} = 250 \times 4 = 1000$ W

$\therefore$ Constant losses = No load input – No load armature copper loss

$= 1000 - 2.7 = 997.3$ W

On full load, $I_L = 60$ A and $I_{sh} = 1$ A

$$\therefore \quad I_a = I_L - I_{sh} = 59 \text{ A}$$

$\therefore$ Full load armature copper loss $= I_a^2 R_a = 59^2 \times 0.3 = 1044.3$ W

$\therefore$ Total loss on full load = Constant losses + $I_a^2 R_a$ loss

$= 997.3 + 1044.3 = 2041.6$ W

Total input on full load $= V I_L = 250 \times 60$

$\therefore \quad P_{in} = 15000$ W

$\therefore \quad P_{out} = P_{in} - \text{Total loss} = 15000 - 2041.6 = 12958.4$ W

$$\therefore \quad \% \eta = \frac{P_{out}}{P_{in}} \times 100 = \frac{12958.4}{15000} \times 100 = \mathbf{86.389\ \%}$$

Example 2.20 : *Calculate the flux in a 6 pole d.c. generator with 780 armature conductors, generating 500 V when running at 1000 r.p.m. if the armature is a) Lap wound b) Wave wound.* **[JNTU : Nov.-2005, (Set-1), May-2005, (Set-2)]**

Solution : $P = 6,\ Z = 780,\ E_g = 500$ V, $N = 1000$ r.p.m.

a) Lap wound, $A = P = 6$

$$\therefore \quad E_g = \frac{\phi PNZ}{60\ A} \quad \text{i.e. } 500 = \frac{\phi \times 6 \times 1000 \times 780}{60 \times 6}$$

$\therefore \quad \phi = \mathbf{0.03846\ Wb}$

b) Wave wound, $A = 2$

$$\therefore \quad E_g = \frac{\phi PNZ}{60\ A} \quad \text{i.e. } 500 = \frac{\phi \times 6 \times 1000 \times 780}{60 \times 2}$$

$\therefore \quad \phi = \mathbf{0.01282\ Wb}$

Example 2.21 : *Calculate the e.m.f. generated by a 6 pole lap wound armature with 65 slots and 12 conductors per slot, when driven at 1000 r.p.m. The flux per pole is 0.02 Wb.* **[JNTU : March 2006 (Set-1, set-4), Nov.-2005 (Set-3), May-2005 (Set-3)]**

Solution : $P = 6,\ \phi = 0.02$ Wb, $N = 1000$ r.p.m., $A = P$ as lap wound

$Z = \text{Slots} \times \text{conductors/slot} = 65 \times 12 = 780$

$$\therefore \quad E_g = \frac{\phi PNZ}{60\ A} = \frac{0.02 \times 6 \times 1000 \times 780}{60 \times 6} = \mathbf{260\ V}$$

Example 2.22 : *A 4 pole d.c. generator has a wave wound armature with 792 conductors. The flux per pole is 0.0121 Wb. Determine the speed at which it should be run to generate 240 V on no load.* **[JNTU : Nov.-2004 (Set-2), Nov.-2005 (Set-2)]**

Solution : $P = 4$, $A = 2$ as wave, $Z = 792$, $\phi = 0.0121$ Wb, $E_g = 240$ V

$$\therefore \quad E_g = \frac{\phi PNZ}{60\ A} \quad \text{i.e.} \quad 240 = \frac{0.0121 \times 4 \times N \times 792}{60 \times 2}$$

$\therefore \quad N = \mathbf{751.3148\ r.p.m.}$ ... Speed

Example 2.23 : *A 250 V, short shunt compound generator is delivering 80 A. The armature, series and shunt field resistances are 0.05, 0.03 and 100 Ω respectively. Calculate the induced voltage allowing a brush drop of 2 V.*
[JNTU : March-2006 (Set-3), May-2005 (Set-1)]

Solution : The generator is shown in the Fig. 2.64.

The current through R_{se} is $I_L = 80$ A as the generator is short shunt.

The drop across R_{sh} is the sum of the drop across R_{se} and V_t.

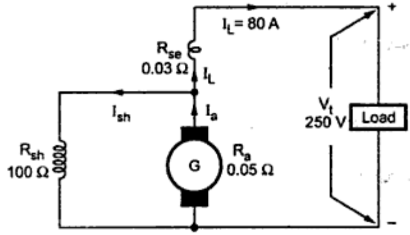


Fig. 2.64

$\therefore \quad I_{sh}R_{sh} = V_t + I_L R_{se}$

i.e. $\quad 100\, I_{sh} = 250 + 80 \times 0.03$

$\therefore \quad I_{sh} = 2.524$ A

$\therefore \quad I_a = I_L + I_{sh} = 80 + 2.524 = 82.524$ A

$\therefore \quad E_g = V_t + I_a R_a + I_L R_{se}$ + Brush drop

$= 250 + 82.524 \times 0.05 + 80 \times 0.03 + 2$

$= \mathbf{258.5262\ V}$ ... Induced e.m.f.

Example 2.24 : *A 4 pole, lap wound generator has 56 coils with 6 turns per coil. The speed is 1150 r.p.m. What must be the flux per pole in order to generator 265 V ? How many commutator bars are required for this generator ?* **[JNTU : May-2005 (Set-4)]**

Solution : $P = 4$, Lap hence $A = P$, $N = 1150$ r.p.m., $E_g = 265$ V

Total turns = Number of coils × Turns/coil = 56 × 6 = 336

$\therefore \quad Z = 2 \times$ total turns $= 2 \times 336 = 672$ ... 2 conductors → 1 turn

$$\therefore \quad E_g = \frac{\phi PNZ}{60\,A} \quad \text{i.e.} \quad 265 = \frac{\phi \times 4 \times 1150 \times 672}{60 \times 4}$$

$\therefore \quad \phi = \mathbf{0.02057\ Wb}$ ... Flux per pole

Number of commutator bars = Number of coils = **56.**

Example 2.25 : *When driven at 1000 r.p.m. with flux/pole of 0.02 Wb, a d.c. generator has an e.m.f. of 200 V. If the speed is increased to 1100 r.p.m. and at the same time flux/pole is reduced to 0.019 Wb/pole, what is the induced e.m.f. ?*
[JNTU : Nov.-2004 (Set-2), Nov.-2005 (Set-2)]

Solution : For a d.c. generator,

$$E_g = \frac{\phi PNZ}{60\,A} \quad \text{i.e.} \quad E_g \propto \phi N \qquad \text{... PZ/60 A constant}$$

$N_1 = 1000$ r.p.m., $\phi_1 = 0.02$ Wb, $E_{g1} = 200$ V, $N_2 = 1100$ r.p.m., $\phi_2 = 0.019$ Wb

$$\therefore \quad \frac{E_{g1}}{E_{g2}} = \frac{N_1}{N_2} \times \frac{\phi_1}{\phi_2} \quad \text{i.e.} \quad \frac{200}{E_{g2}} = \frac{1000}{1100} \times \frac{0.02}{0.019}$$

$$\therefore \quad E_{g2} = \mathbf{209\ V} \qquad \text{... New induced e.m.f.}$$

Example 2.26 : *A 4 pole, 240 V, wave connected shunt motor gives 11.19 kW when running at 1000 r.p.m. and drawing armature and field current of 50 A and 1 A respectively. It has 540 conductors. Its resistance is 0.1 Ω. Assuming a drop of 1 V per brush, calculate : a) Total torque b) Useful torque c) Useful flux/pole d) Rotational losses e) Efficiency.* **[JNTU : May-2005 (Set-4), Nov.-2004 (Set-3)]**

Solution : $P = 4$, $V = 240$ V, $A = 2$ as wave, $N = 1000$ r.p.m., $P_{out} = 11.19$ kW

$I_a = 50$ A, $I_{sh} = 1$ A, $R_a = 0.1\,\Omega$, $Z = 540$

a)
$$E_b = V - I_a R_a - \text{Brush drop}$$

$$E_b = 240 - 50 \times 0.1 - 2 \times 1 = 233\ \text{V}$$

$$\therefore \quad T = \frac{E_b I_a}{\omega} = \frac{E_b I_a}{\left(\frac{2\pi N}{60}\right)} = \frac{233 \times 50 \times 60}{2\pi \times 1000}$$

$$= \mathbf{111.2493\ Nm} \qquad \text{... Total torque}$$

b)
$$T_{sh} = \text{useful torque} = \frac{P_{out}}{\omega} = \frac{11.19 \times 10^3}{\left(\frac{2\pi \times 1000}{60}\right)}$$

$$= \mathbf{106.8566\ Nm}$$

c)
$$E_b = \frac{\phi PNZ}{60\ A} \quad \text{i.e.} \quad 233 = \frac{\phi \times 4 \times 1000 \times 540}{60 \times 2}$$

$$\therefore \quad \phi = \mathbf{0.013\ Wb} \qquad \text{... Flux per pole}$$

d)
$$\text{Rotational losses} = (T - T_{sh}) \times \omega = (T - T_{sh}) \times \frac{2\pi N}{60}$$

$$= (111.2493 - 106.8566) \times \frac{2\pi \times 1000}{60} = \mathbf{460\ W}$$

e)
$$P_{in} = VI_L = V(I_a + I_{sh}) = 240 \times 51 = 12240\ \text{W}$$

$$\therefore \quad \%\,\eta = \frac{P_{out}}{P_{in}} \times 100 = \frac{11.19 \times 10^3}{12240} \times 100 = \mathbf{91.42\ \%}$$

Example 2.27 : *A 250 V d.c. shunt motor on no load runs at 1000 r.p.m. and takes 5 A. The total armature and shunt field resistances are 0.2 Ω and 250 Ω respectively. Calculate the speed when loaded and taking a current of 50 A if armature reaction weakens the field by 3 %.* **[JNTU : March-2006 (Set-3, Set-4), Nov.-2005 (Set-1)]**

Solution : $V = 250\text{ V}, N_0 = 1000\text{ r.p.m.}, I_{L0} = 5\text{ A}, R_a = 0.2\,\Omega, R_{sh} = 250\,\Omega$

$$I_{sh} = \frac{V}{R_{sh}} = \frac{250}{250} = 1\text{ A}$$

$$\therefore \quad I_{a0} = I_{L0} - I_{sh} = 5 - 1 = 4\text{ A}$$

$$\therefore \quad E_{b0} = V - I_{a0}R_a = 250 - 4 \times 0.2 = 249.2\text{ V}$$

on load, $I_{L1} = 50\text{ A}, \ \phi_1 = \phi_0 - 3\ \%\text{ of }\phi_0 = 0.97\,\phi_0$

I_{sh} remains constant as long as V and R_{sh} are constant.

$$\therefore \quad I_{a1} = I_{L1} - I_{sh} = 50 - 1 = 49\text{ A}$$

$$\therefore \quad E_{b1} = V - I_{a1}R_a = 250 - 49 \times 0.2 = 240.2\text{ V}$$

$$N \propto \frac{E_b}{\phi} \qquad \text{... Speed equation}$$

$$\therefore \quad \frac{N_0}{N_1} = \frac{E_{b0}}{E_{b1}} \times \frac{\phi_1}{\phi_0} \quad \text{i.e.} \quad \frac{1000}{N_1} = \frac{249.2}{240.2} \times \frac{0.97\,\phi_0}{\phi_0}$$

$$\therefore \quad N_1 = \mathbf{993.6953\ r.p.m}$$

Example 2.28 : *Determine the value of torque in kgm developed by the armature of a 6 pole, wave wound motor having 492 conductors, 30 mWb flux per pole when the total armature current is 40 A.* **[JNTU : March-2006 (Set-2), May-2005 (Set-1)]**

Solution : $P = 6, \ A = 2 \text{ as wave}, \ Z = 492, \ \phi = 30\text{ mWb}, \ I_a = 40\text{ A}$

$$T = \frac{1}{2\pi}\phi I_a \frac{PZ}{A}\text{ Nm}$$

$$\therefore \quad T = \frac{1}{2\pi} \times 30 \times 10^{-3} \times 40 \times \frac{6 \times 492}{2} = 281.8952\text{ Nm}$$

$$\therefore \quad T = \frac{281.8952}{9.81}\text{ kgm} = \mathbf{28.7355\ kgm} \qquad \text{... } 1\text{ N} = \frac{1}{9.81}\text{ kg}$$

Example 2.29 : *A 4 pole machine running at 1000 r.p.m. has an armature with 80 slots having 8 conductors per slot. The flux per pole is 6×10^{-2} Wb. Determine the induced e.m.f. as a d.c. generator, if the coils are lap connected. If the current per conductor is 50 A, determine the electrical power output of the machine.* **[JNTU : Nov.-2007, (Set-1)]**

Solution : P = 4, N = 1000 r.p.m., 80 slots, $\phi = 6\times 10^{-2}$ Wb, 8 conductors/slot

$$A = P = 4 \qquad \text{... Lap connected}$$

$$Z = \text{Slots} \times \text{conductors/slot} = 80 \times 8 = 640$$

$$\therefore \quad E_g = \frac{\phi PNZ}{60A} = \frac{6\times 10^{-2}\times 4\times 1000\times 640}{60\times 4} = \mathbf{640\ V} \qquad \text{... Induced e.m.f.}$$

As coils are lap connected, parallel paths are 4 and all conductors in each parallel path are in series, carrying a current of 50 A.

$$\therefore \quad I_a = A \times \text{current per parallel path}$$

$$= 4 \times 50$$

$$= 200\ A$$

$$\therefore \quad P = \text{Electrical power output} = E_g \times I_a$$

$$= 640 \times 200 = \mathbf{128\ kW}$$

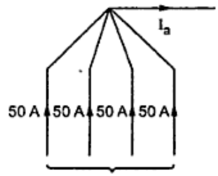


Fig. 2.65

Example 2.30 : *A long shunt compound generator delivers a load current of 50 A at 500 V and has armature, series field and shunt field resistances of 0.05 Ω, 0.03 Ω and 250 Ω respectively. Calculate the generated voltage and the armature current. Allow 1 V per brush for contact drop.* **[JNTU : Nov.-2007, (Set-2)]**

Solution : The generator is shown in the Fig. 2.66.

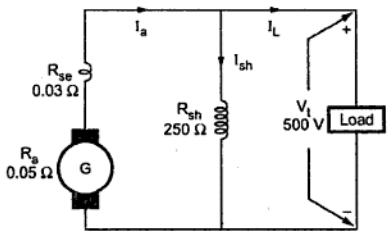


Fig. 2.66

$$I_L = 50\ A, \quad V_L = 500\ V \quad \text{... Given}$$

$$I_{sh} = \frac{V_t}{R_{sh}} = \frac{500}{250} = 2\ A$$

$$\therefore \quad I_a = I_L + I_{sh} = 2 + 50 = \mathbf{52\ A}$$

This is the armature current

$$\therefore \quad E_g = V_t + I_a R_a + I_a R_{se} + \text{Brush drop}$$

$$= 500 + 52 \times 0.05 + 52 \times 0.03 + 2 \times 1 = \mathbf{506.16\ V}$$

This is generated voltage.

Example 2.31 : *The armature of 8 pole d.c. generator has 960 conductors and runs at 400 r.p.m. The flux per pole is 40 mWb.*

a) Calculate the induced e.m.f., if armature is lap wound.

b) At what speed it must be driven to generate 400 V, if the armature is wave connected. **[JNTU : Nov.-2008 (Set-1)]**

Solution : $P = 8, \; N = 400 \text{ r.p.m.}, \; \phi = 40 \text{ mWb}, \; Z = 960$

a) Lap wound, $A = P = 8$

$$E_g = \frac{\phi PNZ}{60\,A} = \frac{40 \times 10^{-3} \times 8 \times 400 \times 960}{60 \times 8} = \mathbf{256\ V}$$

b) Wave connected, $A = 2$, $E_g = 400$ V

$$\therefore \quad E_g = \frac{\phi PNZ}{60\,A} \quad \text{i.e. } 400 = \frac{40 \times 10^{-3} \times 8 \times N \times 960}{60 \times 2}$$

$$\therefore \quad N = \mathbf{156.25\ r.p.m}$$

Example 2.32 : *A 250 V shunt motor takes a total current of 20 A. The shunt field and armature resistances are 200 Ω and 0.3 Ω respectively. Determine : a) The back e.m.f. b) Gross mechanical power developed.* **[JNTU : Nov.-2004 (Set-1), Nov.-2005, (Set-4)]**

Solution : $V = 250 \text{ V}, \; R_a = 0.3 \; \Omega, \quad I_L = 20 \text{ A}, \; R_{sh} = 200 \; \Omega$

$$\therefore \quad I_{sh} = \frac{V}{R_{sh}} = \frac{250}{200} = 1.25 \text{ A} \qquad \text{... Shunt motor}$$

$$\therefore \quad I_a = I_L - I_{sh} = 20 - 1.25 = 18.75 \text{ A}$$

a) $$E_b = V - I_a R_a = 250 - 18.75 \times 0.3 = \mathbf{244.375\ V}$$

b) $$P_m = E_b I_a = 244.375 \times 18.75 = \mathbf{4582.0312\ W}$$

Example 2.33 : *A 4 pole, long shunt, lap wound generator supplies 25 kW at a terminal voltage of 500 V. The armature resistance is 0.03 Ω, series field resistance is 0.04 Ω and shunt field resistance is 200 Ω. The brush drop may be taken as 1 V. Determine :*

a) The generated e.m.f. b) Copper and iron losses c) Efficiency at full load. **[JNTU : Nov.-2008, (Set-2)]**

Solution : The generator is shown in the Fig. 2.67.

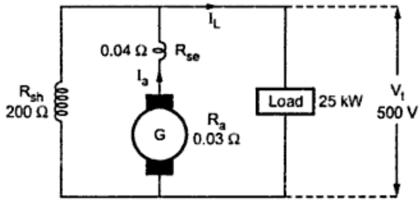


Fig. 2.67

$$I_{sh} = \frac{V_t}{I_{sh}} = \frac{500}{200} = 2.5 \text{ A}$$

$$I_L = \frac{P_L}{V_t} = \frac{25 \times 10^3}{500} = 50 \text{ A}$$

$$\therefore \quad I_a = I_L + I_{sh} = 52.5 \text{ A}$$

Brush drop is 1 V per brush hence total brush drop = 2 V

a) $$E_g = V_t + I_a R_a + I_a R_{se} + V_{brush} = 500 + 52.5\,(0.03 + 0.04) + 2$$

$$= \mathbf{505.675 \text{ V}} \quad \text{... Generated e.m.f.}$$

b) $$\text{Armature copper loss} = I_a^2 R_a = (52.5)^2 \times 0.03 = \mathbf{82.6875 \text{ W}}$$

$$\text{Series field copper loss} = I_a^2 R_{se} = (52.5)^2 \times 0.04 = \mathbf{110.25 \text{ W}}$$

$$\text{Shunt field copper loss} = I_{sh}^2 R_{sh} = (2.5)^2 \times 200 = \mathbf{1250 \text{ W}}$$

$$P_{in} = E_g \times I_a = 505.675 \times 52.5 = 26547.9375 \text{ W}$$

$$P_{out} = 25 \text{ kW}$$

$$\therefore \quad \text{Total losses} = P_{in} - P_{out} = 1547.9375 \text{ W}$$

$$\text{Now total losses} = \text{Copper losses} + \text{Iron losses}$$

$$\therefore \quad 1547.9375 = 82.6875 + 110.25 + 1250 + \text{Iron losses}$$

$$\therefore \quad \text{Iron losses} = \mathbf{105 \text{ W}}$$

c) $$\%\eta = \frac{P_{out}}{P_{in}} \times 100 = \frac{25 \times 10^3}{26547.9375} \times 100 = \mathbf{94.1692 \%}$$

Example 2.34 : *A 20 kW, 250 V d.c. shunt generator has armature and field resistance of 0.1 Ω and 125 Ω respectively. Calculate the total armature power developed when running i) As a generator delivering 20 kW output ii) As a motor taking 20 kW input.*

[JNTU : Nov.-2004, (Set-2), May-2005 (Set-3)]

Solution : i) As a generator

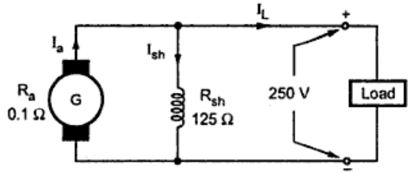


Fig. 2.68 (a)

$$P_{out} = 20 \text{ kW}, \quad V_t = 250 \text{ V}$$

$$\therefore \quad I_L = \frac{P_{out}}{V_t} = \frac{20\times 10^3}{250}$$

$$= 80 \text{ A}$$

$$I_{sh} = \frac{V_t}{R_{sh}} = \frac{250}{125} = 2 \text{ A}$$

$$\therefore \quad I_a = I_L + I_{sh} = 80 + 2 = 82 \text{ A}$$

$$E_g = V_t + I_a R_a = 250 + 82 \times 0.1 = 258.2$$

$$\therefore \quad P_g = E_g \times I_a = 258.2 \times 82 = \mathbf{21.172 \text{ kW}}$$ ... Armature power developed

ii) As a motor

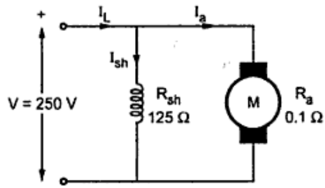


Fig. 2.68 (b)

$$P_{in} = 20 \text{ kW}, \quad V = 250 \text{ V}$$

$$\therefore \quad I_L = \frac{P_{in}}{V} = \frac{20\times 10^3}{250} = 80 \text{ A}$$

$$I_{sh} = \frac{V}{R_{sh}} = \frac{250}{125} = 2 \text{ A}$$

$$\therefore \quad I_a = I_L - I_{sh} = 80 - 2 = 78 \text{ A}$$

$$\therefore \quad E_b = V - I_a R_a = 250 - 78 \times 0.1 = 242.2 \text{ V}$$

$$\therefore \quad P_a = E_b I_a = 242.2 \times 78 = \mathbf{18.8916 \text{ kW}}$$ ... Armature power developed

Example 2.35 : *A 4 pole d.c. generator runs at 750 r.p.m. and generates an e.m.f. of 240 V. The armature is wave wound and has 792 conductors. If the total flux per pole is 0.0145 Wb, what is the leakage coefficient ?* **[JNTU : Nov.-2004 (Set-3)]**

Solution : $P = 4$, $N = 750$ r.p.m., $E_g = 240$ V, $A = 2$ as wave, $Z = 792$

$$E_g = \frac{\phi PNZ}{60 A} \quad \text{i.e. } 240 = \frac{\phi \times 4 \times 750 \times 792}{60 \times 2}$$

$$\therefore \quad \phi = 0.01212 \text{ Wb}$$ ... Useful flux per pole

$$\boxed{\lambda = \text{Leakage coefficient} = \frac{\text{Total flux}}{\text{Useful flux}}}$$

$$\therefore \quad \lambda = \frac{0.0145}{0.01212} = \mathbf{1.196}$$

Example 2.36 : *A 1500 kW, 550 V, 10 pole generator runs at 150 r.p.m. There are 2500 lap connected conductors and the full load copper losses are 25 kW. The air gap flux density has a uniform value of 0.9 Wb/m². Calculate : a) The no load terminal voltage b) The area of the pole shoe.* **[JNTU : Nov.-2004 (Set-3)]**

Solution :

$$P_{out} = 1500 \text{ kW}, \; V_t = 550 \text{ V}, \; P = 10, \; A = P \text{ as lap}$$

$$N = 150 \text{ r.p.m.}, \; Z = 2500, \; P_{cu} = 25 \text{ kW}, \; B = 0.9 \text{ Wb/m}^2$$

$$I_L = \frac{P_{out}}{V_t} = \frac{1500 \times 10^3}{550} = 2727.2727 \text{ A}$$

As R_{sh} is not given, neglect I_{sh} hence $\mathbf{I_a = 2727.2727 \text{ A}}$

a) $$P_{cu} = \text{Armature copper loss} = I_a^2 R_a$$

$\therefore$ $$25 \times 10^3 = (2727.2727)^2 \times R_a \quad \text{i.e.} \quad R_a = 0.003361 \; \Omega$$

$\therefore$ $$E_g = V_t + I_a R_a = 550 + 2727.2727 \times 3.3611 \times 10^{-3} = \mathbf{559.1667 \text{ V}}$$

The is no load terminal voltage.

b) $$E_g = \frac{\phi PNZ}{60 A} \quad \text{i.e.} \quad 559.1667 = \frac{\phi \times 10 \times 150 \times 2500}{60 \times 10}$$

$\therefore$ $$\phi = 0.08946 \text{ Wb}$$

Now $$B = \frac{\phi}{A} \quad \text{i.e.} \quad 0.9 = \frac{0.08946}{A}$$

$\therefore$ $$\mathbf{A = 0.0994 \text{ m}^2} \quad \text{... Area of pole shoe}$$

Review Questions

1. *Explain with a neat sketch, the construction of a d.c. machine.*
2. *Which part of a d.c. machine is laminated ? Why ?*
3. *What is the difference between lap type and wave type of armature winding ?*
4. *Derive from first principles an expression for the e.m.f. of a d.c generator.*
5. *State the different types of d.c. generators and state the applications of each type.*
6. *In a particular d.c. machine, if P = 8, Z = 400, N = 300 r.p.m. and ϕ = 100 mWb, calculate generated e.m.f. if winding is connected in (i) Lap fashion (ii) Wave fashion.*
   (Ans. : (i) 200 V (ii) 800 V)
7. *A 110 V, d.c. shunt generator delivers a load of 50 A. The armature resistance is 0.2 Ω and field resistance is 55 Ω. The generator is driven at 1800 r.p.m. It has 6 poles with 360 conductors connected in lap fashion. Calculate*
   i) The no load voltage (ii) The flux per pole. **(Ans. : (i) 120.4 V (ii) 0.011 Wb)**

8. *A long shunt compound generator delivers a load current of 50 A at 500 V. It has armature, series field and shunt field resistance of 0.05 Ω, 0.03 Ω and 250 Ω respectively. Calculate the generated e.m.f.* **(Ans. : 504.16 V)**
9. *A d.c. machine has 8 poles, lap connected armature with 960 conductors and flux per pole is 40 mWb. It is driven at 400 r.p.m. Calculate the generated e.m.f. If now lap connected armature is replaced by wave connected, calculate the speed at which it should be driven to generate 400 V.* **(Ans. : 250 V, 156 r.p.m.)**
10. *A 4 pole, 100 V d.c. shunt generator with lap connected armature having field and armature resistances of 50 Ω and 0.1 Ω respectively, supplied 60, 100 V, 40 W lamps. All lamps are connected in parallel. Calculate the total armature current and generated e.m.f. Assume brush drop to be 1 V/brush.* **(Ans. : 26 A, 104.6 V)**
11. *A short shunt compound d.c. generator supplied 7.5 kW at 230 V. The shunt field, series field and armature resistances are 100 Ω, 0.3 Ω and 0.4 Ω respectively. Calculate the induced e.m.f. and the load resistance.* **(Ans. : 253.8 V, 7 Ω)**
12. *What is the difference between a generator and a motor?*
13. *Explain the principle of working of a d.c. motor.*
14. *State the voltage and power equation of a d.c. motor explaining the importance of each term.*
15. *What is back e.m.f ? Explain the significance of a back e.m.f.*
16. *Derive the expression for the electromagnetic torque developed in a d.c. motor.*
17. *Sketch and explain the speed-current, speed-torque and toque-current characteristics of a shunt motor, series motor and compound motor.*
18. *Why a d.c. series motor cannot be started on no load ?*
19. *Explain the suitability of a d.c. series motor for a traction.*
20. *Explain clearly the necessity of a starter for a d.c. shunt motor.*
21. *Draw a neat sketch and explain a 3 point starter.*
22. *State the significance of no volt coil and the over load release in the operation of a starter.*
23. *Stating the advantages, explain a 4 point starter used for a d.c. shunt motor.*
24. *A 440 volts d.c. shunt motor has an armature resistance of 0.5 Ω and field resistance of 180 Ω. Determine back e.m.f. when giving an E output of 7.46 kW at 85 % efficiency.* **(Ans. : 431.25 V)**
25. *A 240 V d.c. shunt motor has an armature resistance of 0.2 Ω. The rated full load current is 80 A. Calculate the current at the instant of starting if the motor is started direct on line and express it in terms of the full load current. Calculate the value of starting resistance to limit the starting current to 150 % of rated current.* **(Ans. : 1200 A, 0.133 Ω)**
26. *A d.c. shunt machine has rating of 10 kW, 500 V. The armature resistance is 0.1 Ω. Calculate the induced e.m.f. if machine is made to run as i) Generator ii) Motor.* **(Ans. : 502.5 V, 499.85 V)**
27. *A d.c. shunt motor connected to a 230 V supply takes a line current of 12 A, at some load. If $R_a = 1\ \Omega$ and $R_{sh} = 230\ \Omega$, calculate the back e.m.f.* **(Ans. : 219 V)**

28. *A 4 pole, 250 V, series motor has a wave connected armature with 1254 conductors. The flux per pole is 22 mWb when motor is taking 50 A. Armature and field resistance are 0.2 and 0.2 Ω respectively. Calculate its speed.* **(Ans. : 250.1 r.p.m.)**

29. *Determine the total torque developed in a 250 V, 4 pole d.c. shunt motor with lap winding, accomodated in 60 slots, each containing 20 conductors. The armature current is 50 A and the flux per pole is 23 mWb.* **(Ans. : 219.5 N-m)**

30. *A 250 V, d.c. shunt motor takes 6 A line current on no load and runs at 1000 r.p.m. The field resistance is 250 Ω and armature resistance is 0.2 Ω. If the full load line current is 26 A, calculate the full load speed assuming constant flux.* **(Ans. : 984 r.p.m.)**

31. *A 4 pole, d.c. shunt motor takes 22.5 A from a 250 V supply. R_a = 0.5 Ω and R_{sh} = 125 Ω. The armature is wave wound with 300 conductors. If the flux per pole is 0.02 Wb, calculate i) Speed ii) Torque developed iii) Gross power developed.* **(Ans. : 1199 r.p.m, 39.1 N-m, 4.915 kW)**

❑❑❑

3 Transformers

3.1 Introduction

The main advantage of alternating currents over direct currents is that, the alternating currents can be easily transfe rable from low voltage to high or high voltage to low. Alternating voltages can be raised or lowered as per requirements in the different stages of electrical network as generation, transmission, distribution and utilization. This is possible with a static device called **transformer**. The transformer works on the principle of mutual induction. It transfers an electric energy from one circuit to other when there is no electrical connection between the two circuits. Thus we can define transformer as below :

> **Key Point** : *The transformer is a static piece of apparatus by means of which an electrical power is transformed from one alternating current circuit to another with the desired change in voltage and current, without any change in the frequency.*

The use of transformers in transmission system is shown in the Fig. 3.1.

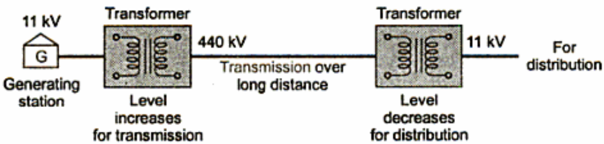


Fig. 3.1 Use of transformers in transmission system

3.2 Principle of Working

The principle of **mutual induction** states that when two coils are inductively coupled and if current in one coil is changed uniformly then an e.m.f. gets induced in the other coil. This e.m.f. can drive a current, when a closed path is provided to it. The transformer works on the same principle. In its elementary form, it consists of two inductive coils which are electrically separated but linked through a common magnetic circuit. The two coils have high mutual inductance. The basic transformer is shown in the Fig. 3.2.

One of the two coils is connected to a source of alternating voltage. This coil in which electrical energy is fed with the help of source is called **primary winding (P)**. The other winding is connected to load. The electrical energy transformed to this winding is drawn out to the load.

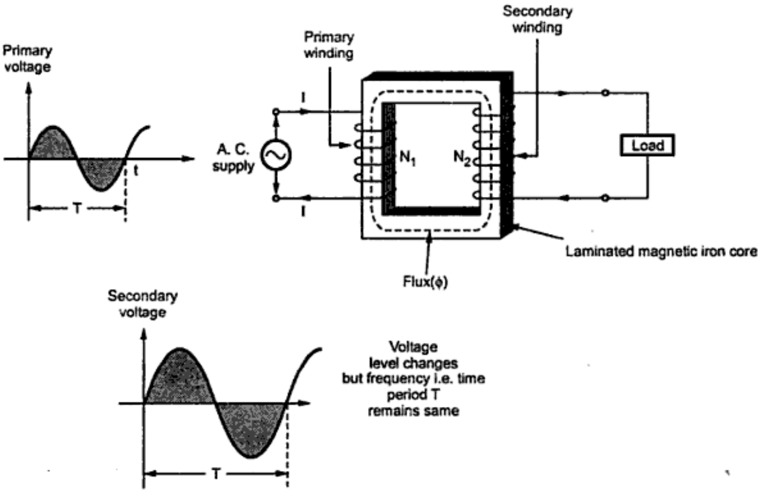


Fig. 3.2 Basic transformer

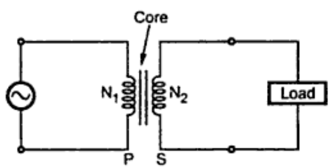


Fig. 3.3 Symbolic representation

This winding is called **secondary winding (S)**. The primary winding has N_1 number of turns while the secondary winding has N_2 number of turns. Symbolically the transformer is indicated as shown in the Fig. 3.3.

When primary winding is excited by an alternating voltage, it circulates an alternating current. This current produces an alternating flux (ϕ) which completes its path through common magnetic core as shown dotted in the Fig. 3.2. Thus an alternating, flux links with the secondary winding. As the flux is alternating, according to Faraday's law of an electromagnetic induction, mutually induced e.m.f. gets developed in the secondary winding. If now load is connected to the secondary winding, this e.m.f. drives a current through it.

Thus though there is no electrical contact between the two windings, an electrical energy gets transferred from primary to the secondary.

Key Point: *The frequency of the mutually induced e.m.f. is same as that of the alternating source which is supplying energy to the primary winding.*

3.2.1 Can D.C. Supply be used for Transformers ?

The d.c. supply can not be used for the transformers.

The transformer works on the principle of mutual induction, for which current in one coil must change uniformly. If d.c. supply is given, the current will not change due to constant supply and transformer will not work.

Practically winding resistance is very small. For d.c., the inductive reactance X_L is zero as d.c. has no frequency. So total impedance of winding is very low for d.c. Thus winding will draw very high current if d.c. supply is given to it. This may cause the burning of windings due to extra heat generated and may cause permanent damage to the transformer.

There can be saturation of the core due to which transformer draws very large current from the supply when connected to d.c.

Thus d.c. supply should not be connected to the transformers.

3.3 Construction

There are two basic parts of a transformer i) Magnetic core ii) Winding or coils.

The core of the transformer is either square or rectangular in size. It is further divided into two parts. The vertical portion on which coils are wound is called **limb** while the top and bottom horizontal portion is called **yoke** of the core. These parts are shown in the Fig. 3.4 (a).

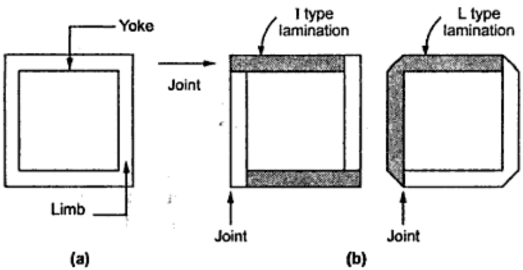


Fig. 3.4 Construction of transformer

Core is made up of laminations. Because of laminated type of construction, eddy current losses get minimised. Generally high grade silicon steel laminations [0.3 to 0.5 mm thick] are used. These laminations are insulated from each other by using insulation like varnish. All laminations are varnished. Laminations are overlapped so that to avoid the air gap at the joints. For this generally 'L' shaped or 'I' shaped laminations are used which are shown in the Fig. 3.4 (b).

The cross-section of the limb depends on the type of coil to be used either circular or rectangular. The different cross-sections of limbs, practically used are shown in the Fig. 3.5.

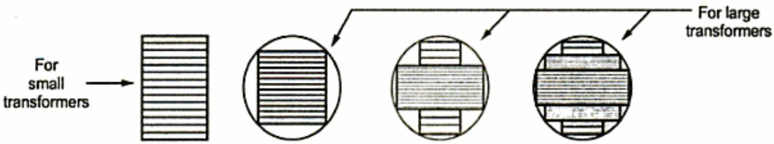


Fig. 3.5 Different cross-sections

3.3.1 Types of Windings

The coils used are wound on the limbs and are insulated from each other. In the basic transformer shown in the Fig. 3.2, the two windings wound are shown on two different limbs i.e. primary on one limb while secondary on other limb. But due to this leakage flux increases which affects the transformer performance badly. Similarly it is necessary that the windings should be very closes to each other to have high mutual inductance. To achieve this, the two windings are split into number of coils and are wound adjacent to each other on the same limb. A very common arrangement is cylindrical concentric coils as shown in the Fig. 3.6.

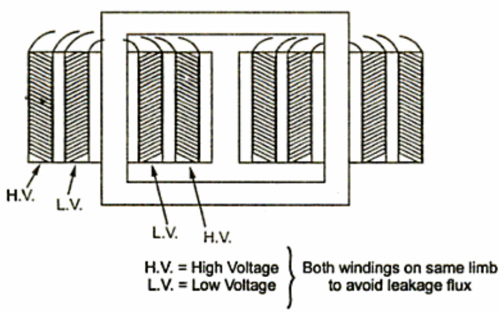


Fig. 3.6 Cylindrical concentric coils

Such cylindrical coils are used in the core type transformer. These coils are mechanically strong. These are wound in the helical layers. The different layers are insulated from each other by paper, cloth or mica. The low voltage winding is placed near the core from ease of insulating it from the core. The high voltage is placed after it.

The other type of coils which is very commonly used for the shell type of transformer is **sandwich coils.** Each high voltage portion lies between the two low voltage portion sandwiching the high voltage portion. Such subdivision of windings into small portions reduces the leakage flux. Higher the degree of subdivision, smaller is the reactance. The **sandwich coil** is shown in the Fig. 3.7. The top and bottom coils are low voltage coils. All the portions are insulated from each other by paper.

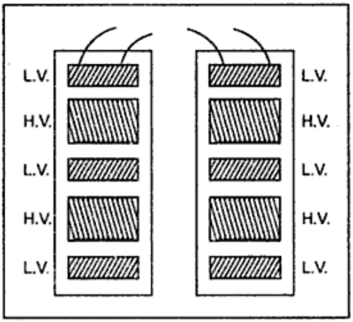


Fig. 3.7 Sandwich coils

3.4 Types of Transformers

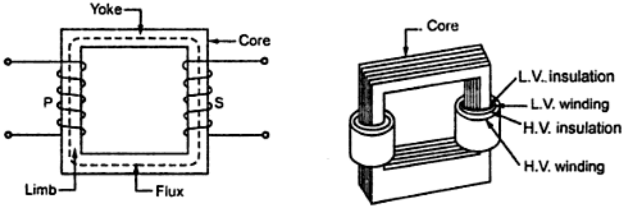
The classification of the transformers is based on the relative arrangement or disposition of the core and the windings. There are three main types of the transformers which are :

1. Core type 2. Shell type and 3. Berry type

3.4.1 Core Type Transformer

It has a single magnetic circuit. The core is rectangular having two limbs. The winding encircles the core. The coils used are of cylindrical type. As mentioned earlier, the coils are wound in helical layers with different layers insulated from each other by paper or mica. Both the coils are placed on both the limbs. The low voltage coil is placed inside near the core while high voltage coil surrounds the low voltage coil. Core is made up of large number of thin laminations.

As the windings are uniformly distributed over the two limbs the natural cooling is more effective. The coils can be easily removed by removing the laminations of the top yoke, for maintenance.



(a) Representation (b) Construction

Fig. 3.8 Core type transformer

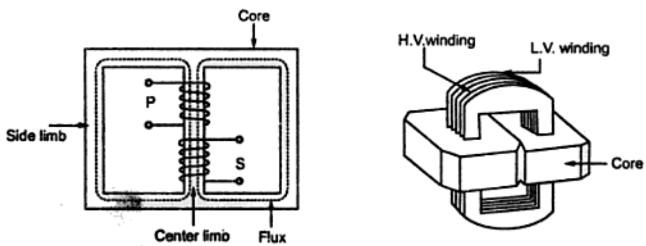
The Fig. 3.8 (a) shows the schematic representation of the core type transformer while the Fig. 3.8 (b) shows the view of actual construction of the core type transformer.

3.4.2 Shell Type Transformer

It has a double magnetic circuit. The core has three limbs. Both the windings are placed on the central limb. The core encircles most part of the windings. The coils used are generally multilayer disc type or sandwich coils. As mentioned earlier, each high voltage coil is in between two low voltage coils and low voltage coils are nearest to top and bottom of the yokes.

The core is laminated. While arranging the laminations of the core, the care is taken that all the joints at alternate layers are staggered. This is done to avoid narrow air gap at the joint, right through the cross-section of the core. Such joints are called over lapped or imbricated joints. Generally for very high voltage transformers, the shell type construction is preferred. As the windings are surrounded by the core, the natural cooling does not exist. For removing any winding for maintenance, large number of laminations are required to be removed.

The Fig. 3.9 (a) shows the schematic representation while the Fig. 3.9 (b) shows the outaway view of the construction of the shell type transformer.



(a) Representation (b) Construction

Fig. 3.9 Shell type transformer

3.4.3 Berry Type Transformer

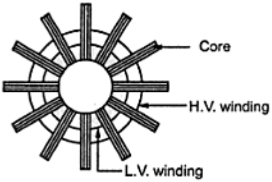


Fig. 3.10 Berry type transformer

This has distributed magnetic circuit. The number of independent magnetic circuits are more than 2. Its core construction is like spokes of a wheel. Otherwise it is symmetrical to that of shell type.

Diagramatically it can be shown as in the Fig. 3.10.

The transformers are generally kept in tightly fitted sheet metal tanks. The tanks are constructed of specified high quality steel plate cut, formed and welded into the rigid structures. All the joints are painted with a solution of light blue chalk which turns dark in the presence of oil, disclosing even the minutest leaks. The tanks are filled with the special insulating oil. The entire transformer assembly is immersed in the oil. The oil serves two functions : i) Keeps the coils cool by circulation and ii) Provides the transformers an additional insulation.

The oil should be absolutely free from alkalies, sulphur and specially from moisture. Presence of very small moisture lowers the dielectric strength of oil, affecting its performance badly. Hence the tanks are sealed air tight to avoid the contact of oil with atmospheric air and moisture. In large transformers, the chambers called **breathers** are provided. The breathers prevent the atmospheric moisture to pass on to the oil. The breathers contain the silica gel crystals which immediately absorb the atmospheric moisture. Due to long and continuous use, the sludge is formed in the oil which can contaminate the oil. Hence to keep such sludge separate from the oil in main tank, an air tight metal drum is provided, which is placed on the top of tank. This is called **conservator.**

3.4.4 Comparison of Core and Shell Type

Sr. No.	Core Type	Shell Type
1.	The winding encircles the core.	The core encircles most part of the winding.
2.	It has single magnetic circuit.	It has a double magnetic circuit.
3.	The core has two limbs.	The core has three limbs.
4.	The cylindrical coils are used.	The multilayer disc or sandwich type coils are used.
5.	The windings are uniformly distributed on two limbs hence natural cooling is effective.	The natural cooling does not exist as the windings are surrounded by the core.
6.	The coils can be easily removed from maintenance point of view.	The coils can not be removed easily.
7.	Preferred for low voltage transformers.	Preferred for high voltage transformers.

3.5 E.M.F. Equation of a Transformer

When the primary winding is excited by an alternating voltage V_1, it circulates alternating current, producing an alternating flux ϕ. The primary winding has N_1 number of turns. The alternating flux ϕ linking with the primary winding itself induces an e.m.f. in it denoted as E_1. The flux links with secondary winding through the common magnetic core. It produces induced e.m.f. E_2 in the secondary winding. This is mutually induced e.m.f. Let us derive the equations for E_1 and E_2.

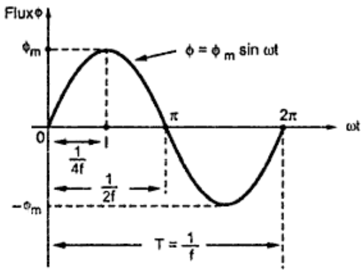


Fig. 3.11 Sinusoidal flux

The primary winding is excited by purely sinusoidal alternating voltage. Hence the flux produced is also sinusoidal in nature having maximum value of ϕ_m as shown in the Fig. 3.11.

The various quantities which affect the magnitude of the induced e.m.f. are :

ϕ = Flux

ϕ_m = Maximum value of flux

N_1 = Number of primary winding turns

N_2 = Number of secondary winding turns

f = Frequency of the supply voltage

E_1 = R.M.S. value of the primary induced e.m.f.

E_2 = R.M.S. value of the secondary induced e.m.f.

From Faraday's law of electromagnetic induction the average e.m.f. induced in each turn is proportional to the average rate of change of flux.

$\therefore$ Average e.m.f. per turn = Average rate of change of flux

$$\therefore \quad \text{Average e.m.f. per turn} = \frac{d\phi}{dt}$$

$$\text{Now} \quad \frac{d\phi}{dt} = \frac{\text{Change in flux}}{\text{Time required for change in flux}}$$

Consider the $1/4^{th}$ cycle of the flux as shown in the Fig. 3.11. Complete cycle gets completed in $1/f$ seconds. In $1/4^{th}$ time period, the change in flux is from 0 to ϕ_m.

$$\therefore \quad \frac{d\phi}{dt} = \frac{\phi_m - 0}{\left(\frac{1}{4f}\right)} \quad \text{as dt for } 1/4^{th} \text{ time period is } 1/4f \text{ seconds}$$

$$= \quad 4 \text{ f } \phi_m \text{ Wb/sec}$$

$\therefore$ **Average e.m.f. per turn** $= \quad 4 \text{ f } \phi_m$ volts

As ϕ is sinusoidal, the induced e.m.f. in each turn of both the windings is also sinusoidal in nature. For sinusoidal quantity,

$$\text{Form factor} = \frac{\text{R.M.S. value}}{\text{Average value}} = 1.11$$

$$\therefore \quad \text{R.M.S. value} = 1.11 \times \text{Average value}$$

$\therefore$ R.M.S. value of induced e.m.f. per turn

$$= 1.11 \times 4 \text{ f } \phi_m$$

$$= 4.44 \text{ f } \phi_m$$

There are N_1 number of primary turns hence the R.M.S. value of induced e.m.f. of primary denoted as E_1 is,

$$E_1 = N_1 \times 4.44 \text{ f } \phi_m \quad \text{volts}$$

While as there are N_2 number of secondary turns the R.M.S value of induced e.m.f. of secondary denoted E_2 is,

$$E_2 = N_2 \times 4.44 \text{ f } \phi_m \quad \text{volts}$$

The expressions of E_1 and E_2 are called e.m.f. equations of a transformer.

Thus e.m.f. equations are,

$$\boxed{E_1 = 4.44 \text{ f } \phi_m N_1 \quad \text{volts}} \quad \dots (1)$$

$$\boxed{E_2 = 4.44 \text{ f } \phi_m N_2 \quad \text{volts}} \quad \dots (2)$$

3.6 Ratios of a Transformer

Consider a transformer shown in Fig. 3.12 indicating various voltages and currents.

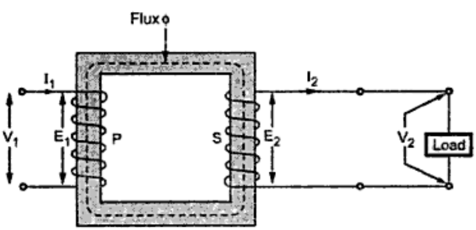


Fig. 3.12 Ratios of transformer

3.6.1 Voltage Ratio

We know from the e.m.f. equations of a transformer that

$$E_1 = 4.44 \, f \, \phi_m \, N_1 \quad \text{and} \quad E_2 = 4.44 \, f \, \phi_m \, N_2$$

Taking ratio of the two equations we get,

$$\boxed{\frac{E_2}{E_1} = \frac{N_2}{N_1} = K}$$

This ratio of secondary induced e.m.f. to primary induced e.m.f. is known as voltage transformation ratio denoted as K.

Thus,
$$\boxed{E_2 = K \, E_1 \qquad \text{where} \quad K = \frac{N_2}{N_1}}$$

1. If $N_2 > N_1$ i.e. $K > 1$, we get $E_2 > E_1$ then the transformer is called **step-up transformer.**
2. If $N_2 < N_1$ i.e. $K < 1$, we get $E_2 < E_1$ then the transformer is called **step-down transformer.**
3. If $N_2 = N_1$ i.e. $K = 1$, we get $E_2 = E_1$ then the transformer is called **isolation transformer** or **1:1 transformer.**

3.6.2 Ideal Transformer

A transformer is said to be ideal if it satisfies following properties :

i) It has no losses.

ii) Its windings have zero resistance.

iii) Leakage flux is zero i.e. 100 % flux produced by primary links with the secondary.

iv) Permeability of core is so high that negligible current is required to establish the flux in it.

Key Point: *For an ideal transformer, the primary applied voltage V_1 is same as the primary induced e.m.f. E_1 as there are no voltage drops.*

Similarly the secondary induced e.m.f. E_2 is also same as the terminal voltage V_2 across the load. Hence for an **ideal transformer** we can write,

$$\boxed{\frac{E_2}{E_1} = \frac{V_2}{V_1} = K}$$

No transformer is ideal in practice but the value of E_1 is almost equal to V_1 for properly designed transformer.

3.6.3 Current Ratio

For an ideal transformer there are no losses. Hence the product of primary voltage V_1 and primary current I_1, is same as the product of secondary voltage V_2 and the secondary current I_2.

So $\qquad V_1 I_1 = \text{input VA} \quad \text{and} \quad V_2 I_2 = \text{output VA}$

For an ideal transformer,

$$V_1 I_1 = V_2 I_2$$

$$\therefore \quad \boxed{\frac{V_2}{V_1} = \frac{I_1}{I_2} = K}$$

Key Point: *Hence the currents are in the inverse ratio of the voltage transformation ratio.*

3.6.4 Volt-Ampere Rating

When electrical power is transferred from primary winding to secondary there are few power losses in between. These power losses appear in the form of heat which increase the temperature of the device. Now this temperature must be maintained below certain limiting value as it is always harmful from insulation point of view. As current is the main cause in producing heat, the output maximum rating is generally specified as the product of output voltage and output current i.e. $V_2 I_2$. This always indicates that when transformer is operated under this specified rating, its temperature rise will not be excessive. The copper losses depend on current and iron losses depend on voltage. These losses are independent of the load power factor $\cos\phi_2$. Hence though the output power depends on $\cos\phi_2$, the transformer losses are functions of V and I and the rating of the transformer is specified as the product of voltage and current called **VA rating.** This rating is generally expressed in kVA (kilo volt amperes rating).

Now $$\frac{V_1}{V_2} = \frac{I_2}{I_1} = K$$

$$\therefore \quad V_1 I_1 = V_2 I_2$$

$$\boxed{\text{kVA rating of a transformer} = \frac{V_1 I_1}{1000} = \frac{V_2 I_2}{1000}}$$

If V_1 and V_2 are the terminal voltages of primary and secondary then from specified kVA rating we can decide full load currents of primary and secondary, I_1 and I_2. This is the **safe maximum current limit** which may carry, keeping temperature rise below its limiting value.

$$I_1 \text{ full load} = \frac{\text{kVA rating} \times 1000}{V_1} \quad \text{... (1000 to convert kVA to VA)}$$

$$I_2 \text{ full load} = \frac{\text{kVA rating} \times 1000}{V_2}$$

Key Point: *The full load primary and secondary currents indicate the safe maximum values of currents which transformer windings can carry.*

These values indicate, how much maximum load can be connected to a given transformer of a specified kVA rating.

Example 3.1 : *The maximum flux density in the core of 240/2400 V, 50 Hz, single phase transformer is 1 Wb/m². If the e.m.f. per turn is 8 V, determine :*

i) The primary and secondary turns and ii) Area of the core [Nov.-2008 (Set - 1)]

Solution : $B_m = 1\ \text{Wb/m}^2$, $E_1 = 240$ V, E/turn = 8 V, f = 50 Hz

$$E_1 = E/\text{turn} \times N_1 \quad \text{i.e.} \quad 240 = 8 \times N_1$$

$$\therefore \quad N_1 = \mathbf{30}$$

$$\therefore \quad \frac{N_1}{N_2} = \frac{E_1}{E_2} \quad \text{i.e.} \quad \frac{30}{N_2} = \frac{240}{2400}$$

$$N_2 = \mathbf{300}$$

$$E_1 = 4.44\ \phi_m\ f\ N_1 \quad \text{i.e.} \quad 240 = 4.44\ \phi_m \times 50 \times 30$$

$$\therefore \quad \phi_m = 0.03636\ \text{Wb}$$

$$\text{And} \quad B_m = \frac{\phi_m}{a} \quad \text{i.e.} \quad a = \frac{\phi_m}{B_m} = \mathbf{0.03636\ m^2} \quad \text{...Area}$$

Example 3.2 : *The primary winding of a 50 Hz single phase transformer has 480 turns and is fed from 5400 V supply. The secondary winding has 20 turns. Find the peak value of the flux in the core and the secondary voltage.* [Nov.- 2006 (Set-1)]

Solution : f = 50 Hz , $N_1 = 480$, $N_2 = 20$, $E_1 = 5400$ V

$$E_1 = 4.44\ f\ \phi_m\ N_1 \quad \text{i.e.} \quad 5400 = 4.44 \times 50 \times \phi_m \times 480$$

$$\therefore \quad \phi_m = \mathbf{0.0506\ Wb} \quad \text{...Peak value of flux}$$

$$\frac{E_1}{E_2} = \frac{N_1}{N_2} \quad \text{i.e.} \quad E_2 = \frac{N_2}{N_1} \times E_1 = \frac{20}{480} \times 5400$$

$$\therefore \quad E_2 = \mathbf{225\ V} \quad \text{...Secondary voltage}$$

3.7 Ideal Transformer on No Load

Consider an ideal transformer on no load as shown in the Fig. 3.13. The supply voltage is V_1 and as it is an no load the secondary current $I_2 = 0$.

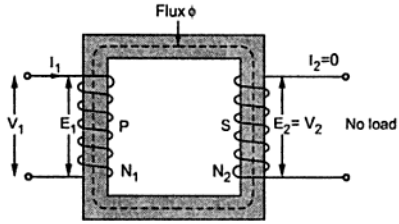


Fig. 3.13 Ideal transformer on no load

The primary draws a current I_1 which is just necessary to produce flux in the core. As it is magnetising the core, it is called **magnetising current** denoted as I_m. As the transformer is ideal, the winding resistance is zero and it is purely inductive in nature. The magnetising current I_m is very small and lags V_1 by 30° as the winding is purely inductive. This I_m produces an alternating flux ϕ which is in phase with I_m .

The flux links with both the winding producing the induced e.m.f.s E_1 and E_2 in the primary and secondary windings respectively. According to Lenz's law, the induced e.m.f. opposes the cause producing it which is supply voltage V_1. Hence E_1 is in antiphase with V_1 but equal in magnitude. The induced E_2 also opposes V_1 hence in antiphase with V_1 but its magnitude depends on N_2. Thus E_1 and E_2 are in phase.

The phasor diagram for the ideal transformer on no load is shown in the Fig. 3.14.

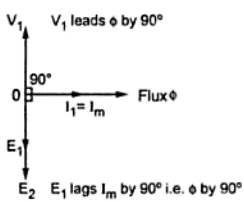


Fig. 3.14 Phasor diagram for ideal transformer on no load

It can be seen that flux ϕ is reference. I_m produces ϕ hence in phase with ϕ. V_1 leads I_m by 90° as winding is purely inductive so current has to lag voltage by 90°.

E_1 and E_2 are in phase and both opposing supply voltage V_1.

The power input to the transformer is $V_1 I_1 \cos (V_1 \wedge I_1)$ i.e. $V_1 I_m \cos (90°)$ i.e. zero. This is because on no load output power is zero and for ideal transformer there are no losses hence input power is also zero. Ideal no load p.f. of transformer is zero lagging.

3.8 Practical Transformer on No Load

Actually in practical transformer iron core causes hysteresis and eddy current losses as it is subjected to alternating flux. While designing the transformer the efforts are made to keep these losses minimum by,

1. Using high grade material as silicon steel to reduce hysteresis loss.
2. Manufacturing core in the form of laminations or stacks of thin laminations to reduce eddy current loss.

Apart from this there are iron losses in the practical transformer. Practically primary winding has certain resistance hence there are small primary copper loss present.

Thus the primary current under no load condition has to supply the iron losses i.e. hysteresis loss and eddy current loss and a small amount of primary copper loss. This current is denoted as I_o.

Now the no load input current I_o has two components :

1. A purely reactive component I_m called magnetising component of no load current required to produce the flux. This is also called **wattless component.**
2. An active component I_c which supplies total losses under no load condition called **power component** of no load current. This is also called **wattful component** or **core loss component** of I_o.

The total no load current I_o is the vector addition of I_m and I_c.

$$\boxed{\bar{I}_o = \bar{I}_m + \bar{I}_c} \quad \dots (1)$$

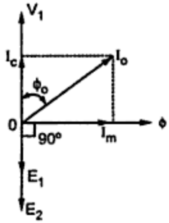


Fig. 3.15 Practical transformer on no load

In practical transformer, due to winding resistance, no load current I_o is no longer at 90° with respect to V_1. But it lags V_1 by angle ϕ_o which is less than 90°. Thus $\cos \phi_o$ is called **no load power factor** of practical transformer.

The phasor diagram is shown in the Fig. 3.15. It can be seen that the two components of I_o are,

$$\boxed{I_m = I_o \sin \phi_o} \quad \text{... (2)}$$

This is magnetising component lagging V_1 exactly by 90°.

$$\boxed{I_c = I_o \cos \phi_o} \quad \text{... (3)}$$

This is core loss component which is in phase with V_1.

The magnitude of the no load current is given by,

$$\boxed{I_o = \sqrt{I_m^2 + I_c^2}} \quad \text{... (4)}$$

while ϕ_o = No load primary power factor angle

The total power input on no load is denoted as W_o and is given by,

$$\boxed{W_o = V_1 I_o \cos \phi_o = V_1 I_c} \quad \text{... (5)}$$

It may be noted that the current I_o is very small, about 3 to 5 % of the full load rated current. Hence the primary copper loss is negligibly small hence I_c is called core loss or iron loss component. Hence power input W_o on no load always represents the iron losses, as copper loss is negligibly small. The iron losses are denoted as P_i and are constant for all load conditions.

$$\therefore \quad \boxed{W_o = V_1 I_o \cos \phi_o = P_i = \text{Iron loss}} \quad \text{... (6)}$$

➡ **Example 3.3 :** *The no load current of a transformer is 10 A at a power factor of 0.25 lagging, when connected to 400 V, 50 Hz supply. Calculate,*

a) Magnetising component of no load current

b) Iron loss and c) Maximum value of flux in the core.

Assume primary winding turns as 500.

Solution : The given values are, I_o = 10 A, $\cos \phi_o$ = 0.25, V_1 = 400 V and f = 50 Hz

a) $I_m = I_o \sin \phi_o$ = Magnetising component

$\phi_o = \cos^{-1}(0.25) = 75.522°$

$\therefore \quad I_m = 10 \times \sin(75.522°) = \mathbf{9.6824\ A}$

b) P_i = Iron loss = Power input on no load

$= W_o = V_1 I_o \cos \phi_o = 400 \times 10 \times 0.25$

$= \mathbf{1000\ W}$

c) On no load, $E_1 = V_1 = 400$ V and $N_1 = 500$

Now $E_1 = 4.44\ f\ \phi_m\ N_1$

$\therefore \quad 400 = 4.44 \times 50 \times \phi_m \times 500$

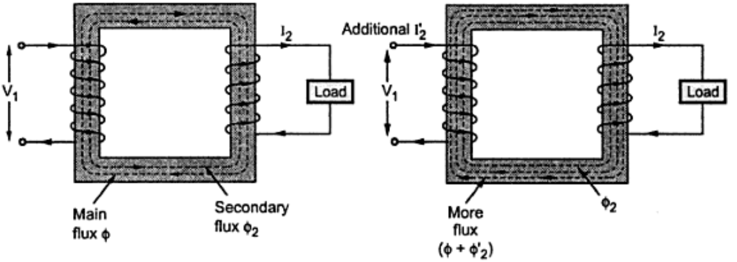
$\therefore \quad \phi_m =$ **3.6036 mWb**

3.9 Transformer on Load

When the transformer is loaded, the current I_2 flows through the secondary winding. The magnitude and phase of I_2 is determined by the load. If load is inductive, I_2 lags V_2. If load is capacitive, I_2 leads V_2 while for resistive load, I_2 is in phase with V_2.

There exists a secondary m.m.f. $N_2\ I_2$ due to which secondary current sets up its own flux ϕ_2. This flux opposes the main flux ϕ which is produced in the core due to magnetising component of no load current. Hence the m.m.f. $N_2 I_2$ is called **demagnetising ampere-turns**. This is shown in the Fig. 3.16 (a).

The flux ϕ_2 momentarily reduces the main flux ϕ, due to which the primary induced e.m.f. E_1 also reduces. Hence the vector difference $\overline{V}_1 - \overline{E}_1$ increases due to which **primary draws more current from the supply.** This additional current drawn by primary is due to the load hence called load component of primary current denoted as I_2' as shown in the Fig. 3.16 (b).



(a) ϕ_2 opposes ϕ

(b) Primary draws more current

Fig. 3.16 Transformer on load

This current I_2' is in antiphase with I_2. The current I_2' sets up its own flux ϕ_2' which opposes the flux ϕ_2 and helps the main flux ϕ. This flux ϕ_2' neutralises the flux ϕ_2 produced by I_2. The m.m.f. i.e. ampere turns $N_1\ I_2'$ balances the ampere turns $N_2 I_2$. Hence the net flux in the core is again maintained at constant level.

Key Point : *Thus for any load condition, no load to full load the flux in the core is practically constant.*

The load component current I_2' always neutralises the changes in the load. As practically flux in core is constant, the core loss is also constant for all the loads. Hence the transformer is called **constant flux machine**.

As the ampere turns are balanced we can write,

$$N_2 I_2 = N_1 I_2'$$

$$\therefore \quad \boxed{I_2' = \frac{N_2}{N_1} I_2 = K\, I_2} \quad \text{... (1)}$$

Thus when transformer is loaded, the primary current I_1 has two components :

1. The no load current I_o which lags V_1 by angle ϕ_o. It has two components I_m and I_c.
2. The load component I_2' which is in antiphase with I_2. And phase of I_2 is decided by the load.

Hence primary current I_1 is vector sum of I_o and I_2'.

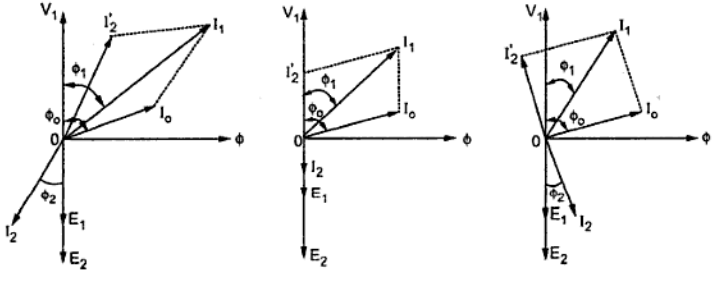
$$\therefore \quad \boxed{\bar{I}_1 = \bar{I}_o + \bar{I}_2'} \quad \text{... (2)}$$

Assume inductive load, I_2 lags E_2 by ϕ_2, the phasor diagram is shown in the Fig. 3.17 (a).

Assume purely resistive load, I_2 in phase with E_2, the phasor diagram is shown in the Fig. 3.17 (b).

Assume capacitive load, I_2 leads E_2 by ϕ_2, the phasor diagram is shown in the Fig. 3.17 (c).

Note that I_2' is always in antiphase with I_2.



(a) Inductive load (b) Resistive load (c) Capacitive load

Fig. 3.17

Actually the phase of I_2 is with respect to V_2 i.e. angle ϕ_2 is angle between I_2 and V_2. For the ideal case, E_2 is assumed equal to V_2 neglecting various drops.

The current ratio can be verified from this discussion. As the no load current I_o is very small, neglecting I_o we can write,

$$I_1 \equiv I_2'$$

Balancing the ampere-turns,

$$N_1 I_2' = N_1 I_1 = N_2 I_2$$

$$\therefore \quad \frac{N_2}{N_1} = \frac{I_1}{I_2} = K$$

Under full load conditions when I_o is very small compared to full load currents, the ratio of primary and secondary current is constant.

Example 3.4 : *A 400/200 V transformer takes 1 A at a power factor of 0.4 on no load. If the secondary supplies a load current of 50 A at 0.8 lagging power factor, calculate the primary current.*

Solution : The given values are,

$$I_o = 1\text{ A},\ \cos\phi_o = 0.4,\ I_2 = 50\text{ A and } \cos\phi_2 = 0.8$$

$$K = \frac{E_2}{E_1} = \frac{200}{400} = 0.5$$

$$\therefore \quad I_2' = KI_2 = 0.5 \times 50 = 25\text{ A}$$

The angle of I_2' is to be decided from $\cos\phi_2 = 0.8$.

Now $\quad \cos\phi_2 = 0.8$

$$\therefore \quad \phi_2 = 36.86°$$

I_2' is in antiphase with I_2 which lags E_2 by 36.86°

Consider the phasor diagram shown in the Fig. 3.18. The flux ϕ is the reference.

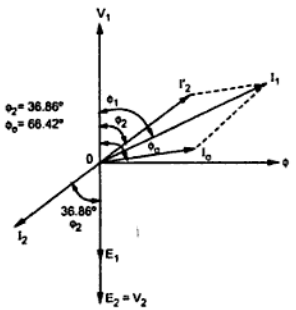


Fig. 3.18

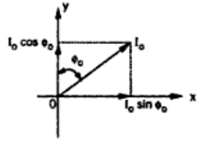


Fig. 3.18 (a)

Now $\cos \phi_o = 0.4$

$\therefore \quad \phi_o = 66.42°$

$$\bar{I}_1 = \bar{I}_2' + \bar{I}_o \qquad \text{... Vector sum}$$

Resolve I_o and I_2' into two components, along reference ϕ and in quadrature with ϕ in phase with V_1.

$$\text{x component of } I_o = I_o \sin \phi_o = 0.9165 \text{ A}$$

$$\text{y component of } I_o = I_o \cos \phi_o = 0.4 \text{ A}$$

$$\therefore \quad \bar{I}_o = 0.9165 + j\,0.4 \text{ A}$$

$$\text{x component of } I_2' = I_2' \sin \phi_2 = 25 \sin (36.86°) = 15 \text{ A}$$

$$\text{y component of } I_2' = I_2' \cos \phi_2 = 25 \times 0.8 = 20 \text{ A}$$

$$\therefore \quad I_2' = 15 + j\,20 \text{ A}$$

$$\bar{I}_1 = 0.9165 + j\,0.4 + 15 + j\,20 = 15.9165 + j\,20.4 \text{ A}$$

Thus the two components of I_1 are as shown in the Fig. 3.18 (c).

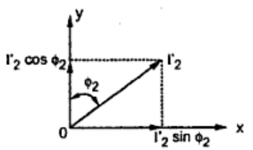


Fig. 3.18 (b)

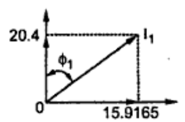


Fig. 3.18 (c)

$$\therefore \quad I_1 = \sqrt{(15.9165)^2 + (20.4)^2} = \mathbf{25.874 \text{ A}}$$

This is the primary current magnitude.

while $\tan \phi_1 = \dfrac{15.9165}{20.4}$

$$\therefore \quad \phi_1 = 37.96°$$

Hence the primary power factor is,

$$\cos \phi_1 = \cos (37.96°) = \mathbf{0.788 \text{ lagging}}$$

> **Key Point** : *Remember that ϕ_1 is angle between V_1 and I_1 and as V_1 is vertical, ϕ_1 is measured with respect V_1. So do not convert rectangular to polar as it gives angle with respect to x-axis and we want it with respect to y-axis.*

3.10 Effect of Winding Resistances

A practical transformer windings possess some resistances which not only cause the power losses but also the voltage drops. Let us see what is the effect of winding resistances on the performance of the transformer.

Let R_1 = Primary winding resistance in ohms

R_2 = Secondary winding resistance in ohms

Now when current I_1 flows through primary, there is voltage drop $I_1 R_1$ across the winding. The supply voltage V_1 has to supply this drop. Hence primary induced e.m.f. E_1 is the vector difference between V_1 and $I_1 R_1$.

$$\therefore \quad \bar{E}_1 = \bar{V}_1 - \overline{I_1 R_1} \quad \text{... (1)}$$

Similarly the induced e.m.f. in secondary is E_2. When load is connected, current I_2 flows and there is voltage drop $I_2 R_2$. The e.m.f. E_2 has to supply this drop. The vector difference between E_2 and $I_2 R_2$ is available to the load as a terminal voltage V_2.

$$\therefore \quad \bar{V}_2 = \bar{E}_2 - \overline{I_2 R_2} \quad \text{... (2)}$$

The drops $I_1 R_1$ and $I_2 R_2$ are purely resistive drops hence are always in phase with the respective currents I_1 and I_2.

3.10.1 Equivalent Resistance

The resistance of the two windings can be transferred to any one side either primary or secondary without affecting the performance of the transformer. The transfer of the resistances on any one side is advantageous as it makes the calculations very easy. Let us see how to transfer the resistances on any one side.

The total copper loss due to both the resistances can be obtained as,

$$\text{Total copper loss} = I_1^2 R_1 + I_2^2 R_2$$

$$= I_1^2 \left[R_1 + \frac{I_2^2}{I_1^2} R_2 \right]$$

$$= I_1^2 \left[R_1 + \frac{1}{K^2} R_2 \right] \quad \text{... (3)}$$

where $\frac{I_2}{I_1} = \frac{1}{K}$ Neglecting no load current.

Now the expression (3) indicates that the total copper loss can be expressed as $I_1^2 R_1 + I_1^2 \cdot \frac{R_2}{K^2}$. This means $\frac{R_2}{K^2}$ is the resistance value of R_2 shifted to primary side which causes same copper loss with I_1 as R_2 causes with I_2. This value of resistance R_2/K^2 which is the value of R_2 referred to primary is called **equivalent resistance of secondary referred to primary**. It is denoted as R_2'.

$$\therefore \quad \boxed{R_2' = \frac{R_2}{K^2}} \quad \text{... (4)}$$

Hence the total resistance referred to primary is the addition of R_1 and R'_2 called equivalent resistance of transformer referred to primary and denoted as R_{1e} .

$$\therefore \quad \boxed{R_{1e} = R_1 + R'_2 = R_1 + \frac{R_2}{K^2}} \quad \text{... (5)}$$

This resistance R_{1e} causes same copper loss with I_1 as the total copper loss due to the individual windings.

$$\therefore \text{ Total copper loss } = I_1^2 R_{1e} = I_1^2 R_1 + I_2^2 R_2 \quad \text{...(6)}$$

So equivalent resistance R_{1e} simplifies the calculations as we have to calculate parameters on one side only.

Similarly it is possible to refer the equivalent resistance to secondary winding.

$$\text{Total copper loss } = I_1^2 R_1 + I_2^2 R_2$$

$$= I_2^2 \left[\frac{I_1^2}{I_2^2} R_1 + R_2 \right]$$

$$= I_2^2 [K^2 R_1 + R_2] \quad \text{... (7)}$$

Thus the resistance $K^2 R_1$ is primary resistance referred to secondary denoted as R'_1.

$$\therefore \quad R'_1 = K^2 R_1 \quad \text{... (8)}$$

Hence the total resistance referred to secondary is the addition of R_2 and R'_1 called **equivalent resistance of transformer referred to secondary** and denoted as R_{2e}.

$$\therefore \quad \boxed{R_{2e} = R_2 + R'_1 = R_2 + K^2 R_1} \quad \text{... (9)}$$

$$\therefore \quad \text{Total copper loss } = I_2^2 R_{2e} \quad \text{... (10)}$$

The concept of equivalent resistance is shown in the Fig. 3.19 (a), (b) and (c).

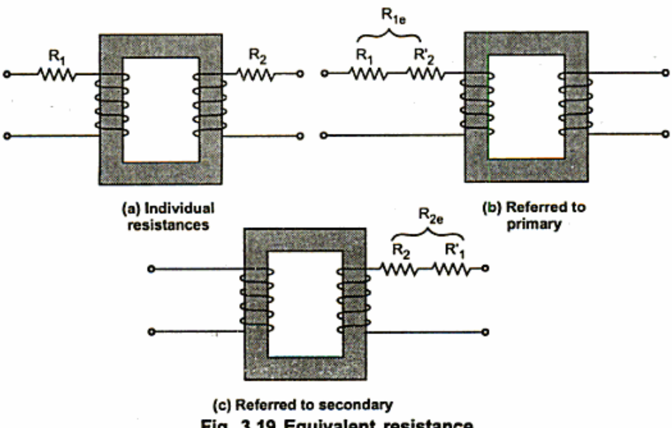


Fig. 3.19 Equivalent resistance

Key Point: *When resistances are transferred to primary, the secondary winding becomes zero resistance winding for calculation purpose. The entire copper loss occurs due to R_{1e}. Similarly when resistances are referred to secondary, the primary becomes resistanceless for calculation purpose. The entire copper loss occurs due to R_{2e}.*

Important Note : When a resistance is to be transferred from the primary to secondary, it must be multiplied by K^2. When a resistance is to be transferred from the secondary to primary, it must be divided by K^2. Remember that K is N_2/N_1.

The result can be cross-checked by another approach. The high voltage winding is always low current winding and hence the resistance of high voltage side is high. The low voltage side is high current side and hence resistance of low voltage side is low. So while transferring resistance from low voltage side to high voltage side, its value must increase while transferring resistance from high voltage side to low voltage side, its value must decrease.

Key Point :

High voltage side → Low current side → High resistance side
Low voltage side → High current side → Low resistance side

Example 3.5 : *A 6600/400 V single phase transformer has primary resistance of 2.5 Ω and secondary resistance of 0.01 Ω. Calculate total equivalent resistance referred to primary and secondary.*

Solution : The given values are,

$$R_1 = 2.5 \ \Omega, \qquad R_2 = 0.01 \ \Omega$$

$$K = \frac{400}{6600} = 0.0606$$

While finding equivalent resistance referred to primary, transfer R_2 to primary as R_2',

$$\therefore \quad R_2' = \frac{R_2}{K^2} = \frac{0.01}{(0.0606)^2} = 2.7225 \ \Omega$$

$$\therefore \quad R_{1e} = R_1 + R_2' = 2.5 + 2.7225 = \mathbf{5.2225 \ \Omega}$$

It can be observed that primary is high voltage hence high resistance side hence while transferring R_2 from low voltage to R_2' on high voltage, its value increases.

To find total equivalent resistance referred to secondary, first calculate R_1',

$$R_1' = K^2 R_1 = (0.0606)^2 \times 2.5 = 0.00918 \ \Omega$$

$$\therefore \quad R_{2e} = R_2 + R_1' = 0.01 + 0.00918 = \mathbf{0.01918 \ \Omega}$$

3.11 Effect of Leakage Reactances

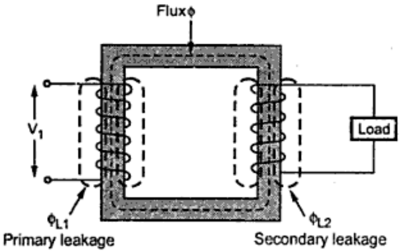


Fig. 3.20 Leakage fluxes

Uptill now it is assumed that the entire flux produced by the primary links with the secondary winding. But in practice it is not possible. Part of the primary flux as well as the secondary flux completes the path through air and links with the respecting winding only. Such a flux is called **leakage flux.** Thus there are two leakage fluxes present as shown in the Fig. 3.20.

The flux ϕ_{L1} is the primary leakage flux which is produced due to primary current I_1. It is in phase with I_1 and links with primary only.

The flux ϕ_{L2} is the secondary leakage flux which is produced due to current I_2. It is in phase with I_2 and links with the secondary winding only.

Due to leakage flux ϕ_{L1} there is self induced e.m.f. e_{L1} in primary. While due to leakage flux ϕ_{L2} there is self induced e.m.f. e_{L2} in secondary. The primary voltage V_1 has to overcome this voltage e_{L1} to produce E_1 while induced e.m.f. E_2 has to overcome e_{L2} to produce terminal voltage V_2. Thus the self induced e.m.f.s are treated as the voltage drops across the fictitious reactances placed in series with the windings. These reactances are called leakage reactances of the winding.

So X_1 = Leakage reactance of primary winding

and X_2 = Leakage reactance of secondary winding

The value of X_1 is such that the drop I_1X_1 is nothing but the self induced e.m.f. e_{L1} due to flux ϕ_{L1}. The value of X_2 is such that the drop I_2X_2 is equal to the self induced e.m.f. e_{L2} due to flux ϕ_{L2}.

Leakage fluxes link with the respective windings only and not to both the windings. To reduce the leakage, as mentioned, in the construction both the winding's are placed on same limb rather than on separate limbs.

3.11.1 Equivalent Leakage Reactance

Similar to the resistances, the leakage reactances also can be transferred from primary to secondary or viceversa. The relation through K^2 remains same for the transfer of reactances as it is studied earlier for the resistances.

Let X_1 is leakage reactance of primary and X_2 is leakage reactance of secondary.

Then the total leakage reactance referred to primary is X_{1e} given by,

$$\boxed{X_{1e} = X_1 + X_2' \quad \text{where} \quad X_2' = \frac{X_2}{K^2}}$$

While the total leakage reactance referred to secondary is X_{2e} given by,

$$\boxed{X_{2e} = X_2 + X_1' \quad \text{where} \quad X_1' = K^2 X_1}$$

And $$K = \frac{N_2}{N_1} = \text{Transformation ratio}$$

3.12 Equivalent Impedance

The transformer primary has resistance R_1 and reactance X_1. While the transformer secondary has resistance R_2 and reactance X_2. Thus we can say that the total impedance of primary winding is Z_1 which is,

$$Z_1 = R_1 + j X_1 \ \Omega \quad \ldots (1)$$

And the total impedance of the secondary winding is Z_2 which is ,

$$Z_2 = R_2 + j X_2 \ \Omega \quad \ldots (2)$$

This is shown in the Fig. 3.21.

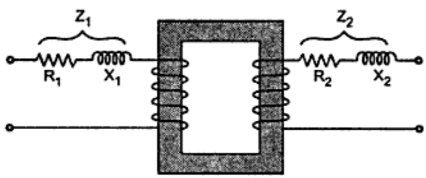


Fig. 3.21 Individual impedances

The individual magnitudes of Z_1 and Z_2 are,

$$Z_1 = \sqrt{R_1^2 + X_1^2} \quad \ldots (3)$$

and $$Z_2 = \sqrt{R_2^2 + X_2^2} \quad \ldots (4)$$

Similar to resistance and reactance, the impedance also can be referred to any one side.

Let Z_{1e} = Total equivalent impedance referred to primary

then $$\boxed{Z_{1e} = R_{1e} + j X_{1e}}$$

$$\therefore \qquad Z_{1e} = Z_1 + Z_2' = Z_1 + \frac{Z_2}{K^2} \qquad \dots (5)$$

Similarly $\quad Z_{2e}$ = Total equivalent impedance referred to secondary

then
$$\boxed{Z_{2e} = R_{2e} + j\, X_{2e}}$$

$$\therefore \qquad Z_{2e} = Z_2 + Z_1' = Z_2 + K^2\, Z_1 \qquad \dots (6)$$

The magnitudes of Z_{1e} and Z_{2e} are,

$$Z_{1e} = \sqrt{R_{1e}^2 + X_{1e}^2} \qquad \dots (7)$$

and
$$Z_{2e} = \sqrt{R_{2e}^2 + X_{2e}^2} \qquad \dots (8)$$

It can be noted that,

$$\boxed{Z_{2e} = K^2\, Z_{1e} \quad \text{and} \quad Z_{1e} = \frac{Z_{2e}}{K^2}} \qquad \dots (9)$$

The concept of equivalent impedance is shown in the Fig. 3.22.

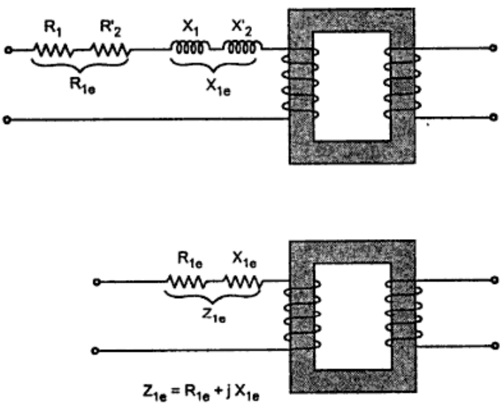
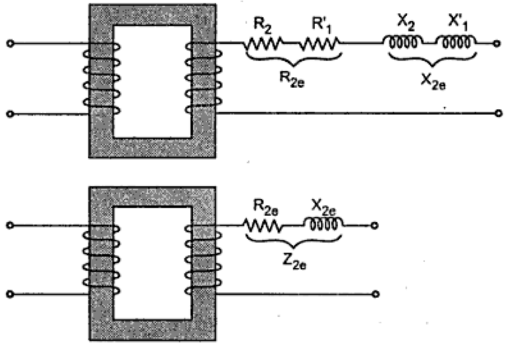


Fig. 3.22 (a) Referred to primary



$Z_{2e} = R_{2e} + j\,X_{2e}$

(b) Referred to secondary

Fig. 3.22 Equivalent impedances

Example 3.6 : *A 220/110 V, 50 Hz, 1.5 kVA transformer has primary and secondary winding resistances of 1 Ω and 2 Ω while reactances of 3 Ω and 5 Ω respectively. Find the total resistance, equivalent reactance and equivalent impedance referred to primary and secondary.* **[May-2005 (Set-1), Nov.-2003 (Set-2)]**

Solution : $R_1 = 1\ \Omega$, $R_2 = 2\ \Omega$, $X_1 = 3\ \Omega$, $X_2 = 5\ \Omega$

$$K = \frac{V_2}{V_1} = \frac{110}{220} = 0.5$$

$$\therefore \quad R_{1e} = R_1 + R_2' = R_1 + \frac{R_2}{K^2} = 1 + \frac{2}{(0.5)^2} = 9\ \Omega$$

$$\therefore \quad X_{1e} = X_1 + X_2' = X_1 + \frac{X_2}{K^2} = 3 + \frac{5}{(0.5)^2} = 23\ \Omega$$

$$\therefore \quad Z_{1e} = \sqrt{R_{1e}^2 + X_{1e}^2} = \sqrt{9^2 + 23^2} = 24.6981\ \Omega$$

$$\therefore \quad R_{2e} = K^2 R_{1e} = 2.25\ \Omega\ ,\ X_{2e} = K^2 X_{1e} = 5.75\ \Omega$$

$$\therefore \quad Z_{2e} = \sqrt{R_{2e}^2 + X_{2e}^2} = K^2 Z_{1e} = 6.1745\ \Omega$$

3.13 Losses in a Transformer

In a transformer, there exists two types of losses.

i) The core gets subjected to an alternating flux, causing **core losses**.

ii) The windings carry currents when transformer is loaded, causing **copper losses**.

3.13.1 Core or Iron Losses

Due to alternating flux set up in the magnetic core of the transformer, it undergoes a cycle of magnetization and demagnetization. Due to hysteresis effect there is loss of energy in this process which is called hysteresis loss.

It is given by, hysteresis loss = $K_h \, B_m^{1.67} \, f \, v$ watts

where K_h = Hysteresis constant depends on material

B_m = Maximum flux density

f = Frequency

v = Volume of the core

The induced e.m.f. in the core tries to set up eddy currents in the core and hence responsible for the eddy current losses. The eddy current loss is given by,

$$\text{Eddy current loss} = K_e \, B_m^2 \, f^2 \, t^2 \text{ watts/unit volume}$$

where K_e = Eddy current constant

t = Thickness of the core

As seen earlier, the flux in the core is almost constant as supply voltage V_1 at rated frequency f is always constant. Hence the flux density B_m in the core and hence both hysteresis and eddy current losses are constants at all the loads. Hence the core or iron losses are also called **constant losses**. The iron losses are denoted as P_i.

The iron losses are minimized by using high grade core material like silicon steel having very low hysteresis loop and by manufacturing the core in the form of laminations.

3.13.2 Copper Losses

The copper losses are due to the power wasted in the form of I^2R loss due to the resistances of the primary and secondary windings. The copper loss depends on the magnitude of the currents flowing through the windings.

$$\text{Total Cu loss} = I_1^2 R_1 + I_2^2 R_2 = I_1^2 (R_1 + R_2') = I_2^2 (R_2 + R_1')$$

$$= I_1^2 R_{1e} = I_2^2 R_{2e}$$

The copper losses are denoted as P_{cu}. If the current through the windings is full load current, we get copper losses at full load. If the load on transformer is half then we get copper losses at half load which are less than full load copper losses. Thus copper losses are called **variable losses**. For transformer VA rating is $V_1 I_1$ or $V_2 I_2$. As V_1 is constant, we can say that copper losses are proportional to the square of the kVA rating.

So, $$\boxed{P_{cu} \propto I^2 \propto (kVA)^2}$$

Thus for a transformer,

$$\text{Total losses} = \text{Iron losses} + \text{Copper losses}$$

$$= P_i + P_{cu}$$

Key Point: *It is seen that the iron losses depend on the supply voltage while the copper losses depend on the current. The losses are not dependent on the phase angle between voltage and current. Hence the rating of the transformer is expressed as a product of voltage and current and called VA* ***rating*** *of transformer. It is not expressed in watts or kilowatts. Most of the times, rating is expressed in kVA.*

3.14 Voltage Regulation of Transformer

Because of the voltage drop across the primary and secondary impedances it is observed that the secondary terminal voltage drops from its no load value (E_2) to load value (V_2) as load and load current increases.

The **regulation** is defined as change in the magnitude of the secondary terminal voltage, when full load i.e. rated load of specified power factor supplied at rated voltage is reduced to no load, with primary voltage maintained constant expressed as the percentage of the rated terminal voltage.

Let E_2 = Secondary terminal voltage **on no load**

V_2 = Secondary terminal voltage **on given load**

then mathematically voltage regulation at given load can be expressed as,

$$\boxed{\%\ \text{Voltage regulation} = \frac{E_2 - V_2}{V_2} \times 100}$$

The ratio ($E_2 - V_2$ / V_2) is called **per unit regulation.**

The secondary terminal voltage does not depend only on the magnitude of the load current but also on the nature of the power factor of the load. If V_2 is determined for full load and specified power factor condition the regulation is called full load regulation.

As load current I_L increases, the voltage drops tend to increase and V_2 drops more and more. In case of lagging power factor $V_2 < E_2$ and we get positive voltage regulation, while for leading power factor $E_2 < V_2$ and we get negative voltage regulation. This is shown in the Fig. 3.23.

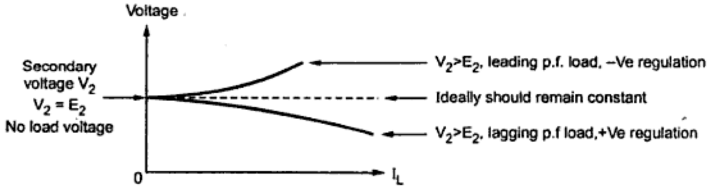


Fig. 3.23 Regulation characteristics

Key Point: *The voltage drop should be as small as possible hence less the regulation better is the performance of a transformer.*

3.14.1 Expression for Voltage Regulation

The voltage regulation is defined as,

$$\% R = \frac{E_2 - V_2}{V_2} \times 100 = \frac{\text{Total voltage drop}}{V_2} \times 100$$

The regulation can be expressed as,

$$\boxed{\% R = \frac{I_2 R_{2e} \cos\phi \pm I_2 X_{2e} \sin\phi}{V_2} \times 100}$$

where I_2 = Full load secondary current

V_2 = No load secondary voltage

R_{2e} = Equivalent resistance referred to secondary

X_{2e} = Equivalent reactance referred to secondary

$\cos\phi$ = Load power factor

+ sign for lagging power factors while – sign for leading power factor loads.

The regulation can be further expressed interms of I_1, V_1, R_{1e} and X_{1e}.

$$\frac{V_2}{V_1} = \frac{I_1}{I_2} = K$$

$$\therefore \quad V_2 = KV_1, \quad I_2 = \frac{I_1}{K}$$

while $$R_{1e} = \frac{R_{2e}}{K^2}, \quad X_{1e} = \frac{X_{2e}}{K^2}$$

Substituting in the regulation expression we get,

$$\% R = \frac{\frac{I_1}{K} \cdot K^2 R_{1e} \cos\phi \pm \frac{I_1}{K} \cdot K^2 X_{1e} \sin\phi}{K V_1} \times 100$$

$$\therefore \quad \% R = \frac{I_1 R_{1e} \cos\phi \pm I_1 X_{1e} \sin\phi}{V_1} \times 100$$

Example 3.7 : *A 250/125 V, 5 kVA single phase transformer has primary resistance of 0.2 Ω and reactance of 0.75 Ω. The secondary resistance is 0.05 Ω and reactance of 0.2 Ω.*

i) Determine its regulation while supplying full load on 0.8 leading p.f.

ii) The secondary terminal voltage on full load and 0.8 leading p.f.

Solution : The given values are,

$R_1 = 0.2\ \Omega$, $X_1 = 0.75\ \Omega$, $R_2 = 0.05\ \Omega$, $X_2 = 0.2\ \Omega$, $\cos\phi = 0.8$ leading

$$K = \frac{E_2}{E_1} = \frac{125}{250} = \frac{1}{2} = 0.5$$

$$(I_2)\text{F.L.} = \frac{\text{kVA}}{V_2} = \frac{5 \times 10^3}{125} = 40 \text{ A} \quad \text{... Full load}$$

$$R_{2e} = R_2 + K^2 R_1 = 0.05 + (0.5)^2 \times 0.2 = 0.1\ \Omega$$

$$X_{2e} = X_2 + K^2 X_1 = 0.2 + (0.5)^2 \times 0.75 = 0.3875\ \Omega$$

i) Regulation on full load, $\cos\phi = 0.8$ leading

$$\sin\phi = 0.6$$

$$\therefore \quad \% R = \frac{I_2 R_{2e} \cos\phi - I_2 X_{2e} \sin\phi}{E_2} \times 100, \quad \text{... } I_2 = I_2(\text{FL}) = 40 \text{ A}$$

$$= \frac{(40 \times 0.1 \times 0.8 - 40 \times 0.3875 \times 0.6)}{125} \times 100$$

$$= \mathbf{-4.88\ \%}$$

ii) On full load, 0.8 p.f. leading the total voltage drop is,

$$\text{Voltage drop} = I_2(\text{FL})\ [R_{2e} \cos\phi - X_{2e} \sin\phi]$$

$$= 40\ [0.1 \times 0.8 - 0.3875 \times 0.6] = -6.1 \text{ V}$$

$$\therefore \quad E_2 - V_2 = -6.1 \quad \text{i.e.} \quad 125 - V_2 = -6.1$$

$$\therefore \quad V_2 = \text{Secondary terminal voltage} = 125 + 6.1 = \mathbf{131.1\ V}$$

3.15 Efficiency of a Transformer

Due to the losses in a transformer, the output power of a transformer is less than the input power supplied.

$$\therefore \quad \text{Power output} = \text{Power input} - \text{Total losses}$$

$$\therefore \quad \text{Power input} = \text{Power output} + \text{Total losses}$$

$$= \text{Power output} + P_i + P_{cu}$$

The efficiency of any device is defined as the ratio of the power output to power input. So for a transformer the efficiency can be expressed as,

$$\eta = \frac{\text{Power output}}{\text{Power input}}$$

$$\therefore \quad \eta = \frac{\text{Power output}}{\text{Power output} + P_i + P_{cu}}$$

Now Power output $= V_2 I_2 \cos\phi$

where $\cos\phi$ = Load power factor

The transformer supplies full load of current I_2 and with terminal voltage V_2.

$$P_{cu} = \text{Copper losses on full load} = I_2^2 R_{2e}$$

$$\therefore \quad \eta = \frac{V_2 I_2 \cos\phi_2}{V_2 I_2 \cos\phi_2 + P_i + I_2^2 R_{2e}}$$

But $V_2 I_2$ = VA rating of a transformer

$$\therefore \quad \eta = \frac{(\text{VA rating}) \times \cos\phi}{(\text{VA rating}) \times \cos\phi + P_i + I_2^2 R_{2e}}$$

$$\therefore \quad \boxed{\%\,\eta = \frac{(\text{VA rating}) \times \cos\phi}{(\text{VA rating}) \times \cos\phi + P_i + I_2^2 R_{2e}} \times 100}$$

This is full load percentage efficiency with,

I_2 = Full load secondary current

But if the transformer is subjected to fractional load then using the appropriate values of various quantities, the efficiency can be obtained.

Let $\boxed{n = \text{Fraction by which load is less than full load} = \dfrac{\text{Actual load}}{\text{Full load}}}$

For example if transformer is subjected to half load then,

$$n = \frac{\text{Half load}}{\text{Full load}} = \frac{(1/2)}{1} = 0.5$$

When load changes, the load current changes by same proportion.

$$\therefore \quad \boxed{\text{New } I_2 = n\,(I_2)\text{ F.L.}}$$

Similarly the output $V_2\, I_2 \cos\phi_2$ also reduces by the same fraction. Thus fraction of VA rating is available at the output.

Similarly as copper losses are proportional to square of current then,

$$\boxed{\text{New } P_{cu} = n^2\,(P_{cu})\text{ F.L.}}$$

Key Point : *So copper losses get reduced by n^2.*

In general for fractional load the efficiency is given by,

$$\boxed{\%\,\eta = \frac{n\,(\text{VA rating})\cos\phi}{n\,(\text{VA rating})\cos\phi + P_i + n^2\,(P_{cu})\text{ F.L.}} \times 100}$$

where n = **Fraction by which load is less than full load.**

Key Point : *For all types of load power factors lagging, leading and unity the efficiency expression does not change, and remains same.*

Example 3.8 : *A 4 kVA, 200/400 V, 50 Hz, single phase transformer has equivalent resistance referred to primary as 0.15 Ω. Calculate,*
i) The total copper losses on full load.
ii) The efficiency while supplying full load at 0.9 p.f. lagging.
iii) The efficiency while supplying half load at 0.8 p.f. leading.
Assume total iron losses equal to 60 W.

Solution : The given values are,

$V_1 = 200$ V, $V_2 = 400$ V, S = 4 kVA , $R_{1e} = 0.15\ \Omega$, $P_i = 60$ W

$$K = \frac{400}{200} = 2$$

$$\therefore \quad R_{2e} = K^2 R_{1e} = (2)^2 \times 0.15 = 0.6\ \Omega$$

$$(I_2)\text{ F.L.} = \frac{\text{kVA}}{V_2} = \frac{4\times10^3}{400} = 10\text{ A}$$

i) Total copper losses on full load,

$$(P_{cu})\text{ F.L.} = [(I_2)\text{ F.L.}]^2 R_{2e} = (10)^2 \times 0.6 = \mathbf{60\ W}$$

ii) $\cos\phi = 0.9$ lagging and full load

$$\therefore \quad \%\eta = \frac{\text{VA rating}\cos\phi}{\text{VA rating}\cos\phi + P_i + (P_{cu})\text{F.L.}} \times 100$$

$$\therefore \quad \eta = \frac{4\times10^3\times0.9}{4\times10^3\times0.9+60+60}\times100 = \mathbf{96.77\ \%}$$

iii) $\cos\phi = 0.8$ leading, half load

As half load, $n = 0.5$

$$(P_{cu})\text{H.L.} = n^2 \times (P_{cu})\text{F.L.} = (0.5)^2\times60 = 15\ \text{W}$$

$$\therefore \quad \%\eta = \frac{n\times(\text{VA rating})\cos\phi}{n\times(\text{VA rating})\cos\phi + P_i + (P_{cu})\text{H.L.}}\times 100$$

$$= \frac{0.5\times4\times10^3\times0.8}{0.5\times4\times10^3\times0.8+60+15}\times100 = \mathbf{95.52\ \%}$$

3.16 Condition for Maximum Efficiency

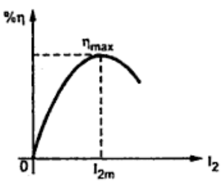


Fig. 3.24

When a transformer works on a constant input voltage and frequency then efficiency varies with the load. As load increases, the efficiency increases. At a certain load current, it achieves a maximum value. If the transformer is loaded further the efficiency starts decreasing. The graph of efficiency against load current I_2 is shown in the Fig. 3.24.

The load current at which the efficiency attains maximum value is denoted as I_{2m} and maximum efficiency is denoted as η_{max}.

Let us determine,

1. Condition for maximum efficiency.
2. Load current at which η_{max} occurs.
3. kVA supplied at maximum efficiency.

The efficiency is a function of load i.e. load current I_2 assuming $\cos\phi_2$ constant. The secondary terminal voltage V_2 is also assumed constant. So for maximum efficiency,

$$\frac{d\eta}{dI_2} = 0$$

Now
$$\eta = \frac{V_2 I_2 \cos\phi_2}{V_2 I_2 \cos\phi_2 + P_i + I_2^2 R_{2e}}$$

$$\therefore \quad \frac{d\eta}{dI_2} = \frac{d}{dI_2}\left[\frac{V_2\ I_2\ \cos\phi_2}{V_2\ I_2\ \cos\phi_2 + P_i + I_2^2\ R_{2e}}\right] = 0$$

$$\therefore\ (V_2\ I_2\ \cos\phi_2 + P_i + I_2^2\ R_{2e})\frac{d}{dI_2}(V_2\ I_2\ \cos\phi_2)$$

$$-(V_2\ I_2\ \cos\phi_2)\cdot\frac{d}{dI_2}(V_2\ I_2\ \cos\phi_2 + P_i + I_2^2\ R_{2e}) = 0$$

$$\therefore\ (V_2\ I_2\ \cos\phi_2 + P_i + I_2^2\ R_{2e})(V_2\ \cos\phi_2) - (V_2\ I_2\ \cos\phi_2)(V_2\ \cos\phi_2 + 2I_2\ R_{2e}) = 0$$

Cancelling $(V_2 \cos\phi_2)$ from both the terms we get,

$$V_2\ I_2\ \cos\phi_2 + P_i + I_2^2\ R_{2e} - V_2\ I_2\ \cos\phi_2 - 2\ I_2^2\ R_{2e} = 0$$

$$\therefore \quad P_i - I_2^2\ R_{2e} = 0$$

$$\therefore \quad P_i = I_2^2\ R_{2e} = P_{cu}$$

So condition to achieve maximum efficiency is that,

$$\boxed{\text{Copper losses} = \text{Iron losses}}$$

3.16.1 Load Current I_{2m} at Maximum Efficiency

For η_{max}, $\quad I_2^2\ R_{2e} = P_i$ but $I_2 = I_{2m}$

$$\therefore \quad I_{2m}^2\ R_{2e} = P_i$$

$$\therefore \quad I_{2m} = \sqrt{\frac{P_i}{R_{2e}}}$$

This is the load current at η_{max}.

Let $\quad (I_2)\text{F.L.} = $ Full load current

$$\therefore \quad \frac{I_{2m}}{(I_2)\text{F.L.}} = \frac{1}{(I_2)\text{F.L.}}\sqrt{\frac{P_i}{R_{2e}}}$$

$$\therefore \quad \frac{I_{2m}}{(I_2)\text{F.L.}} = \sqrt{\frac{P_i}{[(I_2)\ \text{F.L.}]^2\ R_{2e}}}$$

$$= \sqrt{\frac{P_i}{(P_{cu})\ \text{F.L.}}}$$

$$\therefore \quad \boxed{I_{2m} = (I_2)\ \text{F.L.}\sqrt{\frac{P_i}{(P_{cu})\ \text{F.L.}}}}$$

This is the load current at η_{max} interms of full load current.

3.16.2 kVA Supplied at Maximum Efficiency

For constant V_2 the kVA supplied is the function of load current.

$$\therefore \quad \text{kVA at } \eta_{max} = I_{2m} V_2 = V_2 (I_2)\text{F.L.} \times \sqrt{\frac{P_i}{(P_{cu})\text{ F.L.}}}$$

$$\therefore \quad \boxed{\text{kVA at } \eta_{max} = (\text{kVA rating}) \times \sqrt{\frac{P_i}{(P_{cu})\text{ F.L.}}}}$$

Substituting condition for η_{max} in the expression of efficiency, we can write expression for η_{max} as,

$$\% \eta_{max} = \frac{V_2 I_{2m} \cos\phi}{V_2 I_{2m} \cos\phi + 2P_i} \times 100 \qquad \text{as } P_{cu} = P_i$$

i.e.
$$\% \eta_{max} = \frac{\text{kVA for } \eta_{max} \cos\phi}{\text{kVA for } \eta_{max} \cos\phi + 2P_i}$$

Example 3.9 : *A 100 kVA transformer has iron losses of 1.2 kW and full load copper losses of 1.5 kW. Find :*

i) kVA for maximum efficiency *ii) Maximum efficiency at unity p.f.*

[Nov.-2008, (Set-1)]

Solution : $P_i = 1.2$ kW, $(P_{cu})_{F.L.} = 1.5$ kW, kVA = 100

i) $$\text{kVA for } \eta_{max} = \text{kVA} \times \sqrt{\frac{P_i}{(P_{cu})_{F.L.}}} = 100 \times \sqrt{\frac{1.2}{1.5}} = \mathbf{89.4427\ kVA}$$

ii) For η_{max}, $P_{cu} = P_i = 1.2$ kW

$$\therefore \quad \%\eta_{max} = \frac{[\text{kVA for } \eta_{max}] \times \cos\phi}{[\text{kVA for } \eta_{max}] \times \cos\phi + 2P_i} \times 100 \qquad \text{... } \cos\phi = 1$$

$$= \frac{89.4427 \times 10^3 \times 1}{89.4427 \times 10^3 \times 1 + [2 \times 1.2 \times 10^3]} \times 100 = \mathbf{97.386\ \%}$$

Example 3.10 : *A 20 kVA transformer has its maximum efficiency of 0.98 at 15 kVA at unity power factor. The iron loss is 350 W. Calculate the efficiency at full load 0.8 power factor lagging and unity power factor.* **[Nov.-2005 (Set-3), Nov.-2008 (Set-3)]**

Solution : 20 kVA, $\eta_{max} = 0.98$ at 15 kVA and $\cos\phi = 1$, $P_i = 350$ W

$$\text{Load at } \eta_{max} = \text{kVA} \times \sqrt{\frac{P_i}{(P_{cu})_{F.L.}}}$$

$$\therefore \quad 15 = 20 \times \sqrt{\frac{350}{(P_{cu})_{F.L.}}} \qquad \text{i.e.} \quad (P_{cu})_{F.L.} = 622.222 \text{ W}$$

i) $\%\eta$ at $\cos\phi = 0.8$ lag , full load

$$\%\eta_{F.L.} = \frac{VA\cos\phi}{VA\cos\phi + (P_{cu})_{F.L.} + P_i} \times 100 = \frac{20\times10^3\times0.8}{20\times10^3\times0.8 + 622.222 + 350}\times100$$

$$= \mathbf{94.271\ \%}$$

ii) $\%\eta$ at $\cos\phi = 1$, full load

$$\%\eta_{F.L.} = \frac{20\times10^3\times1}{20\times10^3\times1 + 622.222 + 350}\times100 = \mathbf{95.3642\ \%}$$

3.17 Predetermination of Efficiency and Regulation

The efficiency and regulation of a transformer on any load condition and at any power factor condition can be predetermined by indirect loading method. In this method, the actual load is not used on transformer. But the equivalent circuit parameters of a transformer are determined by conducting two tests on a transformer which are,

1. Open circuit test (O.C. test) 2. Short circuit test (S.C. test)

The parameters calculated from these test results are effective in determining the regulation and efficiency of a transformer at any load and power factor condition, without actually loading the transformer. The advantage of this method is that without much power loss the tests can be performed and results can be obtained. Let us discuss in detail how to perform these tests and how to use the results to calculate equivalent circuit parameters.

3.17.1 Open Circuit Test (O.C. Test)

The experimental circuit to conduct O.C. test is shown in the Fig. 3.25.

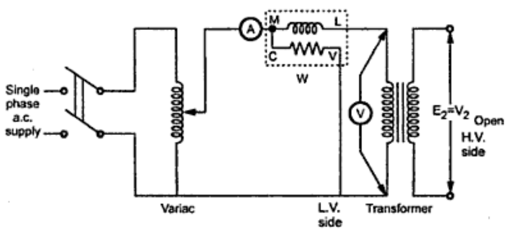


Fig. 3.25 Experimental circuit for O.C. test

The transformer primary is connected to a.c. supply through ammeter, wattmeter and variac. The secondary of transformer is kept open. Usually low voltage side is used as primary and high voltage side as secondary to conduct O.C. test.

The primary is excited by rated voltage, which is adjusted precisely with the help of a variac. The wattmeter measures input power. The ammeter measures input current. The voltmeter gives the value of rated primary voltage applied at rated frequency.

Sometimes a voltmeter may be connected across secondary to measure secondary voltage which is $V_2 = E_2$ when primary is supplied with rated voltage. As voltmeter resistance is very high, though voltmeter is connected, secondary is treated to be open circuit as voltmeter current is always negligibly small.

When the primary voltage is adjusted to its rated value with the help of variac, readings of ammeter and wattmeter are to be recorded.

The observation table is as follows.

V_o volts	I_o amperes	W_o watts
Rated		

$$V_o = \text{Rated voltage}$$

$$W_o = \text{Input power}$$

$$I_o = \text{Input current} = \text{No load current}$$

As transformer secondary is open, it is on no load. So current drawn by the primary is no load current I_o. The two components of this no load current are,

$$\boxed{\begin{aligned} I_m &= I_o \sin \phi_o \\ I_c &= I_o \cos \phi_o \end{aligned}}$$

where $\cos \phi_o$ = No load power factor

And hence power input can be written as,

$$\boxed{W_o = V_o I_o \cos \phi_o}$$

The phasor diagram is shown in the Fig. 3.26.

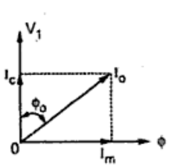


Fig. 3.26

As secondary is open, $I_2 = 0$. Thus its reflected current on primary I_2' is also zero. So we have primary current $I_1 = I_o$. The transformer no load current is always very small, hardly 2 to 4 % of its full load value. As $I_2 = 0$, secondary copper losses are zero. And $I_1 = I_o$ is very low hence copper losses on primary are also very very low. Thus the total copper losses in O.C. test are negligibly small. As against this the input voltage is rated at rated frequency hence flux density in the core is at its maximum value. Hence iron losses are at rated voltage. As output power is zero and copper losses are very low, the total input power is used to supply iron losses.

This power is measured by the wattmeter i.e. W_o. Hence the wattmeter in O.C. test gives iron losses which remain constant for all the loads.

$$\therefore \quad \boxed{W_o = P_i = \text{Iron losses}}$$

Calculations : We know that,

$$W_o = V_o I_o \cos \phi$$

$$\therefore \quad \boxed{\cos \phi_o = \frac{W_o}{V_o I_o} = \text{No load power factor}}$$

Once $\cos \phi_o$ is known we can obtain,

$$I_c = I_o \cos \phi_o$$

and

$$I_m = I_o \sin \phi_o$$

Once I_c and I_m are known we can determine exciting circuit parameters as,

$$R_o = \frac{V_o}{I_c} \ \Omega$$

and

$$X_o = \frac{V_o}{I_m} \ \Omega$$

Key Point : *The no load power factor $\cos \phi_o$ is very low hence wattmeter used must be low power factor type otherwise there might be error in the results. If the meters are connected on secondary and primary is kept open then from O.C. test we get R_o' and X_o' with which we can obtain R_o and X_o knowing the transformation ratio K.*

3.17.2 Short Circuit Test (S.C. Test)

In this test, primary is connected to a.c. supply through variac, ammeter and voltmeter as shown in the Fig. 3.27.

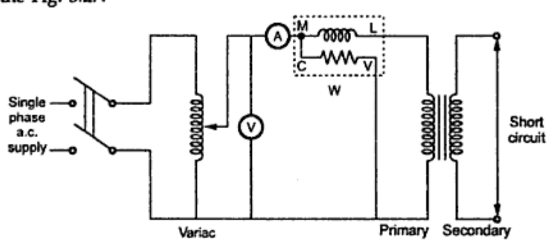


Fig. 3.27 Experimental circuit for S.C. test

The secondary is short circuited with the help of thick copper wire or solid link. As high voltage side is always low current side, it is convenient to connect high voltage side to supply and shorting the low voltage side.

As secondary is shorted, its resistance is very very small and on rated voltage it may draw very large current. Such large current can cause overheating and burning of the transformer. To limit this short circuit current, primary is supplied with low voltage which is just enough to cause rated current to flow through primary which can be observed on an ammeter. The low voltage can be adjusted with the help of variac. Hence this test is also called **low voltage test or reduced voltage test**. The wattmeter reading as well as voltmeter, ammeter readings are recorded. The observation table is as follows.

V_{sc} volts	I_{sc} amperes	W_{sc} watts
	Rated	

Now the currents flowing through the windings are rated currents hence the total copper loss is full load copper loss. Now the voltage applied is low which is a small fraction of the rated voltage. The iron losses are function of applied voltage. So the iron losses in reduced voltage test are very small. Hence the wattmeter reading is the power loss which is equal to full load copper losses as iron losses are very low.

$$\therefore \quad W_{sc} = (P_{cu}) \text{ F.L.} = \text{Full load copper loss}$$

Calculations : From S.C. test readings we can write,

$$W_{sc} = V_{sc} I_{sc} \cos \phi_{sc}$$

$$\therefore \quad \cos \phi_{sc} = \frac{V_{sc} I_{sc}}{W_{sc}} = \text{Short circuit power factor}$$

$$W_{sc} = I_{sc}^2 R_{1e} = \text{Copper loss}$$

$$\therefore \quad \boxed{R_{1e} = \frac{W_{sc}}{I_{sc}^2}}$$

While

$$\boxed{Z_{1e} = \frac{V_{sc}}{I_{sc}} = \sqrt{R_{1e}^2 + X_{1e}^2}}$$

$$\therefore \quad \boxed{X_{1e} = \sqrt{Z_{1e}^2 - R_{1e}^2}}$$

Thus we get the equivalent circuit parameters R_{1e}, X_{1e} and Z_{1e}. Knowing the transformation ratio K, the equivalent circuit parameters referred to secondary also can be obtained.

Important Note : If the transformer is step up transformer, its primary is L.V. while secondary is H.V. winding. In S.C. test, supply is given to H.V. winding and L.V. is shorted. In such case we connect meters on H.V. side which is transformer secondary though for S.C. test purpose H.V. side acts as primary. In such case the parameters calculated from S.C. test readings are referred to secondary which are R_{2e}, Z_{2e} and X_{2e}. So before doing calculations it is necessary to find out where the readings are recorded on transformer primary or secondary and accordingly the parameters are to be determined. In step down transformer, primary is high voltage itself to which supply is given in S.C. test. So in such case test results give us parameters referred to primary i.e. R_{1e}, Z_{1e} and X_{1e}.

Key Point: *In short, if meters are connected to primary of transformer in S.C. test, calculations give us R_{1e} and Z_{1e}. If meters are connected to secondary of transformer in S.C. test calculations give us R_{2e} and Z_{2e}.*

3.17.3 Calculation of Efficiency from O.C. and S.C. Tests

We know that,

From O.C. test, $W_o = P_i$

From S.C. test, $W_{sc} = (P_{cu})$ F.L.

$$\therefore \quad \% \eta \text{ on full load } = \frac{V_2 (I_2) \text{ F.L.} \cos\phi}{V_2 (I_2) \text{ F.L.} \cos + W_o + W_{sc}} \times 100$$

Thus for any p.f. $\cos \phi_2$ the efficiency can be predetermined. Similarly at any load which is fraction of full load then also efficiency can be predetermined as,

$$\% \eta \text{ at any load } = \frac{n \times (\text{VA rating}) \times \cos\phi}{n \times (\text{VA rating}) \times \cos\phi + W_o + n^2 W_{sc}} \times 100$$

where n = Fraction of full load

or

$$\% \eta = \frac{n V_2 I_2 \cos\phi}{n V_2 I_2 \cos\phi + W_o + n^2 W_{sc}} \times 100$$

where $I_2 = n (I_2)$ F.L.

3.17.4 Calculation of Regulation

From S.C. test we get the equivalent circuit parameters referred to primary or secondary.

The rated voltages V_1, V_2 and rated currents (I_1) F.L. and (I_2) F.L. are known for the given transformer. Hence the regulation can be determined as,

$$\boxed{\begin{aligned} \% R &= \frac{I_2 R_{2e} \cos\phi \pm I_2 X_{2e} \sin\phi}{V_2} \times 100 \\ &= \frac{I_1 R_{1e} \cos\phi \pm I_1 X_{1e} \sin\phi}{V_1} \times 100 \end{aligned}}$$

where I_1, I_2 are rated currents for full load regulation.

For any other load the currents I_1, I_2 must be changed by fraction n.

$\therefore$ $\boxed{I_1, I_2 \text{ at any other load } = n\,(I_1)\text{ F.L., } n\,(I_2)\text{ F.L.}}$

Key Point : *Thus regulation at any load and any power factor can be predetermined, without actually loading the transformer.*

Example 3.11 : *The open circuit and short circuit tests on a 10 kVA, 125/250 V, 50 Hz, single phase transformer gave the following results :*
O.C. test : 125 V, 0.6 A, 50 W (on L.V. side)
S.C. test : 15 V, 30 A, 100 W (on H.V. side)
Calculate : i) Copper loss on full load. ii) Full load efficiency at 0.8 leading p.f.
iii) Half load efficiency at 0.8 leading p.f. iv) Regulation at full load, 0.9 leading p.f.

Solution : From O.C. test we can write,

$$W_o = P_i = 50 \text{ W} = \text{Iron loss}$$

From S.C. test we can find the parameters of equivalent circuit. Now S.C. test is conducted on H.V. side i.e. meters are on H.V. side which is transformer secondary. Hence parameters from S.C. test results will be referred to secondary.

$$V_{sc} = 15 \text{ V}, I_{sc} = 30 \text{ A}, W_{sc} = 100 \text{ W}$$

$$\therefore \quad R_{2e} = \frac{W_{sc}}{(I_{sc})^2} = \frac{10}{(30)^2} = 0.111\ \Omega$$

$$Z_{1e} = \frac{V_{sc}}{I_{sc}} = \frac{15}{30} = 0.5\ \Omega$$

$$\therefore \quad X_{2e} = \sqrt{Z_{2e}^2 - R_{2e}^2} = 0.4875\ \Omega$$

i) Copper loss on full load

$$(I_2)\text{ F.L.} = \frac{\text{VA rating}}{V_2} = \frac{10 \times 10^3}{250} = 40 \text{ A}$$

In short circuit test, $I_{sc} = 30$ A and not equal to full load value 40 A.

Hence W_{sc} does not give copper loss on full load.

$$\therefore \quad W_{sc} = P_{cu} \text{ at } 30 \text{ A} = 100 \text{ W}$$

Now $$P_{cu} \propto I^2$$

$$\therefore \quad \frac{P_{cu} \text{ at } 30 \text{ A}}{P_{cu} \text{ at } 40 \text{ A}} = \left(\frac{30}{40}\right)^2$$

$$\therefore \quad \frac{100}{P_{cu} \text{ at } 40 \text{ A}} = \frac{900}{1600}$$

$$\therefore \quad P_{cu} \text{ at } 40 \text{ A} = 177.78 \text{ W}$$

$$\therefore \quad (P_{cu}) \text{ F.L.} = 177.78 \text{ W}$$

ii) Full load η , $\cos \phi_2 = 0.8$

$$\% \eta \text{ on full load} = \frac{V_2(I_2) \text{ F.L. } \cos\phi_2}{V_2(I_2) \text{ F.L.} \cos\phi_2 + P_i + (P_{cu}) \text{ F.L.}} \times 100$$

$$= \frac{250 \times 40 \times 0.8}{250 \times 40 \times 0.8 + 50 + 177.78} \times 100 = \mathbf{97.23\ \%}$$

iii) Half load η , $\cos \phi_2 = 0.8$

$$n = 0.5 \text{ as half load,} \quad (I_2) \text{ H.L.} = \frac{1}{2} \times 40 = 20 \text{ A}$$

$$\therefore \quad \% \eta \text{ on half load} = \frac{V_2(I_2) \text{ H.L. } \cos\phi_2}{V_2(I_2) \text{ H.L.} \cos\phi_2 + P_i + n^2(P_{cu}) \text{ F.L.}} \times 100$$

$$= \frac{n\,(\text{VA rating}) \cos\phi_2}{n\,(\text{VA rating}) \cos\phi_2 + P_i + n^2(P_{cu}) \text{ F.L.}} \times 100$$

$$= \frac{0.5 \times 10 \times 10^3 \times 0.8}{0.5 \times 10 \times 10^3 \times 0.8 + 50 + (0.5)^2 \times 177.78} \times 100$$

$$= \mathbf{97.69\ \%}$$

iv) Regulation at full load, $\cos \phi = 0.9$ leading

$$\% R = \frac{(I_2) \text{ F.L.} R_{2e} \cos\phi - (I_2) \text{ F.L. } X_{2e} \sin\phi}{V_2} \times 100$$

$$= \frac{40 \times 0.111 \times 0.9 - 40 \times 0.4875 \times 0.4358}{250} \times 100$$

$$= \mathbf{-1.8015\ \%}$$

Example 3.12 : *A 50 Hz, single phase transformer has a turns ratio of 6. The resistances are 0.9 Ω and 0.03 Ω and reactances are 5 Ω and 0.13 Ω for high voltage and low voltage windings respectively. Find : i) The voltage to be applied to high voltage side to obtain full load current of 200 A in the low voltage winding on short circuit. ii) The power factor on short circuit.* **[June-2003, (Set-4)]**

Solution : $R_1 = 0.9\ \Omega$, $R_2 = 0.03\ \Omega$, $X_1 = 5\ \Omega$, $X_2 = 0.13\ \Omega$

$$K = \frac{N_2}{N_1} = \frac{1}{6} \text{ as } N_1 : N_2 \text{ is } 6:1.$$

$$\therefore \quad R_{2e} = R_2 + R_1' = R_2 + K^2 R_1 = 0.03 + \left(\frac{1}{6}\right)^2 \times 0.9 = 0.055\ \Omega$$

$$\therefore \quad X_{2e} = X_2 + X_1' = X_2 + K^2 X_1 = 0.13 + \left(\frac{1}{6}\right)^2 \times 5 = 0.26888\ \Omega$$

$$I_{sc} = 200\ \text{A}$$

$$\therefore \quad Z_{2e} = \frac{V_{sc}}{I_{sc}} \quad \text{i.e.} \quad \sqrt{R_{2e}^2 + X_{2e}^2} = \frac{V_{sc}}{200}$$

$$\therefore \quad V_{sc} = 200 \times 0.27444 = 54.8895\ \text{V} \qquad \text{... On secondary}$$

i) $V_1 = \dfrac{V_{sc}}{K} = \dfrac{54.8895}{\left(\frac{1}{6}\right)} =$ 32[illegible]7 V

ii) $$W_{sc} = V_{sc} I_{sc} \cos\phi_{sc} \quad \text{and} \quad W_{sc} = I_{sc}^2 R_{2e}$$

$$\therefore \quad (200)^2 \times 0.055 = 54.8895 \times 200 \times \cos\phi_{sc}$$

$$\therefore \quad \cos\phi_{sc} = \mathbf{0.2\ lagging}$$

3.18 All Day Efficiency of a Transformer

For a transformer, the efficiency is defined as the ratio of output power to input power. This is its power efficiency. But power efficiency is not the true measure of the performance of some special types of transformers such as distribution transformers.

Distribution transformers serve residential and commercial loads. The load on such transformers vary considerably during the period of the day. For most period of the day these transformers are working at 30 to 40 % of full load only or even less than that. But the primary of such transformers is energised at its rated voltage for 24 hours, to provide continuous supply to the consumer. The core loss which depends on voltage, takes place continuously for all the loads. But copper loss depends on the load condition. For no load, copper loss is negligibly small while on full load it is at its rated value. Hence power efficiency cannot give the measure of true efficiency of such transformers. In such transformers, the energy output is calculated in kilo watt hours (kWh). Similarly energy spent in supplying the various losses is also determined in kilo watt hours (kWh). Then ratio of total energy output to total energy input (output + losses) is calculated. Such ratio is called **Energy efficiency** or **All Day Efficiency** of a transformer. Based on this efficiency, the performance of various distribution transformers is compared. All day efficiency is defined as,

$$\% \text{ All day } \eta = \frac{\text{Output energy in kWh during a day}}{\text{Input energy in kWh during a day}} \times 100$$

$$= \frac{\text{Output energy in kWh during a day}}{\text{Output energy + Energy spent for total losses}} \times 100$$

While calculating energies, all energies can be expressed in watt hour (Wh) instead of kilo watt hour (kWh).

Such distribution transformers are designed to have very low core losses. This is achieved by limiting the core flux density to lower value by using a relatively higher core cross-section i.e. larger iron to copper weight ratio. The maximum efficiency in such transformers occur at about 60-70 % of the full load. So by proper designing, high energy efficiencies can be achieved for distribution transformers.

The calculations of all day efficiency for a transformer are illustrated in the Ex. 3.13.

Example 3.13 : *A 400 kVA, distribution transformer has full load iron loss of 2.5 kW and copper loss of 3.5 kW. During a day, its load cycle for 24 hours is,*

6 hours 300 kW at 0.8 p.f.

10 hours 200 kW at 0.7 p.f.

4 hours 100 kW at 0.9 p.f.

4 hours No load

Determine its all day efficiency.

Solution : Given values are,

$$P_i = 2.5 \text{ kW}, \quad (P_{cu})\text{F.L.} = 3.5 \text{ kW}, \quad 400 \text{ kVA}$$

Iron losses are constant for 24 hours. So energy spent due to iron losses for 24 hours is,

$$P_i = 2.5 \times 24 \text{ hours} = 60 \text{ kWh}$$

Total energy output in a day from given load cycle is,

$$\text{Energy output} = 300 \times 6 \text{ hours} + 200 \times 10 \text{ hours} + 100 \times 4 \text{ hours}$$

$$= 4200 \text{ kWh}$$

To calculate energy spent due to copper loss,

i) Load 1 of 300 kW at cos ϕ = 0.8

$$\therefore \quad \text{kVA supplied} = \frac{\text{kW}}{\cos\phi} = \frac{300}{0.8} = 375 \text{ kVA}$$

$$\therefore \quad n = \frac{\text{load kVA}}{\text{kVA rating}} = \frac{375}{400} = 0.9375$$

Copper losses are proportional to square of kVA ratio i.e. n^2.

$$\therefore \quad \text{Load 1 } P_{cu} = n^2 \times (P_{cu})\,\text{F.L.} = (0.9375)^2 \times 3.5$$

$$= 3.076 \text{ kW}$$

$$\therefore \quad \text{Energy spent} = 3.076 \times 6 \text{ hours} = 18.457 \text{ kWh}$$

ii) Load 2 of 200 kW at $\cos\phi = 0.7$

$$\therefore \quad \text{kVA supplied} = \frac{\text{kW}}{\cos\phi} = \frac{200}{0.7} = 285.7142 \text{ kVA}$$

$$\therefore \quad n = \frac{\text{Load kVA}}{\text{kVA rating}} = \frac{285.7142}{400} = 0.7142$$

$$\therefore \quad \text{Load 2 } P_{cu} = n^2 \times (P_{cu})\,\text{F.L.} = (0.7142)^2 \times 3.5$$

$$= 1.7857 \text{ kW}$$

$$\therefore \quad \text{Energy spent} = 1.7857 \times 10 = 17.857 \text{ kWh}$$

iii) Load 3 of 100 kW at $\cos\phi = 0.9$

$$\therefore \quad \text{kVA supplied} = \frac{\text{kW}}{\cos\phi} = \frac{100}{0.9} = 111.111 \text{ kVA}$$

$$\therefore \quad n = \frac{111.111}{400} = 0.2778$$

$$\therefore \quad \text{Load 3 } P_{cu} = n^2 \times (P_{cu})\,\text{F.L.} = (0.2778)^2 \times 3.5 = 0.2701 \text{ kW}$$

$$\therefore \quad \text{Energy spent} = 0.2701 \times 4 = 1.0804 \text{ kWh}$$

iv) No load hence negligible copper losses.

$\therefore$ Total energy spent = Energy spent due to [Iron loss + Total copper loss]

$$= 60 + 18.457 + 17.857 + 1.0804 = 97.3944 \text{ kWh}$$

and Total output = 4200 kWh

$$\therefore \quad \text{All day } \eta = \frac{\text{Total output for 24 hours}}{\text{Total output for 24 hours} + \text{Total energy spent for 24 hours}} \times 100$$

$$= \frac{4200}{4200 + 97.3944} \times 100 = \mathbf{97.73\ \%}$$

Examples with Solutions

Example 3.14 : *The efficiency of a 200 kVA, single phase transformer is 98 % when operating at full load 0.8 lagging p.f. the iron loss in the transformer is 2000 watt. Calculate the i) Full load copper loss ii) Half load copper loss and efficiency at half load.*

Solution : η % = 98 %, S = 200 kVA

$$\cos\phi = 0.8, \text{ Iron loss} = 2000 \text{ W}$$

$$\therefore \qquad \eta\,\% = \frac{200\times10^{3}\times0.8}{200\times10^{3}\times0.8+2000+\text{ Copper loss}}\times100$$

$$0.98\,[200\times10^{3}\times0.8+2000+\text{Copper loss}] = 200\times10^{3}\times0.8$$

i) **Copper loss at full load = 1265.306 watt**

ii) Half load copper loss = $n^2 \times (W_{Cu})_{\text{Full load}}$ where n = 0.5 as half load

$$= (0.5)^2\times1265.306$$

$$= 316.326 \text{ watt}$$

$$\text{Efficiency at half load} = \frac{100\times10^{3}\times0.8\times100}{100\times10^{-3}\times0.8+2000+316.326}$$

$$\eta\,\% = \mathbf{97.186\ \%}$$

Example 3.15 : *A 250 kVA, single phase transformer has 98.135 % efficiency at full load and 0.8 lagging p.f. The efficiency at half load and 0.8 lagging p.f. is 97.751 %. Calculate the iron loss and full load copper loss.*

Solution : a) The output power at full load

$$= 250\times10^{3}\times0.8 = 200\times10^{3} \text{ watt}$$

The input power at full load

$$= \frac{200\times10^{3}}{0.98135}$$

$$\text{The total loss} = \text{Input} - \text{Output}$$

$$= \frac{200\times10^{3}}{0.98135} - 200\times10^{3} = 3800.88$$

$$\therefore \qquad P_i + P_c = 3800.88 \text{ watt} \qquad \text{... (i)}$$

where P_i = Iron loss

P_c = Full load copper loss

b) The power output at half load

$$= 125 \times 10^3 \times 0.8 = 100 \times 10^3$$

The power input at half load

$$= \frac{100 \times 10^3}{0.97751}$$

$$\text{Total loss} = \frac{100 \times 10^3}{0.97751} - 100 \times 10^3 = 2300.74$$

$$P_i + (0.5)^2 P_c = 2300.74$$

$$P_i + 0.25 P_c = 2300.74 \quad \text{...(ii)}$$

From equations (i) and (ii),

$$\mathbf{P_i = 2000.18 \text{ watt}}$$

$$\mathbf{P_c = 1800.69 \text{ watt}}$$

Example 3.16 : *A 100 kVA, 1000 V/ 10,000 V, 50 Hz, single-phase transformer has an iron loss of 1200 W. Find the maximum efficiency at 0.8 power factor lagging if the copper loss is 500 W with 6 A in high voltage winding. Also calculate the corresponding regulation if the equivalent leakage referred to HV is 10 ohms.*

[May-2004 (Set-4), Dec.-2003(Set-4), May-2005 (Set-1)]

Solution : 100 kVA, 1000 V/ 10000 V, $P_i = 1200$ W, $\cos \phi = 0.8$

$$P_{cu} \text{ on } I_2 = 6 \text{ A is } 500 \text{ W}, X_{2e} = 10\ \Omega$$

For η_{max},

$$P_{cu} = P_i = 1200 \text{ W}$$

$$(I_2)\text{F.L.} = \frac{\text{VA rating}}{V_2} = \frac{100 \times 10^3}{10000} = 10 \text{ A}$$

$$\therefore \quad \frac{P_{cu} \text{ on any load}}{(P_{cu})\text{F.L.}} = \left[\frac{(I_2)\text{load}}{(I_2)\text{F.L.}}\right]^2 \quad \text{...As } P_{cu} \propto I^2$$

$$\therefore \quad \frac{500}{(P_{cu})\text{F.L.}} = \left(\frac{6}{10}\right)^2$$

$$\therefore \quad (P_{cu})\text{F.L.} = 1388.88 \text{ W}$$

$$\text{kVA at } \eta_{max} = (\text{kVA rating}) \times \sqrt{\frac{P_i}{(P_{cu})\text{F.L.}}} = 100 \times \sqrt{\frac{1200}{1388.88}}$$

$$= 92.9518 \text{ kVA}$$

$$\therefore \quad \% \eta_{max} = \frac{\text{kVA for } \eta_{max} \cos\phi}{\text{kVA for } \eta_{max} \cos\phi + 2P_i} \times 100$$

$$= \frac{92.9518 \times 10^3 \times 0.8}{92.9518 \times 10^3 \times 0.8 + (2 \times 1200)} \times 100 = \mathbf{96.8734\ \%}$$

$$I_{2m} \text{ at } \eta_{max} = (I_2)\text{F.L.} \times \sqrt{\frac{P_i}{(P_{cu})\text{F.L.}}} = 10 \times \sqrt{\frac{1200}{1388.88}} = 9.2951 \text{ A}$$

$$\therefore \quad P_{cu} \text{ at } \eta_{max} = P_i = I_{2m}^2 R_{2e}$$

$$\therefore \quad 1200 = (9.2951)^2 R_{2e}$$

$$\therefore \quad R_{2e} = 13.889\ \Omega$$

$$\therefore \quad \% R = \frac{I_{2m}[R_{2e} \cos\phi + X_{2e} \sin\phi]}{V_2} \times 100$$

$$= \frac{9.2951[13.889 \times 0.8 + 10 \times 10 \times 0.6]}{10000} \times 100 = \mathbf{1.59\ \%}$$

Example 3.17 : *A 5 kVA, 2300/230 V, 50 Hz transformer was tested for the iron losses with normal excitation and copper losses at full load and these were found to be 40 W and 112 W respectively. Calculate the efficiencies of the transformer at 0.8 power factor for the following kVA outputs :*
1.25, 2.5, 3.75, 5.0, 6.25 and 7.5
Plot efficiency Vs kVA output curve.

Solution : 5 kVA, 2300/230 V, $P_i = 40$ W, P_{cu} (F.L.) = 112 W, $\cos\phi = 0.8$

Sr. No.	kVA output	n = Fraction of full load $= \frac{\text{Actual kVA}}{\text{Total kVA}}$	New P_{cu} = $n^2\ P_{cu}(\text{F.L.})$	$\%\eta = \frac{n[\text{Total VA}] \cos\phi}{n[\text{Total VA}] \cos\phi + P_i + \text{New } P_{cu}} \times 100$
1	1.25	0.25	7	95.51 %
2	2.5	0.5	28	96.711 %
3	3.75	0.75	63	96.668 %
4	5	1	112	96.339 %
5	6.25	1.25	175	95.877 %
6	7.5	1.5	252	95.359 %

The efficiency against kVA output curve is shown in the Fig. 3.28.

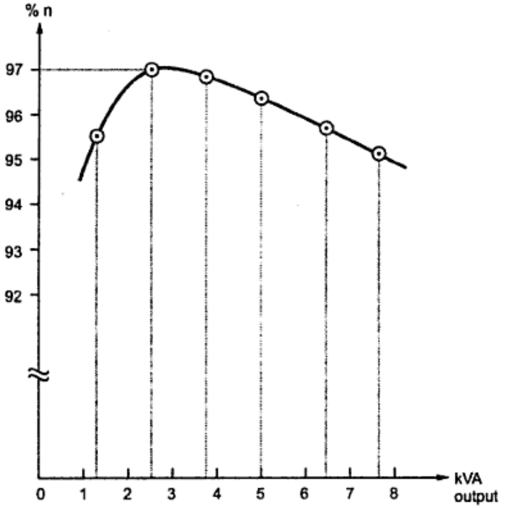


Fig. 3.28

Example 3.18 : *A single phase transformer with 10 : 1 turns ratio and rated at 50 kVA, 2400/240 V, 50 Hz is used to step down the voltage of a distribution system. The low tension voltage is to be kept constant at 240 V. Find the value of load impedance of the low tension side so that transformer will be loaded fully. Find also the value of maximum flux inside the core if the low tension side has 23 turns.* **[JNTU : Nov.-2008 (Set-1)]**

Solution : 50 kVA, $V_1 = 2400$ V, $V_2 = 240$ V, $N_2 = 23$

$$K = \frac{N_2}{N_1} = \frac{1}{10} = 0.1$$

$$I_2(FL) = \frac{VA}{V_2} = \frac{50 \times 10^3}{240} = 208.333 \text{ A}$$

$$R_L = \frac{V_2}{I_2(FL)} = \frac{240}{208.333} = \mathbf{1.152\ \Omega}$$

From the e.m.f equation,

$$240 = 4.44 \times 50 \times \phi_m \times 23$$

$$\therefore \quad \phi_m = \mathbf{47\ mWb}$$

Example 3.19 : *A single phase 2300/230 V, 50 Hz, core type transformer has core section of 0.05 m^2. If the permissible maximum flux density is 1.1 Wb/m^2, calculate the number of turns on primary and secondary.* **[JNTU : Nov.-2008 (Set-4)]**

Solution : $E_1 = 2300$ V, $E_2 = 230$ V, f = 50 Hz, $B_m = 1.1$ Wb/m^2, A = 0.05 m^2

$$B_m = \frac{\phi_m}{A} \quad \text{i.e.} \quad 1.1 = \frac{\phi_m}{0.05}$$

$$\therefore \quad \phi_m = 0.055 \text{ Wb} \quad \text{... Maximum flux in the core}$$

$$E_1 = 4.44\,\phi_m\,f\,N_1 \quad \text{i.e.} \quad 2300 = 4.44 \times 0.055 \times 50 \times N_1$$

$$\therefore \quad N_1 = 188.37 \approx \mathbf{188}$$

$$E_2 = 4.44\,\phi_m f\,N_2 \quad \text{i.e.} \quad 230 = 4.44 \times 0.055 \times 50 \times N_2$$

$$\therefore \quad N_2 = 18.837 \approx \mathbf{19}$$

Example 3.20 : *A 11000/230 V, 150 kVA, 1 phase, 50 Hz transformer has core loss of 1.4 kW and full load copper loss of 1.6 kW. Determine :*

a) kVA load for maximum efficiency and value of maximum efficiency at unity power factor.

b) The efficiency at half load 0.8 power factor leading.

Solution : 150 kVA, $P_i = 1.4$ kW, $P_{cu}(FL) = 1.6$ kW

a) $$\text{kVA for } \eta_{max} = \text{kVA} \times \sqrt{\frac{P_i}{P_{cu}(FL)}} = 150 \times \sqrt{\frac{1.4}{1.6}}$$

$$= \mathbf{140.3121\ kVA}$$

For maximum efficiency, $P_{cu} = P_i = 1.4\,\text{kW}$ and $\cos\phi = 1$

$$\therefore \quad \%\,\eta_{max} = \frac{\text{VA for } \eta_{max} \times \cos\phi}{\text{VA for } \eta_{max} \times \cos\phi + 2P_i} \times 100$$

$$= \frac{140.3121 \times 10^3 \times 1}{140.3121 \times 10^3 \times 1 + 2 \times 1.4 \times 10^3} \times 100 = \mathbf{98.043\ \%}$$

b) At half load, n = 0.5, $\cos\phi = 0.8$

$$\therefore \quad \%\,\eta_{HL} = \frac{n \times VA \times \cos\phi}{n \times VA \times \cos\phi + P_i + [n^2 \times P_{cu}(FL)]} \times 100$$

$$= \frac{0.5 \times 150 \times 10^3 \times 0.8}{0.5 \times 150 \times 10^3 \times 0.8 + 1.4 \times 10^3 + [0.5^2 \times 1.6 \times 10^3]} \times 100 = \mathbf{97.087\ \%}$$

Example 3.21 : *A double wound single phase transformer is required to step down from 1900 V to 240 V at 50 Hz. It is to have 1.5 V/turn. Calculate the required number of turns on the primary and secondary windings respectively. The peak value of the flux density required is 1.5 Wb/m² hence calculate the required cross-sectional area of the core. If the output is 10 kVA, find the secondary current.* **[JNTU : Nov.-2004 (Set -1), May-2005 (Set-3)]**

Solution : $E_1 = 1900$ V, $E_2 = 240$ V, f = 50 Hz, $B_m = 1.5\ \text{Wb/m}^2$,

$$\text{e.m.f/turn} = \frac{E_1}{N_1} = \frac{E_2}{N_2}$$

$$\therefore \quad 1.5 = \frac{1900}{N_1} = \frac{240}{N_2}$$

$$\therefore \quad N_1 = \mathbf{1266.67 \approx 1267}, \quad \mathbf{N_2 = 160}$$

$$E_1 = 4.44\,\phi_m f N_1 \quad \text{i.e.} \quad 1900 = 4.44\,\phi_m \times 50 \times 1267$$

$$\therefore \quad \phi_m = 6.7567 \times 10^{-3}\ \text{Wb}$$

$$B_m = \frac{\phi_m}{A} \quad \text{i.e.} \quad 1.5 = \frac{6.7567 \times 10^{-3}}{A}$$

$$\therefore \quad A = 4.5045 \times 10^{-3}\ \text{m}^2 = \mathbf{45.045\ cm^2} \qquad \text{... Area of core}$$

$$I_2 = \frac{\text{Output VA}}{E_2} = \frac{10 \times 10^3}{240} = \mathbf{41.667\ A}$$

Example 3.22 : *The efficiency of a 250 kVA, single phase transformer is 96 % when delivering full load at 0.8 p.f. lagging and 97.2 % when delivering half load at unity p.f. Determine the efficiency at 75 % of full load at 0.8 p.f. lagging.* **[JNTU : Nov.-2004 (Set-4)]**

Solution : $\%\eta_{FL} = 96\ \%$, $\cos\phi = 0.8$, $\%\eta_{HL} = 97.2\ \%$, $\cos\phi = 1$

$$\%\eta_{FL} = \frac{\text{VA}\cos\phi}{\text{VA}\cos\phi + P_i + P_{cu}(FL)} \times 100 \quad \text{i.e.} \quad 0.96 = \frac{250 \times 10^3 \times 0.8}{250 \times 10^3 \times 0.8 + P_i + P_{cu}(FL)}$$

$$\therefore \quad P_i + P_{cu}(FL) = 8333.333 \qquad \text{... (1)}$$

$$\%\eta_{HL} = \frac{n\text{VA}\cos\phi}{n\,\text{VA}\cos\phi + P_i + n^2 P_{cu}(FL)} \times 100 \qquad \text{... n = 0.5 for half load}$$

$$\therefore \quad 97.2 = \frac{0.5 \times 250 \times 10^3 \times 1}{0.5 \times 250 \times 10^3 \times 1 + P_i + (0.5)^2 P_{cu}(FL)} \times 100$$

$$\therefore \quad P_i + (0.5)^2 P_{cu}(FL) = 3600.823 \qquad \text{.. (2)}$$

Solving equations (1) and (2), P_i = **2023.319 W**, $P_{cu}(FL)$ = **6310.013 W.**

At 75 % of full load, n = 0.75 and $\cos\phi = 0.8$

$$\therefore \quad \%\eta = \frac{nVA\cos\phi}{nVA\cos\phi + P_i + n^2 P_{cu}(FL)} \times 100$$

$$= \frac{0.75\times 250\times 10^3 \times 0.8}{[0.75\times 250\times 10^3\times 0.8] + 2023.319 + [(0.75)^2\times 6310.013]}\times 100$$

$$= \mathbf{96.4179\ \%}$$

Example 3.23 : *In a no load test of a single phase transformer the following data was obtained :*

i) Primary voltage = 220 V ii) Secondary voltage = 110 V

iii) Primary current = 0.5 A iv) Power input = 30 W

v) Resistance of primary winding = 0.6 Ω

Calculate : a) Turns ratio b) Magnetizing component of no load current

c) Working component of no load current d) Iron loss

[JNTU : May-2005 (Set-4), Nov.-2005 (Set-3)]

Solution : $V_o = 220$ V, $I_o = 0.5$ A, $W_o = 30$ W, $R_1 = 0.6\ \Omega$

$$W_o = V_o I_o \cos\phi_o \quad \text{i.e.} \quad \cos\phi_o = \frac{30}{220\times 0.5}$$

$$\therefore \quad \cos\phi_o = 0.27272 \quad \text{i.e.} \quad \phi_o = 74.1733^\circ$$

a) $$K = \text{Turns ratio} = \frac{\text{Secondary voltage}}{\text{Primary voltage}} = \frac{110}{220} = \mathbf{0.5}$$

b) $$I_m = I_o\cos\phi_o = 0.5\times 0.27272 = \mathbf{0.13636\ A.}$$

c) $$I_c = I_o\sin\phi_o = 0.5\times\sin(74.1733^\circ) = \mathbf{0.481\ A.}$$

d) $$P_i = \text{Iron loss} = W_o - \text{No load copper loss}$$

$$= W_o - I_o^2 R_1 = 30 - (0.5)^2\times 0.6 = \mathbf{29.85\ W}$$

Example 3.24 : *In a 50 kVA, 11 kV/400 V transformer, the iron and copper looses are 500 W and 600 W respectively under rated conditions. (a) Calculate the efficiency on full load at unity p.f. (b) Find the load for maximum efficiency.*

[JNTU : Nov.-2005 (Set-2), March-2006 (Set-4)]

Solution : 50 kVA, $P_i = 500$ W, $P_{cu}(FL) = 600$ W

a) $$\cos\phi = 1, \text{ Full load}$$

$$\therefore \quad \%\eta = \frac{VA\cos\phi}{VA\cos\phi + P_i + P_{cu}(FL)}\times 100 = \frac{50\times 10^3\times 1}{50\times 10^3\times 1 + 500 + 600}\times 100$$

$$= \mathbf{97.847\ \%}$$

b) $$\text{kVA at } \eta_{max} = \text{kVA} \times \sqrt{\frac{P_i}{P_{cu}(FL)}} = 50\sqrt{\frac{500}{600}} = 45.64\dot{3}\dot{5} \text{ kVA}$$

$$I_2(FL) = \frac{VA}{V_2} = \frac{50 \times 10^3}{400} = 125 \text{ A}$$

$$\therefore \quad I_{2m} = I_2(FL) \times \sqrt{\frac{P_i}{P_{cu}(FL)}} = 125\sqrt{\frac{500}{600}} = \mathbf{114.1088 \text{ A.}}$$

Example 3.25 : *A single phase transformer has 400 primary and 1000 secondary turns. The net cross-sectional area of the core is 60 cm^2. If the primary winding is connected to a 50 Hz supply at 520 V. Calculate :*

a) Peak value of flux density in the core *b) Transformation ratio*

c) Voltage induced in the secondary *d) E.M.F. induced/turn*

[JNTU : May-2005 (Set-2), March-2006 (Set-3)]

Solution : $N_1 = 400$, $N_2 = 1000$, $A = 60 \text{ cm}^2$, $f = 50$ Hz, $E_1 = 520$ V

a) $$E_1 = 4.44\, \phi_m f N_1 \quad \text{i.e.} \quad 520 = 4.44\, \phi_m \times 50 \times 400$$

$$\therefore \quad \phi_m = 5.8558 \times 10^{-3} \text{ Wb}$$

$$\therefore \quad B_m = \frac{\phi_m}{A} = \frac{5.8558 \times 10^{-3}}{60 \times 10^{-4}} = 0.9759 \text{ Wb/m}^2 \quad \text{... Flux density}$$

b) $$K = \frac{N_2}{N_1} = \frac{1000}{400} = \mathbf{2.5} \quad \text{... Transformation ratio.}$$

c) $$\frac{N_2}{N_1} = \frac{E_2}{E_1} \quad \text{i.e.} \quad \frac{1000}{400} = \frac{E_2}{520}$$

$$\therefore \quad E_2 = \mathbf{1300 \text{ V}} \quad \text{... Secondary induced e.m.f.}$$

d) $$E_1/\text{turn} = \frac{E_1}{N_1} = \frac{520}{400} = \mathbf{1.3 \text{ V/turn}} \quad \text{... Primary}$$

$$E_2/\text{turn} = \frac{E_2}{N_2} = \frac{1300}{1000} = \mathbf{13 \text{ V/turn}} \quad \text{... Secondary}$$

Example 3.26 : *The following readings were obtained from O.C. and S.C. tests on 8 kVA, 400/120 V, 50 Hz transformer.*

O.C. test on L.V. side	120 V	4 A	75 W
S.C. test on H.V. side	9.5 V	20 A	110 W

Calculate the voltage regulation and efficiency at full load, 0.8 p.f. lagging.

[JNTU : Nov.-2004 (Set-2), Nov.-2005 (Set-3), March-2006 (Set-2)]

Solution : From O.C. test, P_i = Iron losses = 75 W

From S.C. test, V_{sc} = 9.5 V, I_{sc} = 20 A , W_{sc} = 110 W

The meters are on H.V. side i.e. primary hence,

$$Z_{1e} = \frac{V_{sc}}{I_{sc}} = \frac{9.5}{20} = 0.475\ \Omega$$

$$R_{1e} = \frac{W_{sc}}{I_{sc}^2} = \frac{110}{(20)^2} = 0.275\ \Omega$$

$$\therefore \quad X_{1e} = \sqrt{Z_{1e}^{\ 2} - R_{1e}^{\ 2}} = 0.3873\ \Omega$$

$$I_1(FL) = \frac{VA}{V_1} = \frac{8\times10^3}{400} = 20\ A = I_{sc}$$

$$\therefore \quad W_{sc} = P_{cu}(FL) = 110\ W \qquad \text{... Copper losses on full load}$$

For $\cos\phi = 0.8, \quad \sin\phi = 0.6$

$$\therefore \quad \%\,R = \frac{I_1(FL)[R_{1e}\cos\phi + X_{1e}\sin\phi]}{V_1} \times 100$$

$$= \frac{20\,[0.275\times0.8+0.3873\times0.6]}{400} \times 100 = \mathbf{2.2619\ \%}$$

$$\%\,\eta_{FL} = \frac{VA\cos\phi}{VA\cos\phi + P_i + P_{cu}(FL)} \times 100$$

$$= \frac{8\times10^3\times0.8}{8\times10^3\times0.8+75+110} \times 100 = \mathbf{97.19\ \%}$$

Example 3.27 : *A 200 kVA single phase transformer is in circuit continuously for 8 hours in a day, the load is 160 kW at 0.8 power factor for 6 hours, the load is 80 kW at unity power factor for 4 hours and for the remaining period of 24 hours it runs on no load. Full load copper losses are 3.02 kW and the iron losses are 1.6 kW. Find all day efficiency.*

[JNTU : Nov.-2008 (Set-3)]

Solution : Load distribution in hours is as given in the table.

6 hours	160 kW	0.8 p.f.
4 hours	80 kW	1 p.f.
14 hours	0 kW	-

P_i = Iron loss = 1.6 kW

$P_{cu}(FL)$ = 3.02 kW

As iron losses are constant for 24 hours, energy spent due to iron losses,

$$P_i = 1.6\times24 = \mathbf{38.4\ kWh}$$

$$\text{Energy Output} = 6\times160+4\times80+14\times0 = 1280\ \text{kWh} \qquad \text{... From given load}$$

To calculate energy spent due to copper loss :

Load 1 : 160 kW, $\cos\phi = 0.8$

$$\therefore \quad kVA = \frac{kW}{\cos\phi} = \frac{160}{0.8} = 200\ kVA \qquad \text{i.e. Full load.}$$

$$\therefore \quad E_1 = P_{cu}(FL) \times hours = 3.02 \times 6 = \mathbf{18.12\ kWh}$$

Load 2 : 80 kW, $\cos\phi = 1$

$$\therefore \quad kVA = \frac{kW}{\cos\phi} = \frac{80}{1} = 80\ kVA$$

$$\therefore \quad n = \text{Fraction of load} = \frac{\text{load kVA}}{\text{kVA rating}} = \frac{80}{200} = 0.4$$

$$\therefore \quad P_{cu} = n^2 \times P_{cu}(FL) = (0.4)^2 \times 3.02 = 0.4832\ kW$$

$$\therefore \quad E_2 = P_{cu} \times hours = 0.4832 \times 4 = \mathbf{1.9328\ \ kWh.}$$

$$\text{Total energy spent} = P_i + E_1 + E_2 = 38.4 + 18.12 + 1.9328 = 58.4528\ kWh$$

$$\therefore \quad \text{All day } \eta = \frac{\text{Total energy output in 24 hours}}{\text{Total energy output for 24 hours} + \text{Total energy spent}} \times 100$$

$$= \frac{1280}{1280 + 58.4528} \times 100 = \mathbf{95.6328\ \%}$$

Review Questions

1. *Explain the principle of working a single phase transformer.*
2. *Explain the construction of a single phase transformer.*
3. *Discuss the difference between core type and shell type of construction.*
4. *Derive from the first principles, the e.m.f. equation for a transformer.*
5. *State the relationships between voltages and currents on primary side and secondary side of a single phase transformer.*
6. *What is kVA rating of a transformer ?*
7. *Explain the various features of an ideal transformer.*
8. *What is the difference between ideal transformer and practical transformer ?*
9. *Draw a no load phasor diagram of a transformer and explain.*
10. *Write a note on various winding parameters of a transformer.*
11. *A 100 V/200 V, 50 Hz, single phase transformer gave the following test results :*
    (No load H.V. side) 1000 V, 0.24 A , 90 W
    (Short circuit L.V. side) 50 V, 5 A , 110 W
    The regulation of the transformer at full load at power factor 0.8 lad is 4.46 %.
    Calculate
    i) Rating of the transformer *ii) Equivalent circuit of transformer with circuit constants*
    iii) kVA load for maximum efficiency
    iv) Voltage to be applied on H.V. side on full at u.p.f. when the secondary terminal voltage is 200 V. **(Ans. : 10 Ω , 8.89 Ω , 191.08 V, 5 A i) 5 kVA iii) 4.52 kVA iv) 1022 volts)**

12. *A single phase transformer has 480 turns on primary and 90 turns on the secondary. The mean length of flux path in the core is 1.8 m and joints are equivalent to an air gap of 1 mm. The maximum value of the flux density is to be 1.1 T when a potential difference of 2200 volts at 50 Hz is applied to the primary. Assume value of magnetic field strength corresponding to the flux density of 1.1 T in the core to be 400 A/m.*
Calculate
i) The cross-section area of the core ii) Maximum value of the magnetizing current
iii) Secondary voltage on no load. **(Ans. : i) 0.01876 m^2 ii) 1.500 A iii) 412.5 volts)**
13. *A resistance connected the secondary of an ideal transformer has a value of 800 ohms as referred to the primary. The same resistance when connected across the primary has a value of 3.125 ohms as referred to secondary. Find the terms ratio of the transformer.* **(Ans. : 4 : 1)**
14. *A 20 kVA, single phase transformer has 200 turns on the primary and 40 turns on the secondary. The primary is connected to 1000 V, 50 Hz supply. Determine the secondary voltage on open circuit and the current flowing through the two windings on full load.* **(Ans. : 200 V, 20 A, 100 A)**
15. *A single phase transformer has 500 primary and 1000 secondary turns. The area of the core is 75 cm^2. If the primary winding is connected to 400 V, 50 Hz supply, calculate the secondary voltage and the peak value of the flux density.* **(Ans. : 0.4804 Wb / m^2)**
16. *A 20 kVA, single phase transformer has 200 turns on the primary and 40 turns on the secondary. The primary is connected to 1000 V, 50 Hz supply. Determine the secondary voltage on open circuit and the current flowing through the two windings on full load.* **(Ans. : 100 A)**
17. *A sinusoidal flux 0.02 Wb links with 55 turns of a transformer secondary coil. Calculate the r.m.s. value of the induced e.m.f. in the secondary. The supply frequency is 50 Hz.* **(Ans. : 244.2 V)**
18. *A 100 kVA, 1100/200 volt single phase transformer has the following parameters;* $r_1 = 0.1\ \Omega$, $x_1 = 0.3\ \Omega$, $r_2 = 0.004\ \Omega$, $x_2 = 0.012\ \Omega$. *Find equivalent resistance and leakage reactance as referred to high voltage winding.* **(Ans. : 0.221 Ω, 0.663 Ω)**
19. *Define regulation, stating an expression to obtain it.*
20. *Enumerate the various losses in a transformer. How these losses can be minimized ?*
21. *What do you understand by efficiency of a transformer ?*
22. *Derive the condition for maximum efficiency.*
23. *Explain the short circuit test and open circuit test on transformer. Why these tests are to be performed ?*
24. *A 5 kVA, 250/500 V, 50 Hz, 1-phase transformer gave the following test results :*
No load 250 V, 0.75 amp, 60 W (L.V. side). Calculate
a) The equivalent circuit constants and insert these on the equivalent circuit diagram.
b) The efficiency at 60 % of full load at which it occurs.
c) The maximum efficiency and the load at which it occurs.
d) The secondary terminal voltage on full load at p.f. 0.8 lagging unity and 0.8 leading.
(Ans. : a) 1041.67 Ω, 351.864 Ω, 0.6 Ω, 1.5 Ω, 1.3747 Ω b) 97.35 % c) 97.65 % d) 486.95 V, 496 V, 503, 4482 V)

25. *A 1-phase, 10 kVA, 500.250 V, 50 Hz transformer has the following constants :*

    Resistance : Primary -0.2 ohm, secondary -0.5 ohm

    Reactance : Primary -0.4 ohm, secondary -0.5 ohm

    Resistance of equivalent exciting circuit referred to primary, R_0 = 1500 ohms

    Reactance of equivalent exciting circuit referred to primary, X_0 = 750 ohms.

    What would be the readings of instruments (placed in primary i.e. H.V. side) when the transformer is connected for the open circuit and short circuit test question

    (Ans. : Open ckt. i) 500 V, ii) I_0 = 0.745 A iii) W_0 = 166.69 W, Short ckt. i) 46.82 V, ii) I_{sc} = 20 A , iii) W_{sc} = 880 W)

26. *A 40 kVA, 400/200 V, 50 Hz, single phase transformer gave the following results :*

    Open circuit test : 400 V, 5 A, 500 W

    Short circuit test : 10 V, 50 A, 150 W

    i) Draw the equivalent circuit of the transformer and shown the values of resistance and reactance interms of primary side.

    ii) Calculate the terminal voltage, when the transformer delivers the solid output at a p.f. of 0.8 lagging on the L.V. Side. **(Ans. : i) $R_{1e} = 0.06\ \Omega$, $Z_{1e} = 0.2\ \Omega$, $X_{1e} = 0.1907\ \Omega$, ii) 191.88 V)**

□□□

4

Alternators

4.1 Introduction

It is known that the electric supply used, nowadays for commercial as well as domestic purposes, is of alternating type.

Similar to d.c. machines, the a.c. machines associated with alternating voltages, are also classified as generators and motors.

The machines generating a.c. e.m.f. are called **alternators or synchronous generators.** While the machines accepting input from a.c. supply to produce mechanical output are called **synchronous motors**. Both these machines work at a specific constant speed called **synchronous speed** and hence in general called **synchronous machines.**

All the modern power stations consists of large capacity three phase alternators. In this chapter, the construction, working principle and the e.m.f. equation of three phase alternator is discussed.

4.2 Difference between D.C. Generator and Alternator

It is seen that in case of a d.c. generator, basically the nature of the induced e.m.f. in the armature conductors is of alternating type. By using commutator and brush assembly it is converted to d.c. and made available to the external circuit. If commutator is dropped from a d.c. generator and induced e.m.f. is tapped outside from an armature directly, the nature of such e.m.f. will be alternating. Such a machine without commutator, providing an a.c. e.m.f. to the external circuit is called an **alternator.** The obvious question is how is it possible to collect an e.m.f. from the rotating armature without commutator ?

> **Key Point :** *So the arrangement which is used to collect an induced e.m.f. from the* ***rotating armature*** *and make it available to the* ***stationary circuit*** *is called* ***slip ring and brush assembly.***

4.2.1 Concept of Slip Rings and Brush Assembly

Whenever there is a need of developing a contact between rotating element and the stationary circuit without conversion of an e.m.f. from a.c. to d.c., the slip rings and brush assembly can be used.

In case of three phase alternators, the armature consist of three phase winding and an a.c. e.m.f. gets induced in these windings. After connecting windings in star or delta, the three ends of the windings are brought out. Across these terminals three phase supply is available. But the armature i.e. these terminals are rotating and hence stationary load cannot be connected directly to them. Hence slip rings, made up of conducting material are mounted on the shaft. Each terminal of winding is connected to an individual slip ring, permanently. Hence three phase supply is now available across the rotating slip rings. The brushes are resting on the slip rings, just making contact.

Key Point : *The brushes are stationary. Hence as brushes make contact with the slip rings, the three phase supply is now available across the brushes which are stationary.*

Hence any stationary load can then be connected across these stationary terminals available from the brushes. The schematic arrangement is shown in the Fig. 4.1.

Not only the induced e.m.f. can be taken out from the rotating winding outside but an induced e.m.f. can be injected to the rotating winding from outside with the help of slip ring and brush assembly. The external voltage can be applied across the brushes, which gets applied across the rotating winding due to the sliprings.

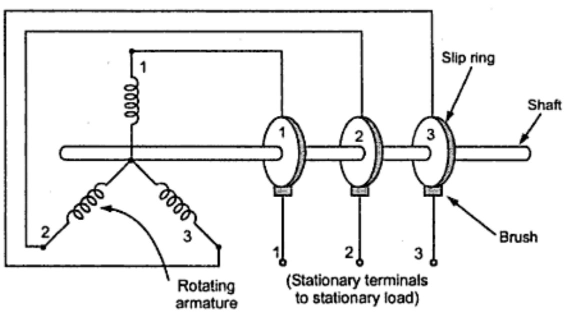


Fig. 4.1 Arrangement of slip rings

Now the induced e.m.f. is basically the effect of the relative motion present between an armature and the field. Such a relative motion is achieved by rotating armature with the help of prime mover, in case of a d.c. generator. As armature is connected to commutator in a d.c. generator, armature must be a rotating member while field as a stationary. But in case of alternators it is possible to have,

i) The rotating armature and stationary field.

ii) The rotating field and stationary armature.

Key Point : *But practically most of the alternators prefer rotating field type construction with stationary armature due to certain advantages.*

4.3 Advantages of Rotating Field over Rotating Armature

1) As everywhere a.c. is used, the generation level of a.c. voltage may be higher as 11 kV to 33 kV. This gets induced in the armature. For stationary armature large space can be provided to accommodate large number of conductors and the insulation.

2) It is always better to protect high voltage winding from the centrifugal forces caused due to the rotation. So high voltage armature is generally kept stationary. This avoids the interaction of mechanical and electrical stresses.

3) It is easier to collect larger currents at very high voltages from a stationary member than from the slip ring and brush assembly. The voltage required to be supplied to the field is very low (110 V to 220 V d.c.) and hence can be easily supplied with the help of slip ring and brush assembly by keeping it rotating.

4) The problem of sparking at the slip rings can be avoided by keeping field rotating which is low voltage circuit and high voltage armature as stationary.

5) Due to low voltage level on the field side, the insulation required is less and hence field system has very low inertia. It is always better to rotate low inertia system than high inertia, as efforts required to rotate low inertia system are always less.

6) Rotating field makes the overall construction very simple. With simple, robust mechanical construction and low inertia of rotor, it can be driven at high speeds. So greater output can be obtained from an alternator of given size.

7) If field is rotating, to excite it by an external d.c. supply two slip rings are enough. One each for positive and negative terminals. As against this, in three phase rotating armature, the minimum number of slip rings required are three and cannot be easily insulated due to high voltage levels.

8) The ventilation arrangement for high voltage side can be improved if it is kept stationary.

Due to all these reasons the most of the alternators in practice use rotating field type of arrangement. For small voltage rating alternators rotating armature arrangement may be used.

4.4 Construction

As mentioned earlier, most of the alternators prefer rotating field type of construction. In case of alternators the winding terminology is slightly different than in case of d.c. generators. In alternators the stationary winding is called 'Stator' while the rotating winding is called 'Rotor'.

Key Point : *Most of the alternators have* ***stator as armature*** *and* ***rotor as field****, in practice.*

Constructional details of rotating field type of alternator are discussed below.

4.5 Stator

The stator is a stationary armature. This consists of a core and the slots to hold the armature winding similar to the armature of a d.c. generator. The stator core uses a laminated construction. It is built up of special steel stampings insulated from each other with varnish or paper. The laminated construction is basically to keep down eddy current losses. Generally choice of material is steel to keep down hysteresis losses.

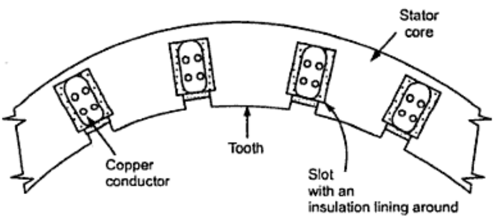


Fig. 4.2 Section of an alternator stator

The entire core is fabricated in a frame made of steel plates. The core has slots on its periphery for housing the armature conductors. Frame does not carry any flux and serves as the support to the core. Ventilation is maintained with the help of holes cast in the frame. The section of an alternator stator is shown in the Fig. 4.2.

4.6 Rotor

There are two types of rotors used in alternators,

i) Salient pole type and ii) Smooth cylindrical type.

4.6.1 Salient Pole Type Rotor

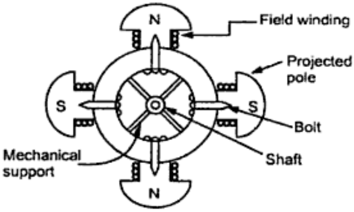


Fig. 4.3 Salient pole type rotor

This is also called **projected pole type as all the poles are projected out from the surface of the rotor.**

The poles are built up of thick steel laminations. The poles are bolted to the rotor as shown in the Fig. 4.3. The pole face has been given a specific shape as discussed earlier in case of d.c. generators. The field winding is provided on the pole shoe. These rotors have large diameters and small axial lengths. The limiting factor for the size of the rotor is the centrifugal force acting on the rotating member of the machine. As mechanical strength of salient pole type is less, this is preferred for low speed alternators ranging from 125 r.p.m. to 500 r.p.m. The prime movers used to drive such rotor are generally water turbines and I.C. engines.

4.6.2 Smooth Cylindrical Type Rotor

This is also called **non salient type** or **non projected pole type of rotor.**

The rotor consists of smooth solid steel cylinder, having number of slots to accommodate the field coil. The slots are covered at the top with the help of steel or manganese wedges. The unslotted portions of the cylinder itself act as the poles. The poles are not projecting out and the surface of the rotor is smooth which maintains uniform air gap between stator and the rotor. These rotors have small diameters and large axial lengths. This is to keep peripheral speed within limits. The main advantage of this type is that these are mechanically very strong and thus preferred for high speed alternators ranging between 1500 to 3000 r.p.m. Such high speed alternators are called **'turboalternators'**. The prime movers used to drive such type of rotors are generally steam turbines, electric motors.

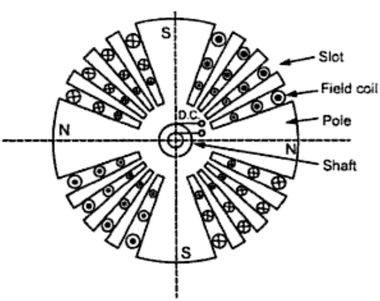


Fig. 4.4 Smooth cylindrical rotor

The Fig. 4.4 shows smooth cylindrical type of rotor.

Let us list down the differences between the two types in tabular form.

4.6.3 Difference between Salient and Cylindrical Type of Rotor

Sr. No.	Salient Pole Type	Smooth Cylindrical Type
1.	Poles are projecting out from the surface.	Unslotted portion of the cylinder acts as poles hence poles are non projecting.
2.	Air gap is nonuniform.	Air gap is uniform due to smooth cylindrical periphery.
3.	Diameter is high and axial length is small.	Small diameter and large axial length is the feature.
4.	Mechanically weak.	Mechanically robust.
5.	Preferred for low speed alternators.	Preferred for high speed alternators i.e. for turboalternators.
6.	Prime mover used are water turbines, I.C. engines.	Prime movers used are steam turbines, electric motors.
7.	For same size, the rating is smaller than cylindrical type.	For same size, rating is higher than salient pole type.
8.	Separate damper winding is provided.	Separate damper winding is not necessary.

4.7 Excitation System

The synchronous machines whether alternator or motor are necessarily separately excited machines. Such machines always require d.c. excitation for their operation. The field systems are provided with direct current which is supplied by a d.c. source at 125 to 600 V. In many cases the exciting current is obtained from a d.c. generator which is mounted on the same shaft of that of alternator. Thus excitation systems are of prime importance. Many of the conventional system involves slip rings, brushes and commutators.

4.7.1 Brushless Excitation System

With the increase in rating of an alternator, the supply of necessary magnetic field becomes difficult as the current values may reach upto 4000 A. If we use conventional excitation systems such as a d.c. generator whose output is supplied to the alternator field through brushes and slip rings, then problems are invariably associated with slip rings,

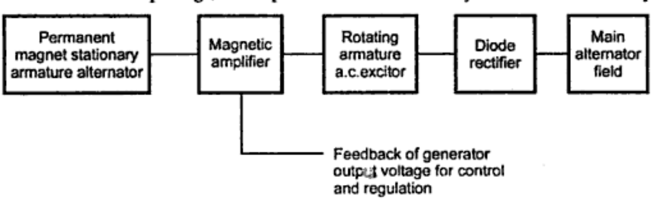


Fig. 4.5 Brushless excitation system

commutators and brushes regarding cooling and maintenance. Thus modern excitation systems are developed which minimizes these problems by avoiding the use of brushes. Such excitation system is called **brushless excitation system** which is shown in the Fig. 4.5.

It consists of silicon diode rectifiers which are mounted on the same shaft of alternator and will directly provide necessary excitation to the field. The power required for rectifiers is provided by an a.c. excitor which is having stationary field but rotating armature.

The field of an excitor is supplied through a magnetic amplifier which will control and regulate the output voltage of the alternator since the feedback of output voltage of alternator is taken and given to the magnetic amplifier. The system can be made self contained if the excitation power for the magnetic amplifier is obtained from a small permanent magnet alternator having stationary armature which is driven from the main shaft. The performance and design of the overall system can be optimized by selecting proper frequency and voltage for a.c. excitor. The additional advantage that can be obtained with this system is that it is not necessary to make arrangement for spare excitors, generator-field circuit breakers and field rheostats.

4.8 Methods of Ventilation

1. Natural ventilation : A fan is attached to either ends of the machine. The ventilating medium is nothing but an atmospheric air which is forced over the machine parts, carrying away the heat. This circulation is possible with or without ventilating ducts. The ventilating ducts if provided may be either axial or radial.

2. Closed circuit ventilating system : An atmospheric air may contain injurious elements like dust, moisture, acidic fumes etc. which are harmful for the insulation of the winding. Hence for large capacity machines, closed circuit system is preferred for ventilation. The ventilating medium used is generally hydrogen. The hydrogen circulated over the machine parts is cooled with the help of water cooled heat exchangers. Hydrogen provides very effective cooling than air which increases the rating of the machine upto 30 to 40 % for the same size. All modern alternators use closed circuit ventilation with the help of hydrogen as a ventilating medium.

4.9 Working Principle

The alternators work on the principle of **electromagnetic induction.** When there is a relative motion between the conductors and the flux, e.m.f. gets induced in the conductors. The d.c. generators also work on the same principle. The only difference in practical alternator and a d.c. generator is that in an alternator the conductors are stationary and field is rotating. But for understanding purpose we can always consider relative motion of conductors with respect to the flux produced by the field winding.

Consider a relative motion of a single conductor under the magnetic field produced by two stationary poles. The magnetic axis of the two poles produced by field is vertical, shown dotted in the Fig. 4.6.

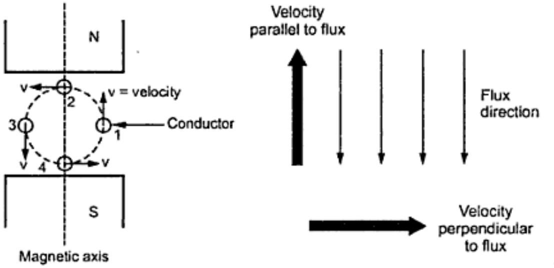


Fig. 4.6 Two pole alternator

Let conductor starts rotating from position 1. At this instant, the entire velocity component is **parallel** to the flux lines. Hence there is no cutting of flux lines by the conductor. So $\frac{d\phi}{dt}$ at this instant is zero and hence induced e.m.f. in the conductor is also zero.

As the conductor moves from position 1 towards position 2, the part of the velocity component becomes perpendicular to the flux lines and proportional to that, e.m.f. gets induced in the conductor. The magnitude of such an induced e.m.f. increases as the conductor moves from position 1 towards 2.

At position 2, the entire velocity component is **perpendicular** to the flux lines. Hence there exists maximum cutting of the flux lines. And at this instant, the induced e.m.f. in the conductor is at its maximum.

As the position of conductor changes from 2 towards 3, the velocity component perpendicular to the flux starts decreasing and hence induced e.m.f. magnitude also starts decreasing. At position 3, again the entire velocity component is parallel to the flux lines and hence at this instant induced e.m.f. in the conductor is zero.

As the conductor moves from position 3 towards 4, the velocity component perpendicular to the flux lines again starts increasing. But the direction of velocity component now is opposite to the direction of velocity component exsisting during the movement of the conductor from position 1 to 2. Hence an induced e.m.f. in the conductor increases but in the opposite direction.

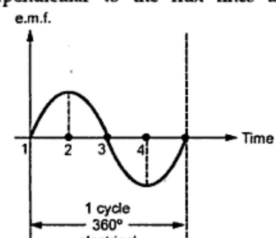


Fig. 4.7 Alternating nature of the induced e.m.f.

At position 4, it achieves maxima in the opposite direction, as the entire velocity component becomes **perpendicular** to the flux lines.

Again from position 4 to 1, induced e.m.f. decreases and finally at position 1, again becomes zero. This cycle continues as conductor rotates at a certain speed.

So if we plot the magnitudes of the induced e.m.f. against the time, we get an alternating nature of the induced e.m.f. as shown in the Fig. 4.7.

This is the working principle of an alternator.

4.9.1 Mechanical and Electrical Angle

We have seen that for 2 pole alternator, one mechanical revolution corresponds to one

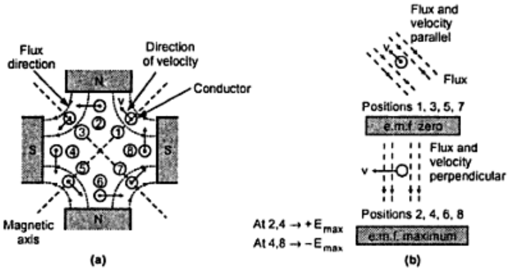


Fig. 4.8 4 Pole alternator

electrical cycle of an induced e.m.f. Now consider 4 pole alternator i.e. the field winding is designed to produce 4 poles. Due to 4 poles, the magnetic axis exists diagonally shown dotted in the Fig. 4.8.

Now in position 1 of the conductor, the velocity component is parallel to the flux lines while in position 2, there is gathering of flux lines and entire velocity component is perpendicular to the flux lines. So at position 1, the induced e.m.f. in the conductor is zero while at position 2, it is maximum. Similarly as conductor rotates, the induced e.m.f. will be maximum at positions 4, 6 and 8 and will be minimum at positions 3, 5 and 7. So during one complete revolution of the conductor, induced e.m.f. will experience four times maxima, twice in either direction and four times zero. This is because of the distribution of flux lines due to existence of four poles.

So if we plot the nature of the induced e.m.f; for one revolution of the conductor, we get the two electrical cycles of the induced e.m.f., as shown in the Fig. 4.9.

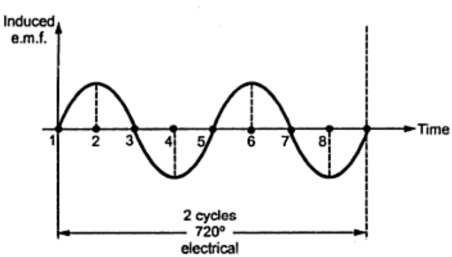


Fig. 4.9 Nature of the induced e.m.f.

Key Point : *Thus the degrees electrical of the induced e.m.f. i.e.* ***number of cycles of the induced e.m.f. depends on the number of poles of an alternator.***

So for a four pole alternator we can write,

$$360^\circ \text{ mechanical} = 720^\circ \text{ electrical}$$

From this we can establish the general relation between degrees mechanical and degrees electrical as,

$$360^\circ \text{ mechanical} = 360^\circ \times \frac{P}{2} \text{ electrical}$$

where P = number of poles

i.e. $$\boxed{1^\circ \text{ mechanical} = \left(\frac{P}{2}\right)^\circ \text{ electrical.}}$$

4.9.2 Frequency of Induced E.M.F.

Let P = Number of poles

N = Speed of the rotor in r.p.m.

and f = Frequency of the induced e.m.f

From the discussion above in section 4.9.1, we can write,

One mechanical revolution of rotor = $\frac{P}{2}$ cycles of e.m.f. electrically

Thus there are P/2 cycles per revolution.

As speed is N r.p.m., in one second, rotor will complete $\left(\frac{N}{60}\right)$ revolutions

But cycles/sec = frequency = f

$\therefore$ Frequency f = (No. of Cycles per revolution) × (No. of revolutions per second)

$$\therefore \quad f = \frac{P}{2} \times \frac{N}{60}$$

$$\therefore \quad \boxed{f = \frac{PN}{120} \quad \text{Hz (cycles per sec).}}$$

So there exists a fixed relationship between three quantities, the number of poles P, the speed of the rotor N in r.p.m. and f the frequency of an induced e.m.f. in Hz (Hertz).

Key Point : *Such a machine bearing a fixed relationship between P, N and f is called* ***synchronous machine*** *and hence alternators are also called* ***synchronous generators.***

4.9.3 Synchronous Speed (N_s)

From the above expression, it is clear that for fixed number of poles, alternator has to be rotated at a particular speed to keep the frequency of the generated e.m.f. constant at the required value. Such a speed is called synchronous speed of the alternator denoted as N_s.

So $$\boxed{N_s = \frac{120\,f}{P}}$$

where f = Required frequency

In our nation, the frequency of an alternating e.m.f. is standard equal to 50 Hz. To get 50 Hz frequency, for different number of poles, alternator must be driven at different speeds called synchronous speeds. Table 4.1 gives the values of the synchronous speeds for the alternators having different number of poles.

Number of poles P	2	4	8	12	24
Synchronous speed N_s in r.p.m.	3000	1500	750	500	250

Table 4.1

From the table, it can be seen that minimum number of poles for an alternator can be two hence maximum value of synchronous speed possible in our nation i.e. for frequency of 50 Hz is 3000 r.p.m.

4.10 Armature Winding

Armature winding of alternators is different from that of d.c. machines. Basically three phase alternators carry three sets of windings arranged in the slots in such a way that there exists a phase difference of 120° between the induced e.m.f.s in them. In a d.c. machine, winding is closed while in alternators winding is open i.e. two ends of each set of winding is brought out. In three phase alternators, the six terminals are brought out which are finally connected in star or delta and then the three terminals are brought out. Each set of windings represents winding per phase and induced e.m.f. in each set is called induced e.m.f. per phase denoted as E_{ph}. All the coils used for one phase must be connected in such a way that their e.m.f.s help each other. And overall design should be in such a way that the waveform of an induced e.m.f. is almost sinusoidal in nature.

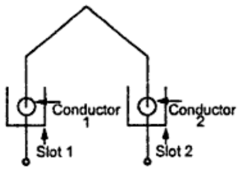
4.10.1 Winding Terminology

i) Conductor : The part of the wire, which is under the influence of the magnetic field and responsible for the induced e.m.f. is called active length of the conductor. The conductors are placed in the armature slots.

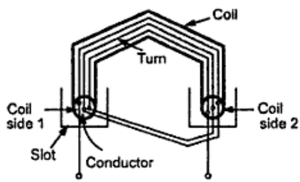
ii) Turn : A conductor in one slot, when connected to a conductor in another slot forms a turn. So two conductors constitute a turn. This is shown in the Fig. 4.10 (a).

iii) Coil : As there are number of turns, for simplicity the number of turns are grouped together to form a coil. Such a coil is called multiturn coil. A coil may consist of single turn called single turn coil. The Fig. 4.10 (b) shows a multiturn coil.

iv) Coil side : When coil consists of many turns, part of the coil in each slot is called coil side of a coil as shown in the Fig. 4.10 (b).



(a) Turn



(b) Multiturn coil

Fig. 4.10

v) Pole pitch : It is centre to centre distance between the two adjacent poles. We have seen that for one rotation of the conductors, 2 poles are responsible for 360° electrical of e.m.f., 4 poles are responsible for 720° electrical of e.m.f. and so on. So **1 pole is responsible for 180° electrical of induced e.m.f.**

Key Point : *So 180° electrical is also called one pole pitch.*

Practically how many slots are under one pole which are responsible for 180° electrical, are measured to specify the pole pitch.

e.g. Consider 2 pole, 18 slots armature of an alternator. Then under 1 pole there are $\frac{18}{2}$ i.e. 9 slots. So pole pitch is 9 slots or 180° electrical. This means 9 slots are responsible to produce a phase difference of 180° between the e.m.f.s induced in different conductors.

This number of slots/pole is denoted as 'n'.

$$\therefore \quad \boxed{\begin{aligned} \text{Pole pitch} &= 180^\circ \text{ electrical} \\ &= \text{Slots per pole (no. of slots/P)} = n \end{aligned}}$$

vi) Slot angle (β) : The phase difference contributed by one slot in degrees electrical is called slot angle β.

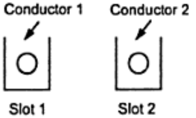
As slots per pole contributes 180° electrical which is denoted as 'n', we can write,

$$\therefore \quad 1 \text{ slot angle} = \frac{180°}{n}$$

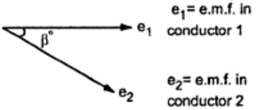
$$\therefore \quad \beta = \frac{180°}{n}$$

In the above example, $n = \frac{18}{2} = 9$, while $\beta = \frac{180°}{n} = 20°$

Note : This means that if we consider an induced e.m.f. in the conductors which are placed in the slots which are adjacent to each other, there will exist a phase difference of β° in between them. While if e.m.f. induced in the conductors which are placed in slots which are 'n' slots distance away, there will exist a phase difference of 180° in between them.



(a) Adjacent slots



(b) Indication of phase difference

Fig. 4.11

4.11 Types of Armature Windings

In general armature winding is classified as,

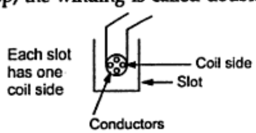
i) Single layer and double layer winding ii) Full pitch and short pitch winding

iii) Concentrated and distributed winding.

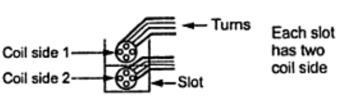
Let us see the details of each classification.

4.11.1 Single Layer and Double Layer Winding

If a slot consists of only one coil side, winding is said to be single layer. This is shown in the Fig. 4.12 (a). While there are two coil sides per slot, one at the bottom and one at the top, the winding is called double layer as shown n the Fig. 4.12 (b).



(a) Single layer



(b) Double layer

Fig. 4.12

A lot of space gets wasted in single layer hence in practice generally double layer winding is preferred.

4.11.2 Full Pitch and Short Pitch Winding

As seen earlier, one pole pitch is 180° electrical. The value of 'n', slots per pole indicates how many slots are contributing 180° electrical phase difference. So if coil side in one slot is connected to a coil side in another slot which is one pole pitch distance away from first slot, the winding is said to be **full pitch winding** and coil is called **full pitch coil.**

For example in 2 pole, 18 slots alternator, the pole pitch is $n = \frac{18}{2} = 9$ slots.

So if coil side in slot No. 1 is connected to coil side in slot No. 10 such that two slots No. 1 and No. 10 are one pole pitch or n slots or 180° electrical apart, the coil is called full pitch coil.

Here we can define one more term related to a coil called coil span.

4.11.2.1 Coil Span

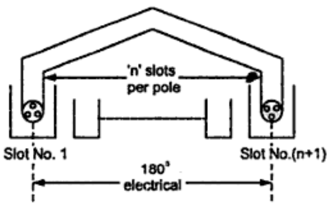


Fig. 4.13 Full pitch coil

It is the distance on the periphery of the armature between two coil sides of a coil. It is usually expressed in terms of number of slots or degrees electrical. So if coil span is 'n' slots or 180° electrical, the coil is called full pitch coil. This is shown in the Fig. 4.13.

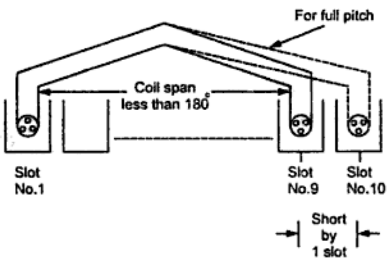


Fig. 4.14 Short pitch coil

As against this if coils are used in such a way that coil span is slightly less than a pole pitch i.e. less than 180° electrical, the coils are called, **short pitched coils or fractional pitched coils**. Generally coils are shorted by one or two slots.

So in 18 slots, 2 pole alternator instead of connecting a coil side in slot No. 1 to slot No.10, it is connected to a coil side in slot No.9 or slot No.8, coil is said to be short pitched coil and winding is called short pitch winding. This is shown in the Fig. 4.14.

4.11.2.2 Advantages of Short Pitch Coils

The various advantages of short pitch coils are :

1. The length required for the end connections of coils is less i.e. inactive length of winding is less. So less copper is required.

2. As copper required is less, it is economical.

3. Short pitching eliminates high frequency harmonics which distort the sinusoidal nature of e.m.f. Hence waveform of an induced e.m.f. is more sinusoidal due to short pitching.

4. As high frequency harmonics get eliminated, eddy current and hysteresis losses which depend on frequency also get minimised.

5. The efficiency increases due to reduced losses.

4.11.3 Concentrated and Distributed Winding

In three phase alternators, we have seen that there are three different sets of windings, each for a phase. So depending upon the total number of slots and number of poles, we have certain slots per phase available under each pole. This is denoted as 'm'.

$$m = \text{Slots per pole per phase} = n/\text{Number of phases}$$

$$= n/3 \text{ (generally no. of phases is 3)}$$

For example in 18 slots, 2 pole alternator we have

$$n = \frac{18}{2} = 9$$

and $$m = \frac{9}{3} = 3$$

So we have 3 slots per pole per phase available. Now let 'x' number of conductors per phase are to be placed under one pole. And we have 3 slots per pole per phase available. But if all 'x' conductors per phase are placed in one slot keeping remaining 2 slots per pole per phase empty then the winding is called **concentrated winding.**

> **Key Point** : *So in concentrated winding all conductors or coils belonging to a phase are placed in one slot under every pole.*

But in practice, an attempt is always made to use all the 'm' slots per pole per phase available for distribution of the winding. So if 'x' conductors per phase are distributed amongst the m slots per phase available under every pole, the winding is called **distributed winding.** So in distributed type of winding all the coils belonging to a phase are well distributed over the 'm' slots per phase, under every pole. Distributed winding makes the waveform of the induced e.m.f. more sinusoidal in nature. Also in concentrated winding due to large number of conductors per slot, heat dissipation is poor.

> **Key Point** : *So in practice, double layer, short pitched and distributed type of armature winding is preferred for the alternators.*

4.12 E.M.F. Equation of an Alternator

Let ϕ = Flux per pole, in Wb

P = Number of poles

N_s = Synchronous speed in r.p.m.

f = Frequency of induced e.m.f. in Hz

Z = Total number of conductors

Z_{ph} = Conductors per phase connected in series

$$\therefore \quad Z_{ph} = \frac{Z}{3} \text{ as number of phases} = 3.$$

Consider a single conductor placed in a slot.

The average value of e.m.f. induced in a conductor $= \dfrac{d\phi}{dt}$

For one revolution of a conductor,

$$e_{avg} \text{ per conductor} = \frac{\text{Flux cut in one revolution}}{\text{Time taken for one revolution}}$$

Total flux cut in one revolution is $\phi \times P$.

Time taken for one revolution is $\dfrac{60}{N_s}$ seconds, as speed is N_s r.p.m.

$$\therefore \; e_{avg} \text{ per conductor} = \frac{\phi P}{\left(\dfrac{60}{N_s}\right)} = \phi \frac{PN_s}{60} \qquad \ldots(1)$$

But

$$f = \frac{PN_s}{120}$$

$$\therefore \quad \frac{PN_s}{60} = 2f$$

Substituting in equation (1),

$$\boxed{e_{avg} \text{ per conductor} = 2\,f\,\phi \quad \text{volts}}$$

Assume full pitch winding for simplicity i.e. this conductor is connected to a conductor which is 180° electrical apart. So these two e.m.f.s will try to set up a current in the same direction i.e. the two e.m.f.s are helping each other and hence resultant e.m.f. per turn will be twice the e.m.f. induced in a conductor.

$$\therefore \quad \text{e.m.f. per turn} = 2 \times (\text{e.m.f. per conductor})$$

$$= 2 \times (2f\,\phi)$$

$$\therefore \quad \boxed{\text{e.m.f. per turn} = 4\,f\,\phi \quad \text{volts}}$$

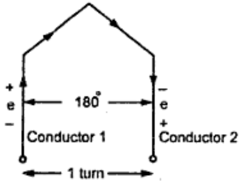


Fig. 4.15 Turn of full pitch coil

Let T_{ph} be the total number of turns per phase connected in series. Assuming concentrated winding , we can say that all are placed in single slot per pole per phase. So induced e.m.f.s in all the turns will be in phase as placed in single slot. Hence net e.m.f. per phase will be algebraic sum of the e.m.f.s per turn.

$$\therefore \quad \text{Average } E_{ph} = T_{ph} \times (\text{Average e.m.f. per turn})$$

$$\therefore \quad \text{Average } E_{ph} = T_{ph} \times 4 f \phi$$

But in a.c. circuits R.M.S. value of an alternating quantity is used for the analysis. The form factor is 1.11 for purely sinusoidal e.m.f.

$$K_f = \frac{\text{R.M.S.}}{\text{Average}} = 1.11 \qquad \text{for sinusoidal}$$

$$\therefore \text{ R.M.S. value of } E_{ph} = K_f \times \text{Average value}$$

$$\therefore \quad E_{ph} = 1.11 \times 4 f \phi T_{ph}$$

$$\boxed{E_{ph} = 4.44 f \phi T_{ph} \quad \text{volts}}$$

This is the basic e.m.f. equation for an induced e.m.f. per phase for full pitch, concentrated type of winding.

where T_{ph} = Number of turns per phase

$$\therefore \quad \boxed{T_{ph} = \frac{Z_{ph}}{2}} \qquad \text{... as 2 conductors constitute 1 turn}$$

But as mentioned earlier, the winding used for the alternators is distributed and short pitch hence e.m.f. induced slightly gets affected. Let us see now the effect of distributed and short pitch type of winding on the e.m.f. equation.

4.12.1 Pitch Factor or Coil Span Factor (K_c)

In practice short pitch coils are preferred. So coil is formed by connecting one coil side to another which is less than one pole pitch away. So actual coil span is less than 180°. The coil is generally shorted by one or two slots.

> **Key Point** : *The angle by which coils are short pitched is called **angle of short pitch** denoted as 'α'.*

α = Angle by which coils are short pitched. As coils are shorted in terms of number of slots i.e. either by one slot, two slots and so on and slot angle is β then angle of short pitch is always a multiple of the slot angle β.

$\therefore$ $\boxed{\begin{array}{ll} & \alpha = \beta \times \text{number of slots by which coils are short pitched.} \\ \text{or} & \alpha = 180^\circ - \text{Actual coil span of the coils} \end{array}}$

This is shown in the Fig. 4.16.

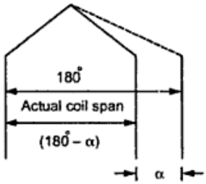


Fig. 4.16 Angle of short pitch

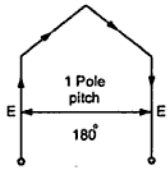


Fig. 4.17 Full pitch coil

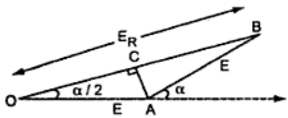


Fig. 4.18 Phasor sum of two e.m.f.s

Now let E be the induced e.m.f. in each coil side. If coil is full pitch coil, the induced e.m.f. in each coil side help each other. Coil connections are such that both will try to set up a current in the same direction in the external circuit. Hence the resultant e.m.f. across a coil will be algebraic sum of the two.

$\therefore \quad E_R = E + E = 2\,E \quad$ ... For full pitch

Now the coil is short pitched by angle 'α', the two e.m.f. in two coil sides no longer remains in phase from external circuit point of view. Hence the resultant e.m.f. is also no longer remains algebraic sum of the two but becomes a phasor sum of the two as shown in the Fig. 4.18.

Obviously E_R in such a case will be less than what it is in case of full pitch coil.

From the geometry of the Fig. 4.18, we can write,

AC is perpendicular drawn on OB bisecting OB.

$$\therefore \quad l(OC) = l(CB) = \frac{E_R}{2}$$

and $\quad \angle BOA = \alpha/2 \quad$ as $\quad l(OA) = l(AB) = E$

$$\therefore \quad \cos(\alpha/2) = \frac{OC}{OA} = \frac{E_R}{2E}$$

$$\therefore \quad E_R = 2\,E\cos(\alpha/2) \quad \text{... for short pitch}$$

This is the resultant e.m.f. in case of a short pitch coil which depends on the angle of short pitch 'α'.

Key Point : *Now the factor by which, induced e.m.f. gets reduced due to short pitching is called **pitch factor** or **coil span factor** denoted by K_c.*

It is defined as the ratio of resultant e.m.f. when coil is short pitch to the resultant e.m.f. when coil is full pitched. It is always less than one.

$$\therefore \quad K_c = \frac{E_R \text{ when coil is short pitched}}{E_R \text{ when coil is full pitched}} = \frac{2E\cos\left(\frac{\alpha}{2}\right)}{2E}$$

$$\therefore \quad \boxed{K_c = \cos\left(\frac{\alpha}{2}\right)}$$

where α = Angle of short pitch

4.12.2 Distribution Factor (K_d)

Similar to full pitch coils, concentrated winding is also rare in practice. Attempt is made to use all the slots available under a pole for the winding which makes the nature of the induced e.m.f. more sinusoidal. Such a winding is called distributed winding.

Consider 18 slots , 2 pole alternator. So slots per pole i.e. n = 9.

$$m = \text{Slots per pole per phase} = 3$$

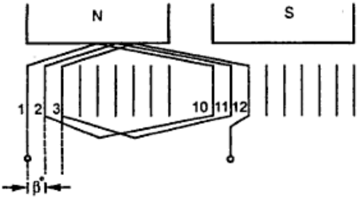
$$\beta = \frac{180°}{9} = 20°$$

Let E = Induced e.m.f. per coil and there are 3 coils per phase.

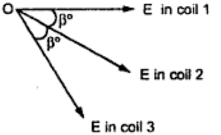
In concentrated type all the coil sides will be placed in one slot under a pole. So induced e.m.f. in all the coils will achieve maxima and minima at the same time i.e. all of them will be in phase. Hence resultant e.m.f. after connecting coils in series will be algebraic sum of all the e.m.f.s as all are in phase.

As against this, in distributed type, coil sides will be distributed, one each in the 3 slots per phase available under a pole as shown in the Fig. 4.19 (a).

Though the magnitude of e.m.f. in each coil will be same as 'E', as each slot contributes phase difference of $\beta°$ i.e. 20° in this case, there will exist a phase difference of $\beta°$ with respect to each other as shown in the Fig. 4.19 (b). Hence resultant e.m.f. will be phasor sum of all of them as shown in the Fig. 4.20. So due to distributed winding, resultant e.m.f. decreases.



(a) Distributed winding



(b) Phase difference between induced e.m.f.

Fig. 4.19

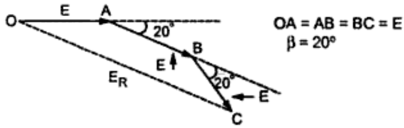


Fig. 4.20 Phasor sum of e.m.f.s

> **Key Point** : *The factor by which there is a reduction in the e.m.f. due to distribution of coils is called* ***distribution factor*** *denoted as* K_d.

Let us see the derivation for its expression.

In general let there be 'n' slots per pole and 'm' slots per pole per phase. So there will be 'm' coils distributed under a pole per phase, connected in series. Let E be the induced e.m.f. per coil. Then all the 'm' e.m.f.s induced in the coils will have successive phase angle difference of $\beta = \frac{180°}{n}$. While finding out the phasor sum of all of them, phasor diagram will approach a shape of a 'm' equal sided polygon circumscribed by a semicircle of radius 'R'.

This is shown in the Fig. 4.21. AB, BC, CD etc., represent e.m.f. per coil. All the ends are joined at 'O' which is centre of the circumscribing semicircle of radius 'R'.

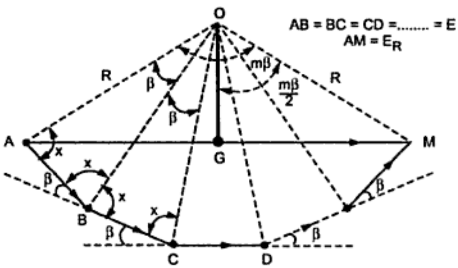


Fig. 4.21 Phasor sum of 'm' e.m.f.s

Angle subtended by each phasor at the origin 'O' is β°. This can be proved as below.

All the triangles OAB, OBC ... are similar and isosceles, as AB = BC = CD = ... = E.

Let the base angles be 'x'.

$$\angle OAB = \angle OBA = \angle OBC = \ldots = x$$

and $\angle AOB = \angle BOC = \ldots = y$ say

Now in ΔOAB, $2x + y = 180°$

while $\angle OBA + \angle OBC + \beta = 180°$ ... (1)

i.e. $2x + \beta = 180°$ ... (2)

Comparing equations (1) and (2), $y = \beta$

So $\angle AOB = \angle BOC = \angle COD = \ldots = \beta$

If 'M' is the last point of the last phasor,

$\angle AOM = m \times \beta = m\beta$

and $AM = E_R$= Resultant of all the e.m.f.s.

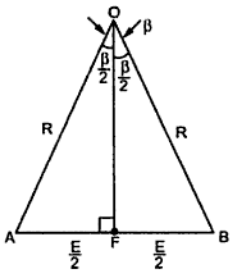


Fig. 4.22

Consider a Δ OAB separately as shown in the Fig. 4.22. Let OF be the perpendicular drawn on AB bisecting angle at apex 'O' as $\beta/2$.

$$l(AB) = E \qquad \therefore \quad l(AF) = \frac{E}{2}$$

and $l(OA) = R$.

$$\therefore \quad \sin\left(\frac{\beta}{2}\right) = \frac{AF}{OA} = \frac{E/2}{R}$$

$$\therefore \quad E = 2R\sin\left(\frac{\beta}{2}\right) \qquad \ldots (3)$$

Now consider Δ OAM as shown in the Fig. 4.23 and OG is the perpendicular drawn from 'O' on its base bisecting $\angle$ AOM.

$$\therefore \quad \angle AOG = \angle GOM = \frac{m\beta}{2}$$

$$\therefore \quad l(AM) = E_R$$

$$\therefore \quad l(AG) = \frac{E_R}{2}$$

$$\therefore \quad \sin\left(\frac{m\beta}{2}\right) = \frac{AG}{OA} = \frac{E_R/2}{R}$$

$$\therefore \quad E_R = 2R\sin\left(\frac{m\beta}{2}\right) \qquad \ldots \text{for distributed}$$

This is the resultant e.m.f. when coils are distributed. If all 'm' coils are concentrated, all would have been in phase giving E_R as algebraic sum of all the e.m.f.s.

$$\therefore \quad E_R = m \times E \qquad \ldots \text{for concentrated}$$

From equation (3), $E = 2R\sin\left(\frac{\beta}{2}\right)$

$$\therefore \qquad E_R = 2\ m\ R\ \sin\left(\frac{\beta}{2}\right)$$

This is resultant e.m.f. when coils are concentrated.

Key Point : *The distribution factor is defined as the ratio of the resultant e.m.f. when coils are distributed to the resultant e.m.f. when coils are concentrated. It is always less than one.*

$$\therefore \qquad K_d = \frac{E_R \text{ when coils are distributed}}{E_R \text{ when coils are concentrated}} = \frac{2\,R\,\sin\left(\frac{m\beta}{2}\right)}{2\,m\,R\sin\left(\frac{\beta}{2}\right)}$$

$$\therefore \qquad \boxed{K_d = \frac{\sin\left(\frac{m\beta}{2}\right)}{m\sin\left(\frac{\beta}{2}\right)}}$$

where m = Slots per pole per phase

β = Slot angle = $\dfrac{180°}{n}$

n = Slots per pole

The distribution factor is also called **breadth factor.**

4.12.3 Generalized Expression for e.m.f. Equation of an Alternator

Considering full pitch, concentrated winding ,

$$E_{ph} = 4.44\ f\ \phi\ T_{ph} \quad \text{volts}$$

But due to short pitch, distributed winding used in practice, this E_{ph} will reduce by factors K_c and K_d. So generalized expression for e.m.f. equation can be written as,

$$\boxed{E_{ph} = 4.44\ K_c\ K_d\ f\ \phi\ T_{ph} \quad \text{volts}}$$

For full pitch coil, $K_c = 1$

For concentrated winding, $K_d = 1$

Key Point : *For short pitch and distributed winding K_c and K_d are always less than unity.*

Example 4.1 : *An armature of a three phase alternator has 120 slots. The alternators has 8 poles. Calculate its distribution factor.*

Solution :

$$n = \frac{\text{slots}}{\text{pole}} = \frac{120}{8} = 15$$

$$m = \text{slots/pole/phase} = \frac{n}{3} = \frac{15}{3} = 5$$

$$\beta = \frac{180°}{n} = \frac{180°}{15} = 12°$$

$$\therefore \quad K_d = \frac{\sin\left(\frac{m\beta}{2}\right)}{m\sin\left(\frac{\beta}{2}\right)} = \frac{\sin\left(\frac{5\times 12}{2}\right)}{5\times\sin\left(\frac{12}{2}\right)} = \mathbf{0.957}$$

Example 4.2 : *In a 4 pole, 3 phase alternator, armature has 36 slots. It is using an armature winding which is short pitched by one slot. Calculate its coil span factor.*

Solution :

$$n = \frac{\text{slots}}{\text{pole}} = \frac{36}{4} = 9$$

$$\beta = \frac{180°}{9} = 20°$$

Now coil is shorted by 1 slot i.e. by 20° to full pitch distance.

$$\therefore \quad \alpha = \text{Angle of short pitch} = 20°$$

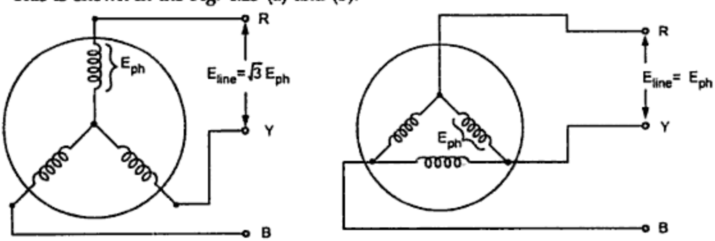
$$\therefore \quad K_c = \cos\left(\frac{\alpha}{2}\right) = \cos(10) = \mathbf{0.9848}$$

4.12.4 Line Value of Induced E.M.F.

If the armature winding of three phase alternator is star connected, then the value of induced e.m.f. across the terminals is $\sqrt{3}\,E_{ph}$ where E_{ph} is induced e.m.f. per phase.

While if it is delta connected, line value of e.m.f. is same as E_{ph}.

This is shown in the Fig. 4.23 (a) and (b).



(a) Star connection (b) Delta connection

Fig. 4.23

Practically most of the alternators are star connected due to following reasons :

1. Neutral point can be earthed from safety point of view.
2. For the same phase voltage, voltage available across the terminals is more than delta connection.
3. For the same terminal voltage, the phase voltage in star is $\frac{1}{\sqrt{3}}$ times line value.

   This reduces strain on the insulation of the armature winding.

➡ **Example 4.3 :** *An alternator runs at 250 r.p.m. and generates an e.m.f. at 50 Hz. There are 216 slots each containing 5 conductors. The winding is distributed and full pitch. All the conductors of each phase are in series and flux per pole is 30 mWb which is sinusoidally distributed. If the winding is star connected, determine the value of induced e.m.f. available across the terminals.*

Solution : $N_s = 250$ r.p.m., f = 50 Hz

$$N_s = \frac{120\,f}{P}$$

$$\therefore \quad 250 = \frac{120 \times 50}{P}$$

$$\therefore \quad P = 24$$

$$\therefore \quad n = \frac{\text{slots}}{\text{pole}} = \frac{216}{24} = 9$$

$$\therefore \quad m = \frac{n}{3} = 3$$

$$\therefore \quad \beta = \frac{180^\circ}{9} = 20^\circ$$

$$\therefore \quad K_d = \frac{\sin\left(\frac{m\beta}{2}\right)}{m\sin\left(\frac{\beta}{2}\right)} = \frac{\sin\left(\frac{3 \times 20}{2}\right)}{3 \times \sin\left(\frac{20}{2}\right)}$$

$$= 0.9597$$

$K_c = 1$ as full pitch coils.

Total number of conductors $Z = 216 \times 5 = 1080$

$$\therefore \quad Z_{ph} = \frac{Z}{3} = \frac{1080}{3} = 360$$

$$T_{ph} = \frac{Z_{ph}}{2} \qquad \text{... 2 conductors constitute 1 turn}$$

$$= \frac{360}{2} = 180$$

$$\therefore \quad E_{ph} = 4.44\ K_c\ K_d\ f\ \phi\ T_{ph}$$

$$= 4.44 \times 1 \times 0.9597 \times 30 \times 10^{-3} \times 50 \times 180$$

$$= 1150.48\ V$$

$$E_{line} = \sqrt{3}\ E_{ph} \qquad \text{...star connection}$$

$$= \sqrt{3} \times 1150.48$$

$$= \mathbf{1992.70\ V}$$

Example 4.4 : *A 3 phase, star connected alternator has the following data : Voltage required to be generated on open circuit = 4000 V, frequency = 50 Hz, speed = 750 r.p.m., slots/pole/phase = 3, conductors/slot = 12. Calculate, i) Number of poles ii) Useful flux per pole. Assume full pitch coil.* **[Nov.-2006 (Set-3)]**

Solution : $E_{line} = 4000$ V, f = 50 Hz, $N_s = 750$ r.p.m., m = 3, $K_c = 1$

$$E_{ph} = \frac{E_{line}}{\sqrt{3}} = \frac{4000}{\sqrt{3}} = 2309.401\ V$$

i) $$N_s = \frac{120f}{P} \qquad \text{i.e.} \qquad 750 = \frac{120 \times 50}{P}$$

$$\therefore \quad P = 8$$

ii) $$n = \text{Slots/pole} = m \times 3 = 9$$

$$\beta = \frac{180^\circ}{n} = \frac{180^\circ}{9} = 20^\circ$$

$$\therefore \quad K_d = \frac{\sin\frac{m\beta}{2}}{m \sin\frac{\beta}{2}} = 0.9598$$

$$\text{Number of slots} = n \times P = 9 \times 8 = 72$$

$$Z = \text{Slots} \times \text{Conductors/slot} = 72 \times 12 = 864$$

$$\therefore \quad Z_{ph} = \frac{Z}{3} = 288 \quad \text{and} \quad T_{ph} = \frac{Z_{ph}}{2} = 144$$

$$\therefore \quad E_{ph} = 4.44\ K_c\ K_d\ \phi\ f\ T_{ph}$$

$$\therefore \quad 2309.401 = 4.44 \times 1 \times 0.9598 \times \phi \times 50 \times 144$$

$$\therefore \quad \phi = \mathbf{0.07526\ Wb} \qquad \text{... flux per pole}$$

4.13 Parameters of Armature Winding

There are three important parameters of an armature winding of an alternator. These are,

1. Armature resistance R_a
2. Armature leakag e reactance X_L
3. Reactance corresponding to armature reaction

Let us discuss these three parameters in detail which will help us to draw an equivalent circuit of an alternator. The equivalent circuit and the concept of synchronous impedance plays an important role in determining the regulation of an alternator.

4.14 Armature Resistance

Every armature winding has its own resistance. The effective resistance of an armature winding per phase is denoted as R_{aph} Ω/ph or R_a Ω/ph.

Generally the armature resistance is measured by applying the known d.c. voltage and measuring the d.c. current through it. The ratio of applied voltage and measured current is the armature resistance. But due to the skin effect, the effective resistance under a.c. conditions is more than the d.c. resistance. Generally the effective armature resistance under a.c. conditions is taken 1.25 to 1.75 times the d.c. resistance.

While measuring the armature resistance, it is necessary to consider how the armature winding is connected whether in star or delta. Consider a star connected armature winding as shown in the Fig. 4.24.

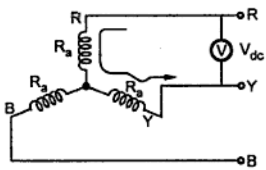


Fig. 4.24 Star connected alternator

When the voltage is applied across any two terminals of an armature winding, then the equivalent resistance is the series combination of the two resistances of two different phase windings.

$\therefore$ R_{RY} = Resistance between R-Y terminals = $R_a + R_a = 2\,R_a$

where R_a = Armature resistance per phase

$$\therefore \quad \boxed{R_a = \frac{R_{RY}}{2} \quad \Omega/\text{ph}}$$

Thus in star connected alternator, the armature resistance per phase is half of the resistance observed across any two line terminals.

Consider the delta connected alternator as shown in the Fig. 4.25. When voltage is applied across any two terminals, then one phase winding appears in parallel with series combination of other two.

Hence the equivalent resistance across the terminals is parallel combination of the resistances R_a and $2R_a$.

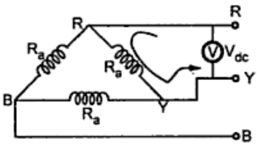


Fig. 4.25 Delta connected alternator

$$\therefore \quad R_{RY} = R_a \parallel 2R_a \quad \Omega/\text{ph} = \frac{R_a \times 2R_a}{R_a + 2R_a} = \frac{2}{3} R_a$$

$$\therefore \quad \boxed{R_a = \frac{3}{2} R_{RY}}$$

Thus in delta connected alternator, the armature resistance per phase is to be calculated from the equivalent resistance observed across any two line terminals.

4.15 Armature Leakage Reactance

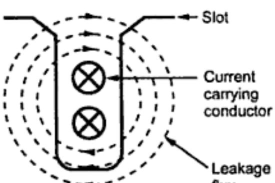


Fig. 4.26 Armature leakage flux

When armature carries a current, it produces its own flux. Some part of this flux completes its path through the air around the conductors itself. Such a flux is called **leakage flux**. This is shown in the Fig. 4.26.

Key Point : *This leakage flux makes the armature winding inductive in nature. So winding possesses a leakage reactance, in addition to the resistance.*

So if 'L' is the leakage inductance of the armature winding per phase, then leakage reactance per phase is given by $X_L = 2\pi f L \ \Omega/\text{ph}$. The value of leakage reactance is much higher than the armature resistance. Similar to the d.c. machines, the value of armature resistance is very very small.

4.16 Armature Reaction

When the load is connected to the alternator, the armature winding of the alternator carries a current. Every current carrying conductor produces its own flux so armature of the alternator also produces its own flux, when carrying a current. So there are two fluxes present in the air gap, one due to armature current while second is produced by the field winding called **main flux**. The flux produced by the armature is called **armature flux**.

> **Key Point** : *So effect of the armature flux on the main flux affecting its value and the distribution is called **armature reaction**.*

The effect of the armature flux not only depends on the magnitude of the current flowing through the armature winding but also depends on the nature of the power factor of the load connected to the alternator.

Let us study the effect of nature of the load power factor on the armature reaction.

4.16.1 Unity Power Factor Load

Consider a purely resistive load connected to the alternator, having unity power factor. As induced e.m.f. E_{ph} drives a current of I_{aph} and load power factor is unity, E_{ph} and I_{aph} are in phase with each other.

If ϕ_f is the main flux produced by the field winding responsible for producing E_{ph} then E_{ph} lags ϕ_f by 90°.

Now current through armature I_a, produces the armature flux say ϕ_a. So flux ϕ_a and I_a are always in the same direction.

This relationship between ϕ_f, ϕ_a, E_{ph} and I_{aph} can be shown in the phasor diagram. Refer to the Fig. 4.27.

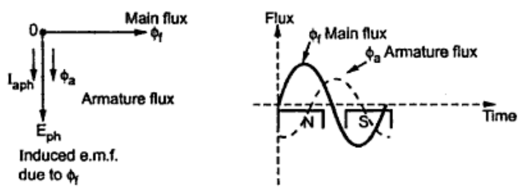


Fig. 4.27 Armature reaction for unity p.f. load

It can be seen from the phasor diagram that there exists a phase difference of 90° between the armature flux and the main flux. The waveforms for the two fluxes are also shown in the Fig. 4.27. From the waveforms it can be seen that the two fluxes oppose each other on the left half of each pole while assist each other on the right half of each pole. Hence average flux in the air gap remains constant but its distribution gets distorted.

Key Point : *Hence such distorting effect of armature reaction under unity p.f. condition of the load is called* ***cross magnetising effect*** *of armature reaction.*

Due to such distortion of the flux, there is small drop in the terminal voltage of the alternator.

4.16.2 Zero Lagging Power Factor Load

Consider a purely inductive load connected to the alternator having zero lagging power factor. This indicates that I_{aph} driven by E_{ph}, lags E_{ph} by 90° which is the power factor angle ϕ.

Induced e.m.f. E_{ph} lags main flux ϕ_f by 90° while ϕ_a is in the same direction as that of I_a. So the phasor diagram and the waveforms are shown in the Fig. 4.28.

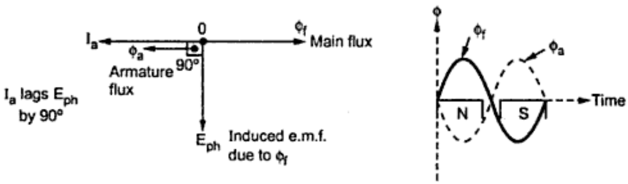


Fig. 4.28 Armature reaction for zero lagging p.f. load

It can be seen from the phasor diagram that the armature flux and the main flux are exactly in opposite direction to each other.

Key Point : *So armature flux tries to cancel the main flux. Such an effect of armature reaction is called* ***demagnetising effect*** *of the armature reaction.*

As this effect causes reduction in the main flux, the terminal voltage drops. This drop in the terminal voltage is more than the drop corresponding to the unity p.f. load.

4.16.3 Zero Leading Power Factor Load

Consider a purely capacitive load connected to the alternator having zero leading power factor. This means that armature current I_{aph} driven by E_{ph}, leads E_{ph} by 90°, which is the power factor angle ϕ.

Induced e.m.f. E_{ph} lags ϕ_f by 90° while I_{aph} and ϕ_a are always in the same direction. The phasor diagram and the waveforms are shown in the Fig. 4.29.

It can be seen from the phasor diagram and waveforms shown in the Fig. 4.29, the armature flux and the main field flux are in the same direction i.e. they are helping each other. This results into the addition in main flux.

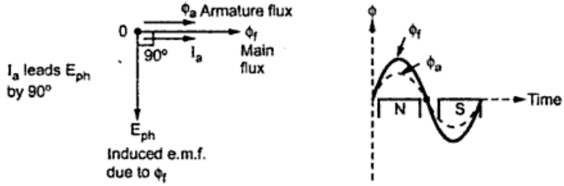


Fig. 4.29 Armature reaction for zero leading p.f. load

Key Point : *Such an effect of armature reaction due to which armature flux assists field flux is called* ***magnetising effect*** *of the armature reaction.*

As this effect adds the flux to the main flux, greater e.m.f. gets induced in the armature. Hence there is increase in the terminal voltage for leading power factor loads.

For intermediate power factor loads i.e. between zero lagging and zero leading the armature reaction is partly cross magnetising and partly demagnetising for lagging power factor loads or partly magnetising for leading power factor loads.

4.16.4 Armature Reaction Reactance (X_{ar})

In all the conditions of the load power factors, there is change in the terminal voltage due to the armature reaction. Mainly the practical loads are inductive in nature, due to demagnetising effect of armature reaction, there is reduction in the terminal voltage. Now this drop in the voltage is due to the interaction of armature and main flux. This drop is not across any physical element.

But to quantify the voltage drop due to the armature reaction, armature winding is assumed to have a fictitious reactance. This fictitious reactance of the armature is called **armature reaction reactance** denoted as X_{ar} Ω/ph. And the drop due to armature reaction can be accounted as the voltage drop across this reactance as $I_a X_{ar}$.

Key Point : *The value of this reactance changes as the load power factor changes, as armature reaction depends on the load power factor.*

4.17 Concept of Synchronous Reactance and Impedance

From the above discussion, it is clear that armature winding has one more parameter which is armature reaction reactance in addition to its resistance and the leakage reactance.

The sum of the fictitious armature reaction reactance accounted for considering armature reaction effect and the leakage reactance of the armature is called **synchronous reactance** of the alternator denoted as X_s.

So $$\boxed{X_s = X_L + X_{ar} \quad \Omega/\text{ph}}$$

As both X_L and X_{ar} are ohmic values per phase, synchronous reactance is also specified as ohms per phase.

Now from this, it is possible to define an impedance of the armature winding. Such an impedance obtained by combining per phase values of synchronous reactance and armature resistance is called **synchronous impedance** of the alternator denoted as Z_s.

So $$\boxed{Z_s = R_a + j\,X_s \quad \Omega/\text{ph}}$$

and $$\boxed{|Z_s| = \sqrt{R_a^{\,2} + (X_s)^2} \quad \Omega/\text{ph}}$$

For getting a standard frequency, alternator is to be driven at synchronous speed. So word synchronous used in specifying the reactance and impedance is referred to the working speed of the alternator. Generally impedance of the winding is constant but in case of alternator, synchronous reactance depends on the load and its power factor condition, hence synchronous impedance also varies with the load and its power factor conditions.

4.18 Equivalent Circuit of an Alternator

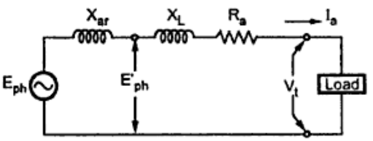


Fig. 4.30 Equivalent circuit

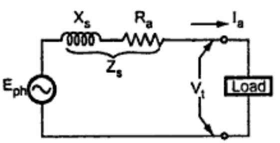


Fig. 4.31 Equivalent circuit of an alternator

From the above discussion it is clear that in all, there are three important parameters of armature winding namely armature resistance R_a, leakage reactance X_L and armature reaction reactance X_{ar}. If E_{ph} is induced e.m.f. per phase on no load condition then on load it changes to E' due to armature reaction as shown in the equivalent circuit. As current I_a flows through the armature, there are two voltage drops across R_a and X_L as $I_a R_a$ and $I_a X_L$ respectively. Hence finally terminal voltage V_t is less than E' by the amount equal to the drops across R_a and X_L.

In practice, the leakage reactance X_L and the armature reaction reactance X_{ar} are combined to get synchronous reactance X_s.

Hence the equivalent circuit of an alternator gets modified as shown in the Fig. 4.31.

Thus in the equivalent circuit shown,

E_{ph} = Induced e.m.f. per phase on no load

$V_{t\,ph}$ = Terminal voltage per phase on load

I_{aph} = Armature resistance per phase

Z_s = Synchronous impedance per phase

and $\boxed{\overline{E}_{ph} = \overline{V}_{tph} + \overline{I_a Z_s}}$ ... Phasor sum

4.19 Voltage Equation of an Alternator

In d.c. generators, we have seen that due to the armature resistance drop and brush drop it is not possible to have all the induced e.m.f. available across the load. The voltage available to the load is called **terminal voltage.** The concept is same in case of alternators. The entire induced e.m.f. cannot be made available to the load due to the various internal voltage drops. So the voltage available to the load is called **terminal voltage** denoted as V_{ph}. In case of three phase alternators as all the phases are identical, the equations and the phasor diagrams are expressed on per phase basis.

So if E_{ph} is the induced e.m.f. per phase in the alternator, there are following voltage drops occur in an alternator.

i) The drop across armature resistance $I_a R_a$ both I_a and R_a are per phase values.

ii) The drop across synchronous reactance $I_a X_s$, both I_a and X_s are per phase values.

After supplying these drops, the remaining voltage of E_{ph} is available as the terminal voltage V_{ph}.

Key Point: *Now drop $I_a R_a$ is always in phase with I_a due to a resistive drop while current I_a lags by 90° with respect to drop $I_a X_s$ as it is a drop across purely inductive reactance.*

Hence all these quantities cannot be added or subtracted algebraically but must be added or subtracted vectorially considering their individual phases. But we can write a voltage equation in its phasor form as,

$$\boxed{\overline{E}_{ph} = \overline{V}_{ph} + \overline{I_a R_a} + \overline{I_a X_s}}$$

This is called **voltage equation** of an alternator.

From this voltage equation, we can draw the phasor diagrams for various load power factor conditions and establish the relationship between E_{ph} and V_{ph}, in terms of armature current i.e. load current and the power factor cos (ϕ).

4.20 Phasor Diagram of a Loaded Alternator

The above voltage equation is to be realised using phasor diagrams for various load power factor conditions. For drawing the phasor diagram, consider all per phase values and remember following steps.

Steps to draw the phasor diagram :

1. Choose current I_a as a reference phasor.
2. Now if load power factor is cos ϕ, it indicates that angle between V_{ph} and I_a is ϕ as V_{ph} is the voltage available to the load.

   So show the phasor V_{ph} in such a way that angle between V_{ph} and I_a is ϕ. For lagging 'ϕ', I_a should lag V_{ph} and for leading 'ϕ', I_a should lead V_{ph}. For unity power factor load ϕ is zero, so V_{ph} and I_a are in phase.
3. Now the drop I_a R_a is a resistive drop and hence will always be in phase with I_a. So phasor I_a R_a direction will be always same as I_a, i.e. parallel to I_a. But as it is to be added to V_{ph}, I_a R_a phasor must be drawn from the tip of the V_{ph} phasor drawn.
4. The drop I_a X_s is drop across purely inductive reactance. In pure inductance, current lags voltage by 90°. So 'I_a X_s' phasor direction will be always such that I_a will lag I_a X_s phasor by 90°. But this phasor is to be drawn from the tip of the I_aR_a phasor to complete phasor addition of V_{ph}, I_a R_a and I_a X_s.

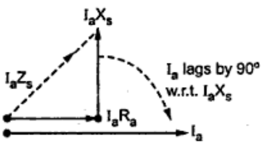


Fig. 4.32

5. Joining the starting point to the terminating point, we get the phasor E_{ph}.

   Whatever may be the load power factor, I_a R_a is a resistive drop, will be in phase with I_a while I_a X_s is purely inductive drop and hence will be perpendicular to I_a in such a way that I_a will lag I_aX_s by 90°. This is shown in the Fig. 4.32.

   By using the above steps, the phasor diagrams for various load power factor conditions can be drawn.

4.20.1 Lagging Power Factor Load

The power factor of the load is cos ϕ lagging so I_a lags V_{ph} by angle ϕ. By using steps discussed above, phasor diagram can be drawn as shown in the Fig. 4.33.

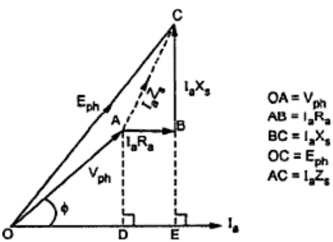


Fig. 4.33 Phasor diagram for lagging p.f. load

To derive the relationship between E_{ph} and V_{ph}, the perpendiculars are drawn on the current phasor from points A and B. These intersect current phasor at points D and E respectively.

Now, $OD = V_{ph} \cos \phi$

$AD = BE = V_{ph} \sin \phi$

$DE = I_a R_a$

Consider Δ OCE, for which we can write,

$$(OC)^2 = (OE)^2 + (EC)^2$$

$$\therefore \quad (E_{ph})^2 = (OD + DE)^2 + (EB + BC)^2$$

$$\therefore \quad (E_{ph})^2 = (V_{ph} \cos\phi + I_a R_a)^2 + (V_{ph} \sin\phi + I_a X_s)^2$$

$$\therefore \quad \boxed{E_{ph} = \sqrt{(V_{ph} \cos\phi + I_a R_a)^2 + (V_{ph} \sin\phi + I_a X_s)^2}}$$

From this equation, the value of induced e.m.f. can be calculated.

4.20.2 Leading Power Factor Load

The power factor of the load is cos ϕ leading. So I_a leads V_{ph} by an angle ϕ. By using steps discussed, the phasor diagram can be drawn as shown in the Fig. 4.34.

To derive the relation between E_{ph} and V_{ph}, the perpendiculars are drawn on current phasor from points A and B. These intersect current phasor at points D and E respectively.

From Δ OAD, $OD = V_{ph} \cos \phi$

$AD = BE = V_{ph} \sin \phi$

$DE = I_a R_a$

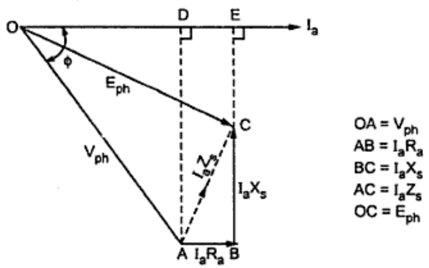


Fig. 4.34 Phasor diagram for leading p.f. load

Consider Δ OCE, for which we can write,

$$(OC)^2 = (OE)^2 + (EC)^2$$

$$\therefore \quad (E_{ph})^2 = (OD + DE)^2 + (BE - BC)^2$$

$$\therefore \quad (E_{ph})^2 = (V_{ph}\cos\phi + I_a R_a)^2 + (V_{ph}\sin\phi - I_a X_s)^2$$

$$\therefore \quad \boxed{E_{ph} = \sqrt{(V_{ph}\cos\phi + I_a R_a)^2 + (V_{ph}\sin\phi - I_a X_s)^2}}$$

It can be observed that the sign of the $I_a X_s$ is negative as against its positive sign for lagging p.f. load. This is because X_s consists of X_{ar} i.e. armature reaction reactance. Armature reaction is demagnetising for lagging while magnetising for leading power factor loads. So sign of $I_a X_s$ is opposite to each other for lagging and leading p.f conditions.

4.20.3 Unity Power Factor Load

The power factor of the load is unity i.e. cos $\phi = 1$. So $\phi = 0$, which means V_{ph} is in phase with I_a. So phasor diagram can be drawn as shown in the Fig. 4.35.

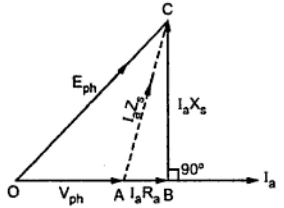


Fig. 4.35 Phasor diagram for unity p.f. load

Consider ΔOBC, for which we can write,

$$(OC)^2 = (OB)^2 + (BC)^2$$

$$\therefore \quad (E_{ph})^2 = (OA + AB)^2 + (BC)^2$$

$$\therefore \quad (E_{ph})^2 = (V_{ph} + I_a R_a)^2 + (I_a X_s)^2$$

$$\therefore \quad \boxed{E_{ph} = \sqrt{(V_{ph} + I_a R_a)^2 + (I_a X_s)^2}}$$

As cos $\phi = 1$, so sin $\phi = 0$ hence does not appear in the equation.

Note : The phasor diagrams can be drawn by considering voltage V_{ph} as a reference phasor. But to derive the relationship, current phasor selected as a reference makes the derivation much more simplified. Hence current is selected as a reference phasor.

It is clear from the phasor diagram that V_{ph} is less than E_{ph} for lagging and unity p.f. conditions due to demagnetising and cross magnetising effects of armature reaction. While V_{ph} is more than E_{ph} for leading p.f. condition due to the magnetising effect of armature reaction.

Thus in general for any power factor condition,

$$(E_{ph})^2 = (V_{ph}\cos\phi + I_a R_a)^2 + (V_{ph}\sin\phi \pm I_a X_s)^2$$

+ sign for lagging p.f. loads

– sing for leading p.f. loads

and V_{ph} = per phase rated terminal voltage

I_a = per phase full load armature current

From this discussion, we can now define the voltage regulation of an alternator.

4.21 Voltage Regulation of an Alternator

Under the load condition, the terminal voltage of alternator is less than the induced e.m.f. E_{ph}. So if load is disconnected, V_{ph} will change from V_{ph} to E_{ph}, if flux and speed is maintained constant. This is because when load is disconnected, I_a is zero hence there are no voltage drops and no armature flux to cause armature reaction. This change in the terminal voltage is significant in defining the voltage regulation.

Key Point : *The* ***voltage regulation*** *of an alternator is defined as the change in its terminal voltage when full load is removed, keeping field excitation and speed constant, divided by the rated terminal voltage.*

So if V_{ph} = Rated terminal voltage

E_{ph} = No load induced e.m.f.

then voltage regulation is defined as,

$$\% \text{ Regulation} = \frac{E_{ph} - V_{ph}}{V_{ph}} \times 100$$

The value of the regulation not only depends on the load current but also on the power factor of the load. For lagging and unity p.f. conditions there is always drop in the terminal voltage hence regulation values are always positive. While for leading capacitive load conditions, the terminal voltage increases as load current increases. Hence regulation is negative in such cases. The relationship between load current and the terminal voltage is called **load characteristics** of an alternator. Such load characteristics for various load power factor conditions are shown in Fig. 4.36.

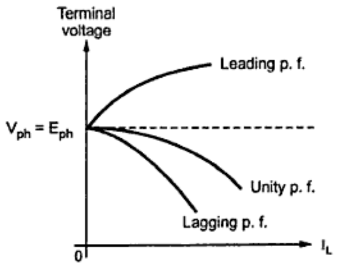


Fig. 4.36 Load characteristics of an alternator

➡ **Example 4.5 :** *A 3 phase, star connected alternator supplies a load of 1000 kW at a power factor of 0.8 lagging with a terminal voltage of 11 kV. Its armature resistance is 0.4 ohms per phase while synchronous reactance is 3 ohms per phase. Calculate the line value of e.m.f. generated and the regulation at this load.*

Solution : P = 100 kW, $\cos\phi = 0.8$ lagging

$$V_L = 11 \text{ kV}, \quad R_a = 0.4\ \Omega, \quad X_s = 3\ \Omega$$

For three phase load, $P = \sqrt{3}\, V_L\, I_L \cos\phi$

$$\therefore \quad 1000\times10^3 = \sqrt{3}\times11\times10^3\times I_L\times0.8$$

$$\therefore \quad I_L = 65.6 \text{ A}$$

Now $I_L = I_a$ as for star connected alternator $I_L = I_{ph}$.

$$\therefore \quad I_{aph} = 65.6 \text{ A} \qquad \text{... full load per phase armature current}$$

For lagging p.f. loads,

$$(E_{ph})^2 = (V_{ph}\cos\phi + I_a R_a)^2 + (V_{ph}\sin\phi + I_a X_s)^2$$

Now $$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{11\times10^3}{\sqrt{3}} \qquad \text{... as star connected}$$

$$= 6350.853 \text{ V}$$

$$\therefore \quad (E_{ph})^2 = (6350.853\times0.8+65.6\times0.4)^2+(6350.853\times0.6+65.6\times3)^2$$

$$\therefore \quad E_{ph} = 6491.47 \text{ V}$$

$$\therefore \quad E_{line} = \sqrt{3}\, E_{ph} = 11243.55 \text{ V} = \mathbf{11.24\ kV}$$

and $\quad \% \text{ Regulation} = \dfrac{E_{ph} - V_{ph}}{V_{ph}} \times 100$

$$= \frac{6491.47 - 6350.853}{6350.853} \times 100$$

$$= \mathbf{2.214\ \%}$$

For lagging p.f. loads, regulation is always positive.

4.22 kVA Rating of an Alternator

The alternators are designed to supply a specific voltage to the various loads. This voltage is called its rated terminal voltage denoted as V_L. The power drawn by the load depends on its power factor. Hence instead of specifying rating of an alternator in watts, it is specified in terms of the maximum apparent power which it can supply to the load. In three phase circuits, the apparent power is $\sqrt{3}\ V_L\ I_L$, measured in VA (volt amperes). This is generally expressed in kilo volt amperes and is called **kVA rating** of an alternator where I_L is the rated full load current which alternator can supply. So for a given rated voltage and kVA rating of an alternator, its full load rated current can be decided.

Consider 60 kVA, 11 kV, three phase alternator.

In this case, kVA rating = 60

but $\quad \boxed{\text{kVA} = \sqrt{3}\, V_L\, I_L \times 10^{-3}}$ $\quad$ ... 10^{-3} to express the product in kilo volt amperes

$\therefore \quad 60 = \sqrt{3} \times 11 \times 10^{3} \times I_L \times 10^{-3}$

$\therefore \quad I_L = 3.15\ \text{A}$

This is the rated full load current of an alternator. But load current is same as the armature current. So from kVA rating, it is possible to determine full load armature current of an alternator which is important in predicting the full load regulation of an alternator for various power factor conditions. Similarly if load condition is different than the full load, the corresponding armature current can be determined from its full load value.

$$I_a \text{ at half load} = \frac{1}{2} \times I_a \text{ at full load.}$$

Key Point : ***Armature curent I_a reduces in the same proportion in which load condition reduces.***

Hence regulation at any p.f. and at any load condition can be determined.

Example 4.6 : *A 1200 kVA, 6600 V, 3 phase, star connected alternator has its armature resistance as 0.25 Ω per phase and its synchronous reactance as 5 Ω per phase. Calculate its regulation if it delivers a full load at i) 0.8 lagging and ii) 0.8 leading p.f.*

Solution : kVA = 1200, $V_L = 6600$ V, $R_a = 0.25\ \Omega$, $X_s = 5\ \Omega$

Now $\quad kVA = \sqrt{3}\, V_L\, I_L \times 10^{-3}$

$\therefore \quad 1200 = \sqrt{3} \times 6600 \times I_L \times 10^{-3}$

$\therefore \quad I_L = 104.97$ A

$\therefore \quad I_{aph} = 104.97$ A ... as star connected.

This is its full load current.

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{6600}{\sqrt{3}}$$

$$= 3810.512 \text{ V}$$

i) For 0.8 lagging p.f. load

$$(E_{ph})^2 = (V_{ph}\cos\phi + I_a R_a)^2 + (V_{ph}\sin\phi + I_a X_s)^2$$

$$\therefore \quad (E_{ph})^2 = (3810.512 \times 0.8 + 104.97 \times 0.25)^2 + (3810.512 \times 0.6 + 104.97 \times 5)^2$$

$$\therefore \quad E_{ph} = 4164.06 \text{ V}$$

$$\therefore \quad \%\text{ Reg} = \frac{E_{ph} - V_{ph}}{V_{ph}} \times 100 = \frac{4166.06 - 3810.512}{3810.512} \times 100$$

$$= +\ 9.33\%$$

ii) For 0.8 leading p.f. load

$$(E_{ph})^2 = (V_{ph}\cos\phi + I_a R_a)^2 + (V_{ph}\sin\phi - I_a X_s)^2$$

$$\therefore \quad (E_{ph})^2 = (3810.512 \times 0.8 + 104.97 \times 0.25)^2 + (3810.512 \times 0.6 - 104.97 \times 5)^2$$

$$\therefore \quad E_{ph} = 3543.47 \text{ V}$$

$$\therefore \quad \%\text{ Reg} = \frac{E_{ph} - V_{ph}}{V_{ph}} \times 100 = \frac{3543.47 - 3810.512}{3810.512} \times 100$$

$$= -\ 7.00\ \%$$

The regulation is negative for leading p.f. loads.

4.23 Methods of Determining the Regulation

The regulation of an alternator can be determined by various methods. In case of small capacity alternators it can be determined by direct loading test while for large capacity alternators it can be determined by synchronous impedance method.

The synchronous impedance method has some short comings. Another method which is popularly used is ampere-turns method. But this method also has certain disadvantages. The disadvantages of these two methods are overcome in a method called zero power factor method. Another important theory which gives accurate results is called Blondel's two reaction theory. Thus there are following methods available to determine the voltage regulation of an alternator,

1. Direct loading method
2. Synchronous impedance method or E.M.F. method
3. Ampere-turns method or M.M.F. method
4. Zero power factor method or Potier triangle method
5. ASA modified form of M.M.F. method
6. Two reaction theory.

But from the syllabus point of view only synchronous impedance method is discussed in this chapter.

4.24 Synchronous Impedance Method or E.M.F. Method

The method is also called E.M.F. method of determining the regulation. The method requires following data to calculate the regulation.

1. The armature resistance per phase (R_a)
2. Open circuit characteristics which is the graph of open circuit voltage against the field current. This is possible by conducting open circuit test on the alternator.
3. Short circuit characteristics which is the graph of short circuit current against field current. This is possible by conducting short circuit test on the alternator.

Let us see, the circuit diagram to perform open circuit as well as short circuit test on the alternator. The alternator is coupled to a prime mover capable of driving the alternator at its synchronous speed. The armature is connected to the terminals of a switch. The other terminals of the switch are short circuited through an ammeter. The voltmeter is connected across the lines to measure the open circuit voltage of the alternator.

The field winding is connected to a suitable d.c. supply with rheostat connected in series. The field excitation i.e. field current can be varied with the help of this rheostat. The circuit diagram is shown in the Fig. 4.37.

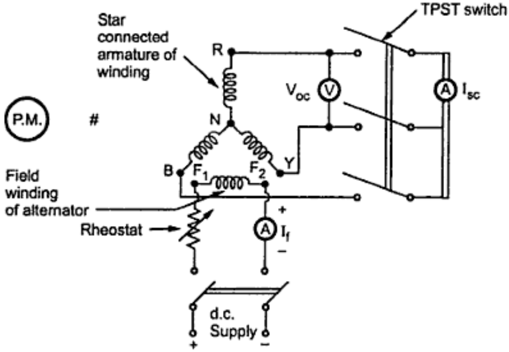


Fig. 4.37 Circuit diagram for open circuit and short circuit test on alternator

4.24.1 Open Circuit Test

Procedure to conduct this test is as follows :

1. Start the prime mover and adjust the speed to the synchronous speed of the alternator.
2. Keeping rheostat in the field circuit maximum, switch on the d.c. supply.
3. The T.P.S.T. switch in the armature circuit is kept open.

4. With the help of rheostat, field current is varied from its minimum value to the rated value. Due to this, flux increases, increasing the induced e.m.f. Hence voltmeter reading, which is measuring line value of open circuit voltage increases. For various values of field current, voltmeter readings are observed.

The observations for open circuit test are tabulated as below.

Sr. No.	I_f A	V_{oc} (line) V	V_{oc} (phase) = V_{oc} (line)/$\sqrt{3}$ V
1.			
2.			
:			
:			

From the above table, graph of $(V_{oc})_{ph}$ against I_f is plotted.

Key Point : *This is called open circuit characteristics of the alternator, called* O.C.C.

This is shown in the Fig. 4.38.

4.24.2 Short Circuit Test

After completing the open circuit test observations, the field rheostat is brought to maximum position, reducing field current to a minimum value. The T.P.S.T. switch is closed. As ammeter has negligible resistance, the armature gets short circuited. Then the field excitation is gradually increased till full load current is obtained through armature winding. This can be observed on the ammeter connected in the armature circuit. The graph of short circuit armature current against field current is plotted from the observation table of short circuit test. This graph is called **short circuit characteristics, S.C.C.** This is also shown in the Fig. 4.38.

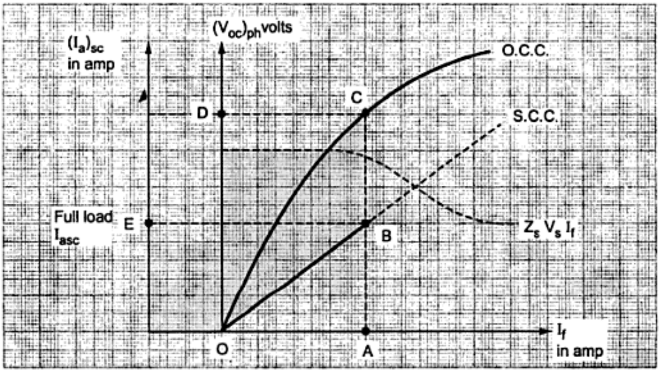


Fig. 4.38 O.C.C. and S.C.C. of an alternator

Observation table for short circuit test :

Sr. No.	I_f A	Short circuit armature current per phase $(I_a)_{sc}$ A
1	0	0
2		

The S.C.C. is a straight line graph passing through the origin while O.C.C. resembles B-H curve of a magnetic material.

Key Point *: As S.C.C. is straight line graph, only one reading corresponding to full load armature current along with the origin is sufficient to draw the straight line.*

4.24.3 Determination of Z_s from O.C.C. and S.C.C.

The synchronous impedance Z_s of the alternator changes as load condition changes. O.C.C. and S.C.C. can be used to determine Z_s for any load and load p.f. conditions.

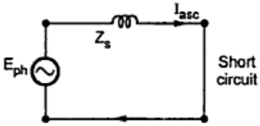


Fig. 4.39

In short circuit test, external load impedance is zero. The short circuit armature current is circulated against the impedance of the armature winding which is Z_s. The voltage responsible for driving this short circuit current is internally induced e.m.f. This can be shown in the equivalent circuit drawn in the Fig. 4.39.

From the equivalent circuit we can write,

$$Z_s = \frac{E_{ph}}{I_{asc}}$$

Now value of I_{asc} is known, which can be observed on the ammeter. But internally induced e.m.f. can not be observed under short circuit condition. The voltmeter connected will read zero which is voltage across short circuit. To determine Z_s it is necessary to determine value of E_{ph} which is driving I_{asc} against Z_s.

Now internally induced e.m.f. is proportional to the flux i.e. field current I_f.

$$E_{ph} \propto \phi \propto I_f \qquad \text{... from e.m.f. equation}$$

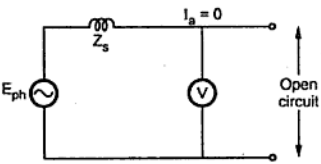


Fig. 4.40

So if the terminals of the alternator are opened without disturbing I_f which was present at the time of short circuited condition, internally induced e.m.f. will remain same as E_{ph}. But now current will be zero. Under this condition equivalent circuit will become as shown in the Fig. 4.40.

It is clear now from the equivalent circuit that as $I_a = 0$ the voltmeter reading $(V_{oc})_{ph}$ will be equal to internally induced e.m.f. (E_{ph}).

$$\therefore \qquad E_{ph} = (V_{oc})_{ph} \text{ on open circuit}$$

This is what we are interested in obtaining to calculate value of Z_s. So expression for Z_s can be modified as,

$$\boxed{Z_s = \left.\frac{(V_{oc})_{ph}}{(I_{asc})_{ph}}\right|_{\text{for same } I_f}}$$

$$\text{Thus in general, } Z_s = \left.\frac{\text{Phase e.m.f. on open circuit}}{\text{Phase current on short circuit}}\right|_{\text{For same excitation}}$$

So O.C.C. and S.C.C. can be used effectively to calculate Z_s.

The value of Z_s is different for different values of I_f as the graph of O.C.C. is non linear in nature.

So suppose Z_s at full load is required then,

$$I_{asc} = \text{Full load current}$$

From S.C.C. determine I_f required to drive this full load short circuit I_a. This is equal to 'OA', as shown in the Fig. 4.40.

Now for this value of I_f, $(V_{oc})_{ph}$ can be obtained from O.C.C. Extend line from point A, till it meets O.C.C. at point C. The corresponding $(V_{oc})_{ph}$ value is available at point D.

$$(V_{oc})_{ph} = OD$$

While $$(I_{asc})_{ph} = OE$$

$$\therefore \quad Z_s \text{ at full load} = \left.\frac{(V_{oc})_{ph}}{\text{Full load } (I_{asc})_{ph}}\right|_{\text{same } I_f \text{ (same excitation)}}$$

$$= \left.\frac{OD}{OE}\right|_{\text{same } I_f = OA \text{ in graph shown}}$$

General steps to determine Z_s at any load condition are :

1. Determine the value of $(I_{asc})_{ph}$ for corresponding load condition. This can be determined from known full load current of the alternator. For half load, it is half of the full load value and so on.
2. S.C.C. gives relation between $(I_{asc})_{ph}$ and I_f. So for $(I_{asc})_{ph}$ required, determine the corresponding value of I_f from S.C.C.
3. Now for this same value of I_f, extend the line on O.C.C. to get the value of $(V_{oc})_{ph}$. This is $(V_{oc})_{ph}$ for same I_f, required to drive the selected $(I_{asc})_{ph}$.
4. The ratio of $(V_{oc})_{ph}$ and $(I_{asc})_{ph}$, for the same excitation gives the value of Z_s at any load conditions.

The graph of synchronous impedance Z_s against excitation current I_f is also shown in the Fig. 4.40.

4.24.4 Regulation Calculations

From O.C.C. and S.C.C., Z_s can be determined for any load condition.

The armature resistance per phase (R_a) can be measured by different methods. One of the method is applying d.c. known voltage across the two terminals and measuring current. So value of R_a per phase is known.

Now $$Z_s = \sqrt{(R_a)^2 + (X_s)^2}$$

$$\therefore \quad \boxed{X_s = \sqrt{(Z_s)^2 - (R_a)^2} \quad \Omega/\text{ph}}$$

So synchronous reactance per phase can be determined.

No load induced e.m.f. per phase, E_{ph} can be determined by the mathematical expression derived earlier.

$$\boxed{E_{ph} = \sqrt{(V_{ph}\ \cos\phi + I_a\ R_a)^2 + (V_{ph}\ \sin\phi \pm I_a\ X_s)^2}}$$

where V_{ph} = Phase value of rated voltage

I_a = Phase value of current depending on the load condition

$\cos\phi$ = p.f. of load

Positive sign for lagging power factor while negative sign for leading power factor while R_a and X_s values are known from the various tests performed.

The regulation then can be determined by using formula,

$$\boxed{\%\ \text{Regulation} = \frac{E_{ph} - V_{ph}}{V_{ph}} \times 100}$$

4.24.5 Advantages and Limitations of Synchronous Impedance Method

The main advantage of this method is the value of synchronous impedance Z_s for any load condition can be calculated. Hence regulation of the alternator at any load condition and load power factor can be determined. Actual load need not be connected to the alternator and hence method can be used for very high capacity alternators.

The main limitation of this method is that the method gives large values of synchronous reactance. This leads to high values of percentage regulation than the actual results. Hence this method is called **pessimistic method.**

➡ **Example 4.7 :** *The open circuit and short circuit test is conducted on a 3 phase, star connected, 866 V, 100 kVA alternator.*

The O.C. test results are,

I_f Amp	1	2	3	4	5	6	7
V_{oc} line Volts	173	310	485	605	728	790	840

The field current of 1 A, produces a short circuit current of 25 A.

The armature resistance per phase is 0.15 Ω . Calculate its full load regulation at 0.8 lagging power factor condition.

Solution : V_L = 866 V, kVA = 100

$$\therefore \quad kVA = \sqrt{3} V_L I_L \times 10^{-3}$$

$$\therefore \quad 100 = \sqrt{3} \times 866 \times I_L \times 10^{-3}$$

$$\therefore \quad I_L = 66.67 \text{ A}$$

$$\therefore \quad I_{aph} \text{ F.L} = I_L = 66.67 \text{ A} \quad \text{...as star connected alternator}$$

$$V_{ph} = \text{Rated terminal voltage per phase} = \frac{V_L}{\sqrt{3}}$$

$$= \frac{866}{\sqrt{3}}$$

$$= 500 \text{ V}$$

For calculation of Z_s on full load, it is necessary to plot O.C.C. and S.C.C. to the scale.

Note : If for same value of I_f, both I_{asc} and V_{oc} can be obtained from the table itself, graph need not be plotted. In some problems, the values of V_{oc} and I_{asc} for same I_f are directly given, in that case too, the graph need not be plotted.

In this problem, I_{asc} = 25 A for I_f = 1 A.

But we want to calculate Z_s for I_{asc} = its full load value which is 66.67 A. So graph is required to be plotted.

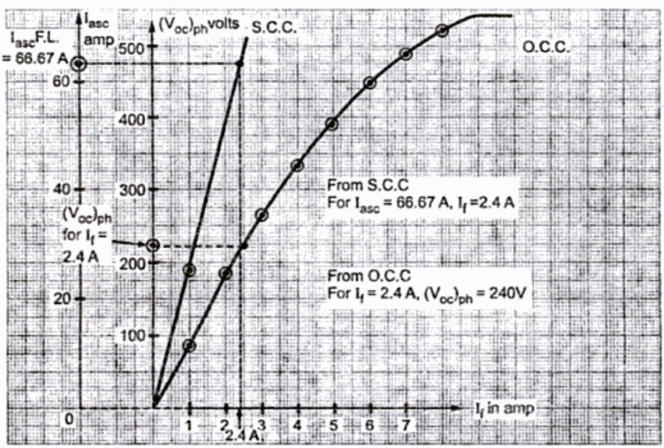


Fig. 4.41

For plotting O.C.C. the line values of open circuit voltage are converted to phase by dividing each value by $\sqrt{3}$.

From S.C.C.

For $I_{asc} = 66.67$ A, $I_f = 2.4$ A

From O.C.C.

For $I_f = 2.4$ A, $(V_{oc})_{ph} = 240$ V

From the graph, Z_s for full load is,

$$Z_s = \left.\frac{(V_{oc})_{ph}}{(I_{asc})_{ph}}\right|_{\text{for same excitation}} = \left.\frac{240}{66.67}\right|_{\text{For } I_f = 2.4 \text{ A}}$$

$$= 3.6 \ \Omega/\text{phase}$$

$$R_a = 0.15 \ \Omega/\text{phase}$$

$$\therefore \quad X_s = \sqrt{(Z_s)^2 - (R_a)^2}$$

$$= 3.597 \ \Omega/\text{phase}$$

$$V_{ph\ F.L.} = 500 \text{ V}$$

$$\cos\phi = 0.8$$

$$\therefore \quad \sin\phi = 0.6 \text{ lagging p.f.}$$

So E_{ph} for full load, 0.8 lagging p.f. condition can be calculated as,

$$(E_{ph})^2 = (V_{ph}\cos\phi + I_a R_a)^2 + (V_{ph}\sin\phi + I_a X_s)^2$$

$$\therefore \quad (E_{ph})^2 = (500 \times 0.8 + 66.67 \times 0.15)^2 + (500 \times 0.6 + 66.67 \times 3.597)^2$$

$$\therefore \quad E_{ph} = 677.86 \text{ V}$$

$$\therefore \quad \% \text{ Regulation} = \frac{E_{ph} - V_{ph}}{V_{ph}} \times 100 = \frac{677.86 - 500}{500} \times 100$$

$$= +\ 35.57\%$$

Example 4.8 : *A 230 V, 3 phase, star connected alternator gives on open circuit, e.m.f. of 230 V, for a field current of 0.38 A. The same field current on short circuit causes an armature current of 12.5 A. The armature resistance measured between two lines is 1.8 ohms. Find the regulation for the current of 10 amps at 0.8 lagging and 0.8 leading power factors.*

Solution : V_L = 230 V, R_a between lines = 1.8 Ω

$$(V_{oc})_{line} = 230 \text{ V}, I_{asc} = 12.5 \text{ A for same } I_f = 0.38 \text{ A}$$

The value of open circuit e.m.f. is always line value unless and until specifically mentioned to be a phase value.

$$\therefore \quad Z_s = \left.\frac{(V_{oc})_{ph}}{(I_{asc})_{ph}}\right|_{\text{for same } I_f}$$

$$(V_{oc})_{ph} = \frac{230}{\sqrt{3}} = 132.79 \text{ V}$$

$$\therefore \quad Z_s = \frac{132.79}{12.5} = 10.623 \ \Omega/\text{ph}$$

R_a between terminals = 1.8 Ω

For star connection, R_a between the terminals is 2 R_a per ph.

$$\therefore \quad 2 R_a \text{ per ph} = 1.8$$

$$\therefore \quad R_a \text{ per ph} = 0.9 \ \Omega$$

$$\therefore \quad X_s = \sqrt{Z_s^2 - R_a^2} = \sqrt{(10.623)^2 - (0.9)^2} = 10.585 \ \Omega/\text{ph}$$

Now regulation is asked for I_a = 10 A

Note : The value of Z_s is calculated for I_a = 12.5 A and not at I_a = 10 A. It will be different for I_a = 10 A. But in this problem the test results are not given hence it is not possible to sketch the graphs to determine Z_s at I_a = 10 A. So value of Z_s calculated is assumed to be same at I_a = 10 A.

> **Key Point** : *As change in Z_s is not significant for small difference in I_a, Z_s can be assumed to be constant for different values of I_a, once calculated for given value of I_a, if sufficient data is not given.*

i) For 0.8 lagging p.f.

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{230}{\sqrt{3}} = 132.79 \text{ V}$$

$$I_a = 10 \text{ A}$$

$$\cos\phi = 0.8 \qquad \text{so } \sin\phi = 0.6$$

$$\therefore \quad (E_{ph})^2 = (V_{ph}\cos\phi + I_a R_a)^2 + (V_{ph}\sin\phi + I_a X_s)^2$$

$$\therefore \quad (E_{ph})^2 = (132.79 \times 0.8 + 10 \times 0.9)^2 + (132.79 \times 0.6 + 10 \times 10.585)^2$$

$$\therefore \quad E_{ph} = 218.39 \text{ V}$$

$\therefore \quad \% \text{ Regulation} = \dfrac{E_{ph} - V_{ph}}{V_{ph}} \times 100 = \dfrac{218.39 - 132.79}{132.79} \times 100$

$= \mathbf{+\ 64.46\ \%}$

ii) For 0.8 leading p.f.

$\therefore \quad (E_{ph})^2 = (V_{ph} \cos\phi + I_a R_a)^2 + (V_{ph} \sin\phi - I_a X_s)^2$

$\therefore \quad (E_{ph})^2 = (132.79 \times 0.8 + 10 \times 0.9)^2 + (132.79 \times 0.6 - 10 \times 10.585)^2$

$\therefore \quad E_{ph} = 118.168 \text{ V}$

$\therefore \quad \% \text{ Reg.} = \dfrac{E_{ph} - V_{ph}}{V_{ph}} \times 100 = \dfrac{118.168 - 132.79}{132.79} \times 100$

$= \mathbf{-\ 11.01\ \%}$

Key Point : *For leading p.f. the regulation is negative as $E_{ph} < V_{ph}$.*

Examples with Solutions

Example 4.9 : *Find the number of armature conductors in series per phase required for 3 phase, 10 pole alternator when driven at a speed of 600 r.p.m. Armature has 90 slots and armature winding is star connected to give induced e.m.f. of 11 kV between the lines. Assume flux per pole as 16 mWb.*

Solution : $P = 10, \quad N_s = 600$ r.p.m., Slots = 90

$\phi = 16 \text{ mWb}, \quad E_{line} = 11 \text{ kV}$

$N_s = \dfrac{120 \times f}{P}$

$\therefore \quad 600 = \dfrac{120 \times f}{10}$

$f = 50 \text{ Hz}$

For star connection, $E_{ph} = \dfrac{E_{line}}{\sqrt{3}} = \dfrac{11 \times 10^3}{\sqrt{3}}$

$\therefore \quad E_{ph} = 6350.853 \text{ V}$

Now $\quad E_{ph} = 4.44\ K_c\ K_d\ \phi\ f\ T_{ph}$

$K_c = 1$ as no information about short pitching is given.

$n = \dfrac{\text{Slots}}{\text{Pole}} = \dfrac{90}{10} = 9$

$$m = \text{Slots/pole/phase}$$

$$= \frac{n}{3} = 3$$

$$\beta = \text{Slot angle} = \frac{180^\circ}{n} = \frac{180^\circ}{9} = 20^\circ$$

$$\therefore \quad K_d = \frac{\sin\left(\frac{m\beta}{2}\right)}{m\sin\left(\frac{\beta}{2}\right)} = \frac{\sin\left(\frac{3\times 20}{2}\right)}{3\sin\left(\frac{20}{2}\right)} = 0.9598$$

$$\therefore \quad 6350.853 = 4.44\times 1\times 0.9598\times 16\times 10^{-3}\times 50\times T_{ph}$$

$$\therefore \quad T_{ph} = 1862.852 \approx 1862$$

$$\therefore \quad Z_{ph} = 2\times T_{ph} = 2\times 1862 = \mathbf{3724}$$

These are armature conductors per phase required to be connected in series.

Example 4.10 : *In a 3 phase, star connected alternator, there are 2 coil sides per slot and 16 turns per coil. Armature has 288 slots on its periphery. When driven at 250 r.p.m. it produces 6600 V between the lines at 50 Hz. The pitch of the coil is 2 slots less than the full pitch. Calculate the flux per pole.*

Solution : $N_s = 250$ r.p.m., $f = 50$ Hz

$$\text{Slots} = 288,\ E_{line} = 6600\ \text{V}$$

$$N_s = \frac{120\times f}{P}$$

$$\therefore \quad 250 = \frac{120\times 50}{P}$$

$$\therefore \quad P = 24$$

$$n = \frac{\text{Slots}}{\text{Pole}} = \frac{288}{24} = 12$$

$$m = \frac{n}{2} = \frac{12}{3} = 4$$

$$\beta = \frac{180^\circ}{n} = \frac{180^\circ}{12} = 15^\circ$$

$$\therefore \quad K_d = \frac{\sin\left(\frac{m\beta}{2}\right)}{m\sin\left(\frac{\beta}{2}\right)} = \frac{\sin\left(\frac{4\times 15}{2}\right)}{4\sin\left(\frac{15}{2}\right)} = 0.9576$$

Now coil is short pitched by 2 slots.

$$\therefore \quad \alpha = \text{Angle of short pitch} = 2 \times \beta = 2 \times 15$$

$$= 30°$$

$$\therefore \quad K_c = \cos\left(\frac{\alpha}{2}\right) = \cos(15) = 0.9659$$

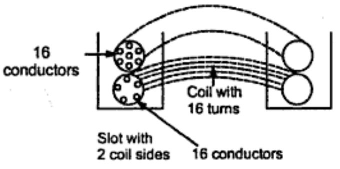


Fig. 4.42

Each coil consists of 16 turns, i.e. in a slot each coil side consists of 16 conductors as shown in the Fig. 4.42. and in each slot there are 2 coil sides. So each slot consists of 16 per coil side × 2 i.e. 32 conductors.

$\therefore$ Conductors/slot = 32

$\therefore$ Total conductors = Slots × conductors/slot

$$\therefore \quad Z = 288 \times 32$$

$$\therefore \quad Z = 9216$$

$$\therefore \quad Z_{ph} = \text{Conductors/phase} = \frac{9216}{3}$$

$$= 3072$$

$$\therefore \quad T_{ph} = \frac{Z_{ph}}{2} = \frac{3072}{2} \qquad \text{... 2 conductors} \rightarrow \text{1 turn}$$

$$= 1536$$

Now $$E_{ph} = \frac{E_{line}}{\sqrt{3}} = \frac{6600}{\sqrt{3}} = 3810.51 \text{ V}$$

$$E_{ph} = 4.44\, K_c\, K_d\, \phi\, f\, T_{ph}$$

$$\therefore \quad 3810.51 = 4.44 \times 0.9659 \times 0.9576 \times \phi \times 50 \times 1536$$

$$\therefore \quad \phi = 0.012 \text{ Wb}$$

$$= \mathbf{12 \text{ mWb}}$$

Example 4.11 : *A 12 pole, three phase, 600 r.p.m., star connected alternator has 180 slots. There are 2 coil sides per slot and total 10 conductors per slot. If flux per pole is 0.05 Wb, determine from first principles,*
i) r.m.s. value of e.m.f. in a conductor ii) r.m.s. value of e.m.f. in a turn
iii) r.m.s. value of e.m.f. in a coil iv) per phase induced e.m.f.
Assume full pitch coils.

Solution : $P = 12, \quad N_s = 600$ r.p.m.

$$\therefore \quad f = \frac{P \times N_s}{120} = \frac{12 \times 600}{120}$$

$$= 60 \text{ Hz}$$

i) Average value of e.m.f. in a conductor = $2 f \phi$

$$\therefore \quad \text{r.m.s. value} = 1.11 \times 2 f \phi = 2.22 \times 60 \times 0.05$$

$$= \mathbf{6.66 \ V}$$

ii) Average value of e.m.f. in a turn = $4 f \phi$

As 2 conductors joined properly form a turn.

$$\therefore \quad \text{r.m.s. value} = 1.11 \times 4 f \phi = 4.44 \times 60 \times 0.05$$

$$= \mathbf{13.32 \ V}$$

iii) Now each slot has 10 conductors and 2 coil sides. So,

$$\text{Conductors/coil side} = \frac{10}{2} = 5$$

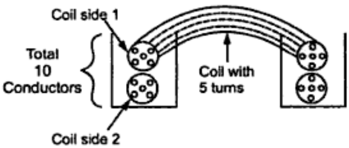


Fig. 4.43

Such coil sides are connected to another coil sides to form a coil. So in a coil there are 5 turns as shown in Fig. 4.43.

$$\therefore \quad \text{R.M.S. value of e.m.f. in a coil} = \frac{\text{R.M.S. value of e.m.f.}}{\text{turn}} \times \frac{\text{Number of turns}}{\text{coil}} = 13.32 \times 5$$

$$= \mathbf{66.6 \ V}$$

iv) Now total conductors $\quad Z = \dfrac{\text{Conductors}}{\text{Slot}} \times \text{Number of slots}$

$$\therefore \quad Z = 10 \times 180 = 1800$$

$$\therefore \quad Z_{ph} = \frac{Z}{3} = \frac{1800}{3} = 600$$

$$\therefore \quad T_{ph} = \frac{Z_{ph}}{2} = \frac{600}{2} = 300$$

And $\quad n = \dfrac{\text{Slots}}{\text{Pole}} = \dfrac{180}{12} = 15$

$$m = \frac{n}{3} = 5$$

$$\beta = \frac{180°}{n} = \frac{180°}{15} = 12°$$

$$\therefore \quad K_d = \frac{\sin\left(\frac{m\beta}{2}\right)}{m \sin\left(\frac{\beta}{2}\right)} = \frac{\sin\left(\frac{5\times 12}{2}\right)}{5\sin\left(\frac{12}{2}\right)}$$

$$= 0.9566$$

$$\therefore \quad E_{ph} = \text{R.M.S. value per turn} \times T_{ph} \times K_d \times K_c$$

$$= 13.32 \times 300 \times 0.9566 \times 1$$

$$= \mathbf{3822.88\ V}$$

$$\text{or} \quad E_{ph} = 4.44\ K_c\ K_d\ \phi\ f\ T_{ph}$$

$$= 4.44 \times 1 \times 0.9566 \times 0.05 \times 60 \times 300$$

$$= \mathbf{3822.88\ V}$$

Example 4.12 : *A 6 pole, 3 phase, 50 Hz alternator has 12 slots per pole, and 4 conductors per slot. The winding is $\frac{5}{6}$ th pitch. The flux per pole is 1.5 Wb. The armature is star connected. Calculate line value of induced e.m.f.*

[JNTU: Dec.-2003 (Set-2), May-2004 (Set-1)]

Solution : P = 6, f = 50 Hz, n = 12 slots/pole, 4 conductors/slot

For full pitch, n = 12 slots/pole

Actual pitch of winding $= \frac{5}{6} \times n = \frac{5}{6} \times 12 = 10$ slots

So winding shorted by = 12 − 10 = 2 slots

$$\therefore \quad \alpha = \text{Short pitch angle} = 2 \text{ slot angles} = 2 \times \beta$$

$$\text{Now} \quad \beta = \frac{180°}{n} = \frac{180°}{12} = 15°$$

$$\therefore \quad \alpha = 2\beta = 30°$$

$$\therefore \quad K_c = \cos\frac{\alpha}{2} = \cos 15° = 0.9659$$

$$m = \frac{n}{3} = \frac{12}{3} = 4 \text{ slots/pole/phase}$$

$$\therefore \quad K_d = \frac{\sin \frac{m\beta}{2}}{m \sin \frac{\beta}{2}} = 0.95766$$

$$\text{Total slots} = n \times P = 12 \times 6 = 72$$

$$\therefore \quad Z = \text{Total conductors} = 72 \times 4 = 288$$

$$\therefore \quad Z_{ph} = \frac{288}{3} = 96$$

$$\therefore \quad T_{ph} = \frac{Z_{ph}}{2} = 48$$

$$\therefore \quad E_{ph} = 4.44\ K_c\ K_d\ \phi\ f\ T_{ph}$$

$$= 4.44 \times 0.9659 \times 0.95766 \times 1.5 \times 50 \times 48$$

$$= 14785.2606\ \text{V} = 14.7852\ \text{kV}$$

$$\therefore \quad E_{line} = \sqrt{3}\ E_{ph} = \mathbf{25.6088\ kV}$$

Example 4.13 : *The coil span for the stator winding of an alternator is 120° (electrical). Find the chording factor of the winding.* **[JNTU : Nov.-2008, (Set-1)]**

Solution : The coil span of 120° is shown in the Fig. 4.44.

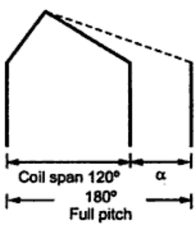


Fig. 4.44

The angle of short pitch is,

$\alpha = 180° - \text{coil span} = 180° - 120°$

$= 60°$

The chording factor is,

$K_c = \cos\frac{\alpha}{2} = \cos 30° = \mathbf{0.866}$

Example 4.14 : *In a 60 kVA, 200 V, single phase alternator, the effective armature resistance and leakage reactance are* $0.016\,\Omega$ *and* $0.07\,\Omega$ *respectively. Calculate the e.m.f. induced in the armature, when the alternator is delivering rated current at a p.f. of a) unity b) 0.7 lagging.* **[JNTU : Nov.-2005, (Set-4)]**

Solution : $V_{ph} = 200$ V, 60 kVA, $R_a = 0.016\,\Omega$, $X_s = 0.07\,\Omega$

Key Point: *The alternator is single phase hence all values are per phase.*

$$VA = V_{ph} I_{ph} \quad \text{i.e. } 60 \times 10^3 = 200 \times I_{ph} \qquad \text{... Single phase}$$

$$\therefore \quad I_{ph} = 300\ \text{A} = I_a \qquad \text{... Full load current}$$

a) $\cos\phi = 1, \quad \sin\phi = 0$

$$\therefore \qquad E_{ph}^2 = (V_{ph}\cos\phi + I_a R_a)^2 + (I_a X_s)^2$$

$$= (200\times 1 + 300\times 0.016)^2 + (300\times 0.07)^2$$

$$\therefore \qquad E_{ph} = \mathbf{205.8738\ V}$$

b) $\cos\phi = 0.7$ lagging, $\sin\phi = 0.714$

$$E_{ph}^2 = (V_{ph}\cos\phi + I_a R_a)^2 + (V_{ph}\sin\phi + I_a X_s)^2$$

$$= (200\times 0.7 + 300\times 0.016)^2 + (200\times 0.7141 + 300\times 0.07)^2$$

$$\therefore \qquad E_{ph} = \mathbf{218.6413\ V}$$

Example 4.15 : *A 550 V, 55 kVA single phase alternator has an effective resistance of $0.2\,\Omega$. A field current of 10 A produces an armature current of 200 A on short circuit and an e.m.f. of 450 V on open circuit. Calculate :*

a) Synchronous reactance b) Full load regulation at 0.8 lagging p.f.

[JNTU : March-2006,(Set-2)]

Solution : $V_{ph} = 550$ V, 55 kVA, $R_a = 0.2\,\Omega$

Key Point: *The alternator is* ***single phase*** *hence all values are per phase.*

$I_f = 10$ A, $I_{asc} = 200$ A, $V_{oc} = 450$ V

$$\therefore \qquad Z_s = \left.\frac{V_{oc}}{I_{asc}}\right|_{\text{same } I_f} = \left.\frac{450}{200}\right|_{I_f = 10\text{ A}} = 2.25\,\Omega$$

a) $$X_s = \sqrt{Z_s^2 - R_a^2} = \sqrt{2.25^2 - 0.2^2} = 2.2411\,\Omega$$

$$VA = V_{ph} I_{ph} \qquad \text{... As single phase}$$

$$\therefore \qquad 55\times 10^3 = 550\times I_{ph}$$

$$\therefore \qquad I_{ph} = 100\text{ A} = I_a \qquad \text{... Full load armature current}$$

b) $\cos\phi = 0.8$ lagging, $\sin\phi = 0.6$

$$E_{ph}^2 = (V_{ph}\cos\phi + I_a R_a)^2 + (V_{ph}\sin\phi + I_a X_s)^2$$

$$= (550\times 0.8 + 100\times 0.2)^2 + (550\times 0.6 + 100\times 2.2411)^2$$

$$\therefore \qquad E_{ph} = \mathbf{720.1652\ V}$$

$$\therefore \qquad \%R = \frac{E_{ph} - V_{ph}}{V_{ph}}\times 100 = \frac{720.1652 - 550}{550}\times 100 = \mathbf{30.94\ \%}$$

Example 4.16 : *The effective resistance of a 2200 V, 50 Hz, 440 kVA, single phase alternator is 0.5 Ω. On short circuit a field current of 40 A gives the full load current of 200 A. The e.m.f. on open circuit with the same field excitation is 1160 V.*
Calculate : a) Synchronous impedance, b) Synchronous reactance c) % regulation at 0.707 p.f. leading. **[JNTU : May-2005, (Set-1, 2), Nov.-2005, (Set-2, 3), March-2006, (Set-4)]**

Solution : $V_{ph} = 2200$ V, f = 50 Hz, 440 kVA, $R_a = 0.5\ \Omega$

Key Point: *The given alternator is* ***single phase*** *hence all values are per phase.*

$I_{aph} = 200\text{ A} = I_{sc},\ V_{oc} = 1160\text{ V},\ I_f = 40\text{ A}$

a) $$Z_s = \left.\frac{V_{oc}}{I_{sc}}\right|_{\text{same } I_f} = \frac{1160}{200} = 5.8\ \Omega$$

b) $$X_s = \sqrt{Z_s^2 - R_a^2} = \sqrt{5.8^2 - 0.5^2} = 5.7784\ \Omega$$

c) $$\cos\phi = 0.707 \text{ leading, } \sin\phi = 0.707$$

$$\therefore \quad E_{ph}^2 = (V_{ph}\cos\phi + I_a R_a)^2 + (V_{ph}\sin\phi - I_a X_s)^2$$

$$= (2200 \times 0.707 + 200 \times 0.5)^2 + (2200 \times 0.707 - 200 \times 5.7784)^2$$

$$\therefore \quad E_{ph} = 1702.9754 \text{ V}$$

$$\therefore \quad \% R = \frac{E_{ph} - V_{ph}}{V_{ph}} \times 100 = \frac{1702.9754 - 2200}{2200} \times 100 = -22.592\ \%$$

Example 4.17 : *The data obtained on 100 kVA, 1100 V, 3 phase alternator is,*
D.C. resistance test : E between lines = 6 V d.c., I in lines = 10 A d.c.
O.C. test : Field current = 12.5 A, Voltage between lines = 420 V.
S.C. test : Field current = 12.5 A, Line current = Rated value.
Calculate the voltage regulation of alternator at 0.8 power factor lagging.
[JNTU : Nov.-2008, (Set-4)]

Solution : Assume star connected alternator.

$$R_a + R_a = \frac{V_{d.c.}}{I_{d.c.}}$$

$$\therefore \quad 2R_a = \frac{6}{10} = 0.6$$

$$\therefore \quad R_a = 0.3\ \Omega / \text{ph}$$

$V_{oc}(\text{line}) = 420$ V, $V_L = 1100$ V, 100 kVA

$$\therefore \quad VA = \sqrt{3}\, V_L I_L$$

i.e. $$100 \times 10^3 = \sqrt{3} \times 1100 \times I_L$$

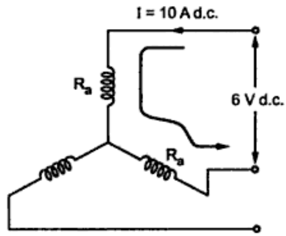


Fig. 4.45 D.C. resistance test

$\therefore \quad I_L = 52.4864 \text{ A} = I_{aph}$ ... Star connection

$\therefore$ Rated armature current = 52.4864 A = I_{asc}

$$\therefore \quad Z_s = \frac{V_{oc}(ph)}{I_{asc}(ph)} = \frac{(420/\sqrt{3})}{52.4864} = 4.62\,\Omega/\text{ph}$$

$$\therefore \quad X_s = \sqrt{Z_s^2 - R_a^2} = \sqrt{(4.62)^2 - (0.3)^2} = 4.6102\,\Omega/\text{ph}$$

For $\cos\phi = 0.8$ lagging, $\sin\phi = 0.6$

$$(E_{ph})^2 = (V_{ph}\cos\phi + I_a R_a)^2 + (V_{ph}\sin\phi + I_a X_s)^2$$

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{1100}{\sqrt{3}} = 635.085 \text{ V}$$

$$\therefore \quad (E_{ph})^2 = [635.085 \times 0.8 + 52.4864 \times 0.3]^2 + [635.085 \times 0.6 + 52.4864 \times 4.6102]^2$$

$$\therefore \quad E_{ph} = 813.9654 \text{ V}$$

$$\therefore \quad \% R = \frac{E_{ph} - V_{ph}}{V_{ph}} \times 100 = \frac{813.9654 - 635.085}{635.085} \times 100 = \mathbf{28.1663\ \%}$$

Example 4.18 : *The O.C. and S.C. test data on a 3 phase, 1 MVA, 3.6 kV, star connected alternator is given below :*

I_f, A	60	70	80	90	100	110
OC V	2560	3000	3360	3600	3800	3960
SC A	180					

The resistance measured between the terminals is 2Ω. Find the % regulation at full load 0.707 p.f. lagging and 0.8 p.f. leading by synchronous impedance method.

[JNTU : May-2005, (Set-3, 4)]

Solution : $2R_a = 2$ i.e. $R_a = 1\,\Omega/\text{ph}$

$V_L = 3.6$ kV, MVA = 1

$$\therefore \quad VA = \sqrt{3}\, V_L I_L$$

$$\therefore \quad 1 \times 10^6 = \sqrt{3} \times 3.6 \times 10^3 \times I_L$$

$\therefore \quad I_L = 160.375 \text{ A} = I_{aph}$ ... Star

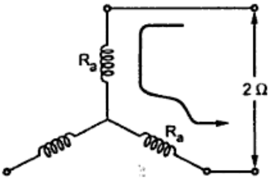


Fig. 4.46

From the test results, obtain the open circuit and short circuit characteristics and obtain V_{oc} for full load I_{sc} of 160.375 A.

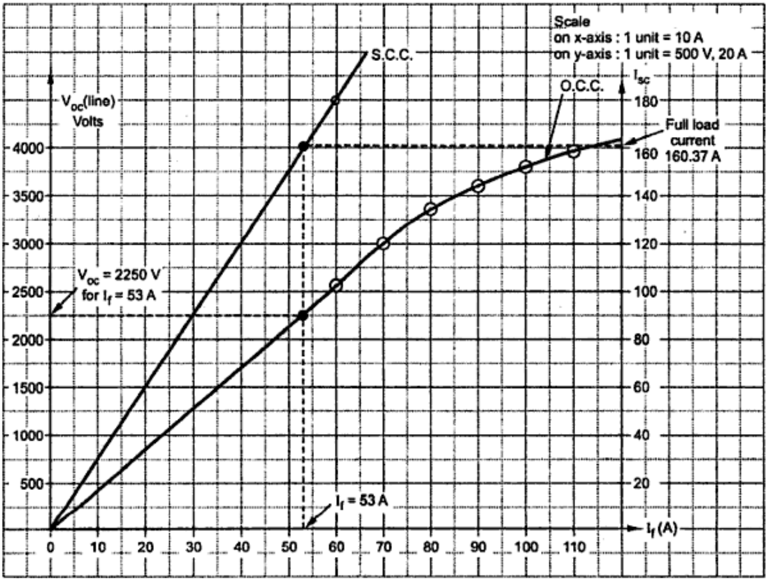


Fig. 4.47

From the graph, for full load short circuit current of 160.37 A, I_f = 53 A and corresponding V_{oc}(line) = 2250 V.

$$\therefore \quad Z_s = \left.\frac{V_{ocph}}{I_{scph}}\right|_{\text{same } I_f} = \left.\frac{\left(\frac{2250}{\sqrt{3}}\right)}{160.37}\right|_{I_f = 53\,A} = 8.1\ \Omega/\text{ph}$$

$$\therefore \quad X_s = \sqrt{Z_s^2 - R_a^2} = \sqrt{8.1^2 - 1^2} = 8.038\ \Omega/\text{ph}$$

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{3.6 \times 10^3}{\sqrt{3}} = 2078.46\ \text{V},\ I_{aph} = 160.37\ \text{A}$$

i) $\cos\phi$ = 0.707 lagging, $\sin\phi$ = 0.707

$$\therefore \quad E_{ph}^2 = (V_{ph}\cos\phi + I_a R_a)^2 + (V_{ph}\sin\phi + I_a X_s)^2$$

$$= (2078.46 \times 0.707 + 160.37 \times 1)^2$$

$$+ (2078.46 \times 0.707 + 160.37 \times 8.038)^2$$

$$\therefore \qquad E_{ph} = 3204.0356 \text{ V}$$

$$\therefore \qquad \% R = \frac{E_{ph} - V_{ph}}{V_{ph}} \times 100 = \frac{3204.0356 - 2078.46}{2078.46} \times 100 = \mathbf{54.15\ \%}$$

ii) $\cos\phi = 0.8$ leading, $\sin\phi = 0.6$

$$\therefore \qquad E_{ph}^2 = (V_{ph}\cos\phi + I_a R_a)^2 + (V_{ph}\sin\phi - I_a X_s)^2$$

Substituting the values, $E_{ph} = 1823.6271$ V

$$\therefore \qquad \% R = \frac{1823.6271 - 2078.46}{2078.46} \times 100 = \mathbf{-12.26\ \%}$$

Example 4.19 : *A 1 MVA, 11 kV, 3 phase, star connected alternator has following O.C.C. test data :*

I_f A	50	110	140	180
V_{oc} (line)V	7000	12500	13750	15000

The short circuit test yielded full load current at a field current of 40 A. The armature resistance per phase is 0.6 Ω. Find the % regulation at half full load at 0.8 p.f. lagging and at full load, 0.9 p.f. leading. **[JNTU : Nov.-2005, (Set-1)]**

Solution : 1 MVA, $V_L = 11$ kV, $R_a = 0.6\,\Omega$

$$VA = \sqrt{3}\, V_L I_L \quad \text{i.e.} \quad 1 \times 10^6 = \sqrt{3} \times 11 \times 10^3 \times I_L$$

$$\therefore \qquad I_L = 52.486 \text{ A} = I_{aph}(\text{full load}) \qquad \text{... Star}$$

Now $I_f = 40$ A for $I_{asc} = 52.486$ A. To find Z_s, plot the O.C.C. and obtain V_{oc} for $I_f = 40$ A. (See Fig. 4.48 on next page.)

From the graph, V_{oc}(line) = 6000 V for $I_f = 40$ A.

$$\therefore \qquad Z_s = \left.\frac{V_{ocph}}{I_{ascph}}\right|_{\text{same } I_f} = \left.\frac{\left(\frac{6000}{\sqrt{3}}\right)}{52.486}\right|_{I_f = 40\text{ A}} = 66\,\Omega$$

$$\therefore \qquad X_s = \sqrt{Z_s^2 - R_a^2} = \sqrt{66^2 - 0.6^2} = 65.99\,\Omega$$

a) $\cos\phi = 0.8$ lagging, $\sin\phi = 0.6$, half load

At half load, $I_{aph} = \frac{1}{2} \times I_{aph}(\text{FL}) = \frac{1}{2} \times 52.486 = 26.243$ A

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{11 \times 10^3}{\sqrt{3}} = 6350.853 \text{ V}$$

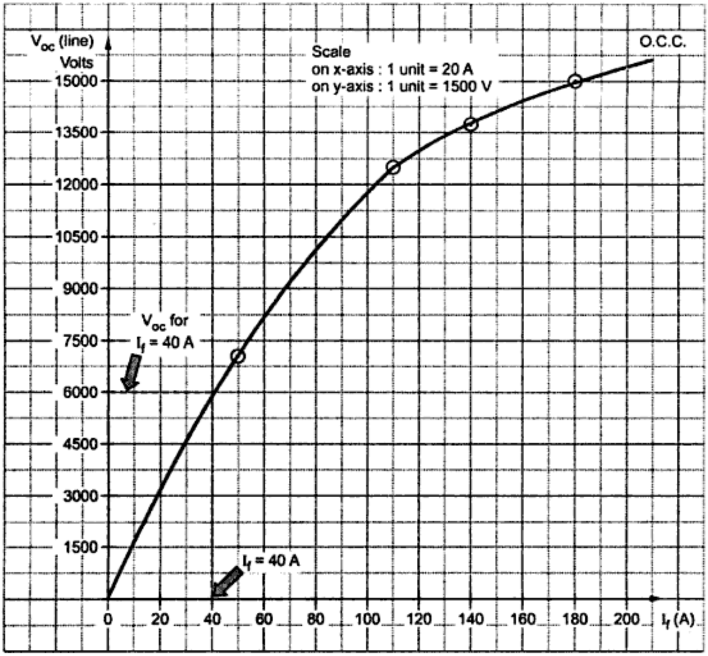


Fig. 4.48

$$E_{ph}^2 = (V_{ph}\cos\phi + I_{aph}R_a)^2 + (V_{ph}\sin\phi + I_{aph}X_s)^2$$

$$= [6350.853 \times 0.8 + 26.243 \times 0.6]^2$$

$$+ [6350.853 \times 0.6 + 26.243 \times 65.99]^2$$

$$\therefore \quad E_{ph} = 7529.3113 \text{ V}$$

$$\% R = \frac{E_{ph} - V_{ph}}{V_{ph}} \times 100 = \frac{7529.3113 - 6350.853}{6350.853} \times 100 = +18.56 \%$$

b) $\cos\phi = 0.9$ leading, $\sin\phi = 0.4358$, Full load so $I_{aph} = 52.486$ A

$$E_{ph}^2 = (V_{ph}\cos\phi + I_{aph}R_a)^2 + (V_{ph}\sin\phi - I_{aph}X_s)^2$$

$$= [6350.853 \times 0.9 + 52.486 \times 0.6]^2$$

$$+ [6350.853 \times 0.4358 - 52.486 \times 65.99]^2$$

$$\therefore \quad E_{ph} = 5789.231 \text{ V}$$

$$\% R = \frac{E_{ph} - V_{ph}}{V_{ph}} \times 100 = \frac{5789.231 - 6350.853}{6350.853} \times 100 = \mathbf{-8.843\ \%}$$

Example 4.20 : *A 3 phase, star connected alternator has an open circuit line voltage of 6599 V. The armature resistance and synchronous reactance are 0.6 Ω and 6 Ω per phase respectively. Find the terminal voltage and voltage regulation if load current is 180 A at a power factor of a) 0.9 lagging b) 0.8 leading* **[JNTU : March-2006, (Set-1)]**

Solution : $R_a = 0.6\ \Omega,\ X_s = 6\,\Omega,\ I_{aph} = 180$ A

$$E_{ph} = \frac{E_{line}}{\sqrt{3}} = \frac{6599}{\sqrt{3}} = 3809.9344 \text{ V}$$

a) $\cos\phi = 0.9$ lagging, $\sin\phi = 0.4358$

$$E_{ph}^2 = (V_{ph}\cos\phi + I_{aph} R_a)^2 + (V_{ph}\sin\phi + I_{aph} X_s)^2$$

$$\therefore \quad (3809.9344)^2 = [V_{ph} \times 0.9 + 180 \times 0.6]^2 + [V_{ph} \times 0.4358 + 180 \times 6]^2$$

$$\therefore \quad 14.5156 \times 10^6 = 0.81\, V_{ph}^2 + 194.4\, V_{ph} + 11664 + 0.1899\, V_{ph}^2 + 941.328\, V_{ph} + 1166400$$

$$\therefore \quad V_{ph}^2 + 1135.728\, V_{ph} - 13337536 = 0$$

$$\therefore \quad V_{ph} = 3128.08,\ -4263.808 \qquad \text{... Neglect negative value}$$

$$\therefore \quad V_{ph} = \mathbf{3128.08\ V} \qquad \text{... Terminal voltage}$$

$$\therefore \quad \% R = \frac{E_{ph} - V_{ph}}{V_{ph}} \times 100 = \frac{3809.9344 - 3128.08}{3128.08} \times 100 = \mathbf{+\,21.7978\ \%}$$

b) $\cos\phi = 0.8$ leading, $\sin\phi = 0.6$

$$\therefore \quad E_{ph}^2 = (V_{ph}\cos\phi + I_{aph} R_a)^2 + (V_{ph}\sin\phi - I_{aph} X_s)^2$$

$$\therefore \quad (3809.9344)^2 = [V_{ph} \times 0.8 + 180 \times 0.6]^2 + [V_{ph} \times 0.6 - 180 \times 6]^2$$

$$\therefore \quad 14.5156 \times 10^6 = 0.64\, V_{ph}^2 + 172.8\, V_{ph} + 11664 + 0.36\, V_{ph}^2 - 1296\, V_{ph} + 1166400$$

$$\therefore \quad V_{ph}^2 - 1123.2\, V_{ph} - 13337536 = 0$$

$$\therefore \quad V_{ph} = \mathbf{4256.5872\ V} \qquad \text{... Neglecting negative value}$$

$$\therefore \quad \% R = \frac{E_{ph} - V_{ph}}{V_{ph}} \times 100 = \frac{3809.9344 - 4256.5872}{4256.5872} \times 100 = \mathbf{-\,10.493\ \%}$$

Example 4.21 : *A 3 phase star connected alternator is rated at 1500 kVA, 12 kV. The armature effective resistance and synchronous reactance are 2 Ω and 10 Ω per phase respectively. Calculate % regulation for a load of 1200 kW at 0.8 p.f. lagging and 0.707 p.f. leading.* **[JNTU : Nov.-2004, (Set - 2)]**

Solution : 1500 kVA, $V_L = 12$ kV, $R_a = 2\Omega$, $X_s = 10\Omega$

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{12 \times 10^3}{\sqrt{3}} = 6928.2032 \text{ V} \quad \text{... Star}$$

$$P_L = \sqrt{3}\, V_L\, I_L \cos\phi$$

a) $\cos\phi = 0.8$ lagging, $\sin\phi = 0.6$

$$\therefore \quad 1200 \times 10^3 = \sqrt{3} \times 12 \times 10^3 \times I_L \times 0.8$$

$$\therefore \quad I_L = 72.168 \text{ A} = I_{aph} \quad \text{... Star}$$

$$E_{ph}^2 = (V_{ph} \cos\phi + I_a R_a)^2 + (V_{ph} \sin\phi + I_a X_s)^2$$

$$= (6928.2032 \times 0.8 + 72.168 \times 2)^2 + (6928.2032 \times 0.6 + 72.168 \times 10)^2$$

$$\therefore \quad E_{ph} = 7492.768 \text{ V}$$

$$\% R = \frac{E_{ph} - V_{ph}}{V_{ph}} \times 100 = \frac{7492.768 - 6928.2032}{6928.2032} \times 100 = \mathbf{+8.148\ \%}$$

b) $\cos\phi = 0.707$ leading, $\sin\phi = 0.707$

$$\therefore \quad 1200 \times 10^3 = \sqrt{3} \times 12 \times 10^3 \times I_L \times 0.707 \quad \text{i.e. } I_L = 81.66 \text{ A} = I_a$$

$$E_{ph}^2 = (V_{ph} \cos\phi + I_a R_a)^2 + (V_{ph} \sin\phi - I_a X_s)^2$$

$$= (6928.2032 \times 0.707 + 81.66 \times 2)^2 + (6928.2032 \times 0.707 - 81.66 \times 10)^2$$

$$\therefore \quad E_{ph} = 6502.2433 \text{ V}$$

$$\% R = \frac{E_{ph} - V_{ph}}{V_{ph}} \times 100 = \frac{6502.2433 - 6928.2032}{6928.2032} \times 100 = \mathbf{-6.148\%}$$

Review Questions

1. *Explain in detail the constructional features of a three phase alternator.*
2. *Discuss the advantages of rotating field type of alternators.*
3. *List the differences between salient type and non-salient type of rotor construction.*
4. *Establish the relationship between the number of poles, frequency and the synchronous speed for a three phase alternator.*
5. *Explain the working principle of a three phase alternator.*
6. *What is the difference between degrees mechanical and degrees electrical ? Explain.*
7. *With the help of neat sketches, explain the various types of windings used in alternators.*
8. *Derive the generalized expression for an induced e.m.f. per phase in three phase alternator.*

9. *Define and state the expressions for*

   a) Pitch factor

   b) Distribution factor

10. *Derive the expressions for the pitch factor and the distribution factor.*
11. *Which are the various parameters of an armature winding ?*
12. *What is the armature reaction ? How power factor of the load affects the armature reaction. Explain with neat sketches.*
13. *Explain the concept of synchronous impedance of a three phase alternator.*
14. *State the voltage equation for a three phase alternator, explaining each term involved in it.*
15. *Which are the various voltage drops, which occur in an alternator when it is loaded ?*
16. *What is voltage regulation of an alternator ?*
17. *Draw the phasor diagram and establish the relationship between induced e.m.f. and the terminal voltage for a three phase alternator for lagging, leading and unity power factor load.*
18. *Explain the synchronous impedance method for calculating the regulation of a three phase alternator.*
19. *A 3 phase, 12 pole, star connected alternator has 72 slots and 6 conductors per slot. If it is driven at 500 r.p.m., calculate the induced e.m.f. per phase. Assume coil span as 150° electrical and flux per pole 40 mWb.* **(Ans. : 596.53 V)**
20. *The stator of a 3 phase, 8 pole alternator has 144 slots and 2 conductors per slot. The winding is arranged in double layer. All the conductors per phase are connected in series if it is driven at 750 r.p.m. speed, calculate induced e.m.f. per phase. Flux per pole is 20 mWb and coil span is 150° elect.* **(Ans. : 196.815 V)**
21. *The particulars of rotating field type alternator are as below : 3 phase, star connected, 4 pole, 1800 r.p.m., 60 slots, 1 coil side per slot and 1 turn per coil, flux per pole 1.5 Wb. Calculate r.m.s. values of induced e.m.f. in i) each conductor ii) each coil iii) each phase.*

    Assume coil span as 13 slots. **(Ans. : i) 199.8 V ii) 390.86 V iii) 3739.43 V)**

22. *A 3 phase, star connected, 8 pole, 750 r.p.m. alternator has 3 slots per pole per phase. The conductors per slot are 8 while flux per pole is 0.62 Wb. Calculate the induced e.m.f. per phase.* **(Ans. : 12.682 kV)**
23. *A 230 V, 3 phase, 50 Hz, star connected alternator has 6 poles with 4 slots per pole per phase. There are 2 coil sides per slot with 2 turns per coil. The coil span is $\frac{5}{6}$ th of full pitch. Calculate flux per pole to produced rated voltage of 230 V between the lines.* **(Ans. : 13.47 mWb)**
24. *Find the number of conductors in series per phase required for an armature of a three phase, 50 Hz, 10 pole alternator with 90 slots. The winding is to be star connected to give a line voltage of 11 kV. The flux per pole is 0.16 Wb.* **(Ans. : 372 conductors per ph)**

25. *A 500 V, 50 kVA, single phase alternator has an effective resistance of 0.2 Ω. A field current of 10 A produces an armature current of 200 A on short circuit and an e.m.f. of 450 V on open circuit. Calculate the full load regulation with 0.8 lagging p.f.* **(Ans. : + 34.32%)**

26. *A 50 kVA, 750 V, 3 phase star connected alternator has open circuit e.m.f. of 125 V for a field current of 12 A. When the terminals are shorted, same field current is driving a current of 38.49 A through the armature. If resistance between the terminals is 1.92 Ω.*

    Calculate full load % regulation at

    i) 0.8 lagging ii) 0.8 leading and iii) Unity p.f. **(Ans. : 15.47%, – 0.376%, +9.47%)**

❑❑❑

5 Three Phase Induction Motors

5.1 Introduction

An electric motor is a device which converts an electrical energy into a mechanical energy. This mechanical energy then can be supplied to various types of loads. The motors can operate on d.c. as well as single and three phase a.c. supply. The motors operating on d.c. supply are called d.c. motors while motors operating on a.c. supply are called a.c. motors. As a.c. supply is commonly available, the a.c. motors are very popularly used in practice. The a.c. motors are classified as single and three phase induction motors, synchronous motors and some special purpose motors. Out of all these types, three phase induction motors are widely used for various industrial applications. Hence this chapter gives the emphasis on the working principle, types and features of three phase induction motors. The important advantages of three phase induction motors over other types are self starting property, no need of starting device, higher power factor, good speed regulation and robust construction. The working principle of three phase induction motors is based on the production of **rotating magnetic field.** Hence before beginning the actual discussion of three phase induction motors, let us discuss the production of rotating magnetic field from a three phase a.c. supply.

5.2 Rotating Magnetic Field (R.M.F.)

The rotating magnetic field can be defined as the field or flux having constant amplitude but whose axis is continuously rotating in a plane with a certain speed. So if the arrangement is made to rotate a permanent magnet, then the resulting field is a rotating magnetic field. But in this method, it is necessary to rotate a magnet physically to produce rotating magnetic field.

But in three phase induction motors such a rotating magnetic field is produced by supplying currents to a set of **stationary** windings, with the help of three phase a.c. supply. The current carrying windings produce the magnetic field or flux. And due to interaction of three fluxes produced due to three phase supply, resultant flux has a constant magnitude and its axis rotating in space, without physically rotating the windings. This type of field is nothing but rotating magnetic field. Let us study how it happens.

5.2.1 Production of R.M.F.

A three phase induction motor consists of three phase winding as its stationary part called **stator**. The three phase stator winding is connected in star or delta. The three phase windings are displaced from each other by 120°. The windings are supplied by a balanced three phase a.c. supply. This is shown in the Fig. 5.1. The three phase windings are denoted as R-R′, Y-Y′ and B-B′.

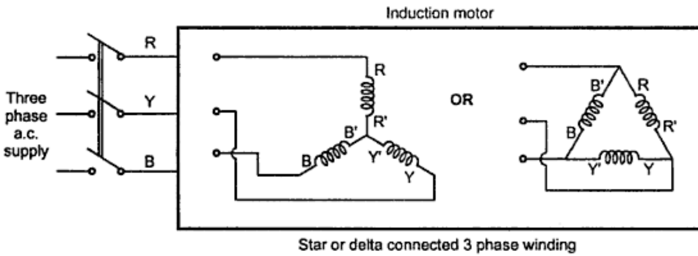


Fig. 5.1

The three phase currents flow simultaneously through the windings and are displaced from each other by 120° electrical. Each alternating phase current produces its own flux which is sinusoidal. So all three fluxes are sinusoidal and are separated from each other by 120°. If the phase sequence of the windings is R-Y-B, then mathematical equations for the instantaneous values of the three fluxes ϕ_R, ϕ_Y and ϕ_B can be written as,

$$\phi_R = \phi_m \sin(\omega t) = \phi_m \sin\theta \qquad \ldots (1)$$

$$\phi_Y = \phi_m \sin(\omega t - 120°) = \phi_m \sin(\theta - 120°) \qquad \ldots (2)$$

$$\phi_B = \phi_m \sin(\omega t - 240°) = \phi_m \sin(\theta - 240°) \qquad \ldots (3)$$

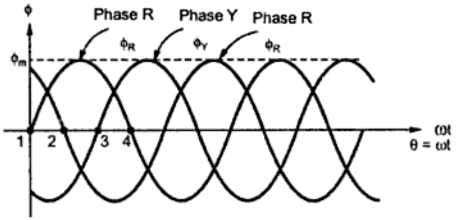
As windings are identical and supply is balanced, the magnitude of each flux is ϕ_m. Due to phase sequence R-Y-B, flux ϕ_Y lags behind ϕ_R by 120° and ϕ_B lags ϕ_Y by 120°. So ϕ_B ultimately lags ϕ_R by 240°. The flux ϕ_R is taken as reference while writing the equations.

The Fig. 5.2 (a) shows the waveforms of three fluxes in space. The Fig. 5.2 (b) shows the phasor diagram which clearly shows the **assumed positive directions** of each flux. Assumed positive direction means whenever the flux is positive it must be represented along the direction shown and whenever the flux is negative it must be represented along the opposite direction to the assumed positive direction.

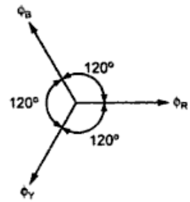
Let ϕ_R, ϕ_Y and ϕ_B be the instantaneous values of three fluxes. The resultant flux ϕ_T is the phasor addition of ϕ_R, ϕ_Y and ϕ_B.

$$\therefore \qquad \bar{\phi}_T = \bar{\phi}_R + \bar{\phi}_Y + \bar{\phi}_B$$

Let us find ϕ_T at the instants 1, 2, 3 and 4 as shown in the Fig. 5.2 (a) which represents the values of θ as 0°, 60°, 120° and 180° respectively. The phasor addition can be performed by obtaining the values of ϕ_R, ϕ_Y and ϕ_B by substituting values of θ in the equations (1), (2) and (3).



(a) Waveforms of three fluxes



(b) Assumed positive directions

Fig. 5.2

Case 1 : $\theta = 0°$

Substituting in the equations (1), (2) and (3) we get,

$$\phi_R = \phi_m \sin 0° = 0$$

$$\phi_Y = \phi_m \sin(-120°) = -0.866\ \phi_m$$

$$\phi_B = \phi_m \sin(-240°) = +0.866\ \phi_m$$

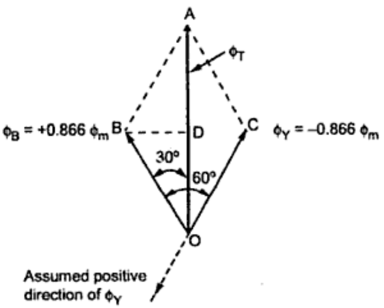


Fig. 5.3 (a) Vector diagram for θ = 0°

The phasor addition is shown in the Fig. 5.3 (a). The positive values are shown in assumed positive directions while negative values are shown in opposite direction to the assumed positive directions of the respective fluxes. Refer to assumed positive directions shown in the Fig. 5.2 (b).

BD is drawn perpendicular from B on ϕ_T. It bisects ϕ_T.

$\therefore$ $$OD = DA = \frac{\phi_T}{2}$$

In triangle OBD, $$\angle BOD = 30^\circ$$

$\therefore$ $$\cos 30^\circ = \frac{OD}{OB} = \frac{\phi_T/2}{0.866\,\phi_m}$$

$\therefore$ $$\phi_T = 2\times 0.866\ \phi_m \times \cos 30^\circ$$

$$= 1.5\ \phi_m$$

So magnitude of ϕ_T is 1.5 ϕ_m and its position is vertically upwards at $\theta = 0°$.

Case 2 : $\theta = 60^\circ$

Equations (1), (2) and (3) give us,

$$\phi_R = \phi_m \sin 60^\circ = +\ 0.866\ \phi_m$$

$$\phi_Y = \phi_m \sin(-60^\circ) = -\ 0.866\ \phi_m$$

$$\phi_B = \phi_m \sin(-180^\circ) = 0$$

So ϕ_R is positive and ϕ_Y is negative and hence drawing in appropriate directions we get phasor diagram as shown in the Fig. 5.3 (b).

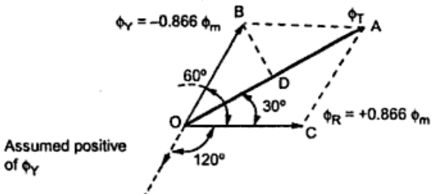


Fig. 5.3 (b) Vector diagram for $\theta = 60°$

Doing the same construction, drawing perpendicular from B on ϕ_T at D we get the same result as,

$$\phi_T = 1.5\ \phi_m$$

But it can be seen that though its magnitude is 1.5 ϕ_m it has rotated through 60° in space, in clockwise direction, from its previous position.

Case 3 : $\theta = 120^\circ$

Equations (1), (2) and (3) give us,

$$\phi_R = \phi_m \sin 120 = +\ 0.866\ \phi_m$$

$$\phi_Y = \phi_m \sin 0 = 0$$

$$\phi_B = \phi_m \sin(-120) = -\ 0.866\ \phi_m$$

From the above discussion we have following conclusions :

a) The resultant of the three alternating fluxes, separated from each other by 120°, has a constant amplitude of 1.5 ϕ_m where ϕ_m is maximum amplitude of an individual flux due to any phase.

b) The resultant always keeps on rotating with a certain speed in space.

Key Point: *This shows that when a three phase stationary windings are excited by balanced three phase a.c. supply then the resulting field produced is* ***rotating magnetic field****. Though nothing is physically rotating, the field produced is rotating in space having constant amplitude.*

5.2.2 Speed of R.M.F.

There exists a fixed relation between frequency f of a.c. supply to the windings, the number of poles P for which winding is wound and speed N r.p.m. of rotating magnetic field. For a standard frequency whatever speed of R.M.F. results is called **synchronous speed,** in case of induction motors. It is denoted as N_s.

$$\therefore \qquad \boxed{N_s = \frac{120\,f}{P} = \textbf{Speed of R.M.F.}}$$

where f = Supply frequency in Hz

P = Number of poles for which winding is wound

This is the speed with which R.M.F. rotates in space. Let us see how to change direction of rotation of R.M.F.

5.2.3 Direction of R.M.F.

The direction of the R.M.F. is always from the axis of the leading phase of the three phase winding towards the lagging phase of the winding. In a phase sequence of R-Y-B, phase R leads Y by 120° and Y leads B by 120°. So R.M.F. rotates from axis of R to axis of Y and then to axis of B and so on. So its direction is clockwise as shown in the Fig. 5.4 (a). This direction can be reversed by interchanging any two terminals of the three phase windings while connecting to the three phase supply. The terminals Y and B are shown interchanged in the Fig. 5.4 (b). In such case the direction of R.M.F. will be anticlockwise.

The three rings made up of conducting material called **slip rings** are mounted on the same shaft with which winding is rotating. Each terminal of winding is connected to an individual slip ring, permanently. Thus three ends R-Y-B of winding are available at the three rotating slip rings. The three brushes are then used. Each brush is resting on the corresponding slip ring, making contact with the slip ring but the brushes are stationary. So rotating three ends R-Y-B are now available at the brushes which are stationary as shown in the Fig. 5.5. Now stationary external circuit can be connected to the brushes which are nothing but the three ends of the winding.

Thus the external stationary circuit can be connected to the rotating internal part of the machine with the help of slip rings and brush assembly. Not only the external circuit can be connected but the voltage also can be injected to the rotating winding, by connecting stationary supply to the brushes externally.

Key Point : *Such slip rings and brush assembly plays an important role in the working of slip ring induction motor.*

Let us see the construction of three phase induction motor.

5.4 Construction

Basically, the induction motor consists of two main parts, namely

1. The part i.e. three phase windings, which is stationary called **stator.**
2. The part which rotates and is connected to the mechanical load through shaft called **rotor.**

The conversion of electrical power to mechanical power takes place in a rotor. Hence rotor develops a driving torque and rotates.

5.4.1 Stator

The stator has a laminated type of construction made up of stampings which are 0.4 to 0.5 mm thick. The stampings are slotted on its periphery to carry the stator winding. The stampings are insulated from each other. Such a construction essentially keeps the iron losses to a minimum value. The number of stampings are stamped together to build the stator core. The built up core is then fitted in a casted or fabricated steel frame. The choice of material for the stampings is generally silicon steel, which minimises the hysteresis loss. The slots on the periphery of the stator core carries a **three phase winding,** connected either in star or delta. This three phase winding is called **stator winding.** It is wound for definite number of poles. This winding when excited by a three phase supply produces a rotating magnetic field as discussed earlier. The choice of number of poles depends on the speed of the rotating magnetic field required.

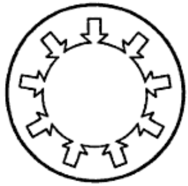


Fig. 5.6 Stator lamination

The radial ducts are provided for the cooling purpose. In some cases, all the six terminals of three phase stator winding are brought out which gives flexibility to the user to connect them either in star or delta. The Fig. 5.6 shows a stator lamination.

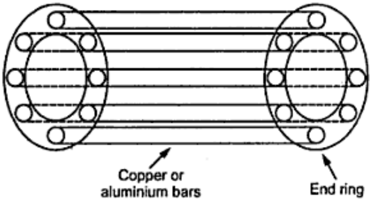
5.4.2 Rotor

The rotor is placed inside the stator. The rotor core is also laminated in construction and uses cast iron. It is cylindrical, with slots on its periphery. The rotor conductors or winding is placed in the rotor slots. The two types of rotor constructions which are used for induction motors are,

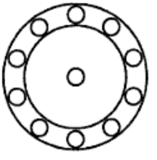
1. Squirrel cage rotor and
2. Slip ring or wound rotor

5.4.2.1 Squirrel Cage Rotor

The rotor core is cylindrical and slotted on its periphery. The rotor consists of uninsulated copper or aluminium bars called rotor conductors. The bars are placed in the slots. These bars are permanently shorted at each end with the help of conducting copper ring called **end ring**. The bars are usually brazed to the end rings to provide good mechanical strength. The entire structure looks like a cage, forming a closed electrical circuit. So the rotor is called squirrel cage rotor. The construction is shown in the Fig. 5.7.



(a) Cage type structure of rotor



(b) Symbolic representation

Fig. 5.7 Squirrel cage rotor

As the bars are permanently shorted to each other through end ring, the entire rotor resistance is very very small. Hence this rotor is also called **short circuited rotor.** As rotor itself is short circuited, no external resistance can have any effect on the rotor resistance. Hence no external resistance can be introduced in the rotor circuit. So slip ring and brush assembly is not required for this rotor. Hence the construction of this rotor is very simple.

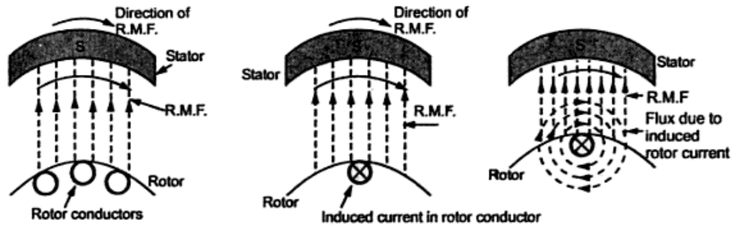
Fan blades are generally provided at the ends of the rotor core. This circulates the air through the machine while operation, providing the necessary cooling. The air gap between stator and rotor is kept uniform and as small as possible.

$$N_s = \frac{120\,f}{P} = \text{Speed of rotating magnetic field.}$$

where f = Supply frequency.

P = Number of poles for which stator winding is wound.

This rotating field produces an effect of rotating poles around a rotor. Let direction of rotation of this rotating magnetic field is **clockwise** as shown in the Fig. 5.9 (a).



(a) (b) (c)

Fig. 5.9

Now at this instant rotor is **stationary** and stator flux R.M.F. is **rotating**. So its obvious that there exists a relative motion between the R.M.F. and rotor conductors. Now the R.M.F. gets cut by rotor conductors as R.M.F. sweeps over rotor conductors. Whenever conductor cuts the flux, e.m.f. gets induced in it. So e.m.f. gets induced in the rotor conductors called **rotor induced e.m.f.** This is electro-magnetic induction. As rotor forms closed circuit, induced e.m.f. circulates current through rotor called **rotor current** as shown in the Fig. 5.9 (b). Let direction of this current is going into the paper denoted by a cross as shown in the Fig. 5.9 (b).

Any current carrying conductor produces its own flux. So rotor produces its flux called **rotor flux**. For assumed direction of rotor current, the direction of rotor flux is clockwise as shown in the Fig. 5.9 (c). This direction can be easily determined using right hand thumb rule. Now there are two fluxes, one R.M.F. and other rotor flux. Both the fluxes interact with each as shown in the Fig. 5.9 (d). On left of rotor conductor, two fluxes are in same direction hence add up to get high flux area. On right side, two fluxes cancel each other to produce low flux area. As flux lines act as stretched rubber band, high flux density area exerts a push on rotor conductor towards low flux density area. So rotor conductor experiences a force from left to right in this case, as shown in the Fig. 5.9 (d), due to **interaction of the two fluxes.**

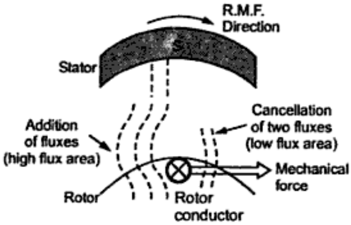


Fig. 5.9 (d)

As all the rotor conductors experience a force, the overall rotor experiences a torque and starts rotating. So **interaction of the two fluxes is very essential for a motoring action.** As seen from the Fig. 5.9 (d), the direction of force experienced is same as that of rotating magnetic field. Hence rotor starts rotating in the same direction as that of rotating magnetic field.

Alternatively this can be explained as : According to Lenz's law the direction of induced current in the rotor is so as to oppose the cause producing it. The cause of rotor current is the induced e.m.f. which is induced because of relative motion present between the rotating magnetic field and the rotor conductors. Hence to oppose the relative motion i.e. to reduce the relative speed, the rotor experiences a torque in the same direction as that of R.M.F. and tries to catch up the speed of rotating magnetic field.

So, N_s = Speed of rotating magnetic field in r.p.m.

N = Speed of rotor i.e. motor in r.p.m.

$N_s - N$ = Relative speed between the two, rotating magnetic field and the rotor conductors.

Thus rotor always rotates in same direction as that of R.M.F.

5.5.1 Can $N = N_s$?

When rotor starts rotating, it tries to catch the speed of rotating magnetic field.

If it catches the speed of the rotating magnetic field, the relative motion between rotor and the rotating magnetic field will vanish ($N_s - N = 0$). In fact the relative motion is the main cause for the induced e.m.f. in the rotor. So induced e.m.f. will vanish and hence there cannot be rotor current and the rotor flux which is essential to produce the torque on the rotor. Eventually motor will stop. But immediately there will exist a relative motion between rotor and rotating magnetic field and it will start. But due to inertia of rotor, this does not happen in practice and rotor continues to rotate with a speed slightly less than the synchronous speed of the rotating magnetic field in the steady state. The **induction motor never rotates at synchronous speed**. The speed at which it rotates is hence called **subsynchronous speed** and motor sometimes called **asynchronous motor**.

$$\therefore \quad N < N_s$$

So it can be said that rotor slips behind the rotating magnetic field produced by stator. The difference between the two is called **slip speed** of the motor.

$N_s - N$ = Slip speed of the motor in r.p.m.

This speed decides the magnitude of the induced e.m.f. and the rotor current, which in turn decides the torque produced. The torque produced is as per the requirements of overcoming the friction and iron losses of the motor along with the torque demanded by the load on the motor.

5.6 Slip of Induction Motor

We have seen that rotor rotates in the same direction as that of R.M.F. but in steady state attains a speed less than the synchronous speed. The difference between the two speeds i.e. synchronous speed of R.M.F. (N_s) and rotor speed (N) is called slip speed. This slip speed is generally expressed as the percentage of the synchronous speed.

So slip of the induction motor is defined as the difference between the synchronous speed (N_s) and actual speed of rotor i.e. motor (N) expressed as a fraction of the synchronous speed (N_s). This is also called absolute slip or fractional slip and is denoted as 's'.

Thus $$\boxed{s = \frac{N_s - N}{N_s}} \quad \text{... (Absolute slip)}$$

The percentage slip is expressed as,

$$\boxed{\% \, s = \frac{N_s - N}{N_s} \times 100} \quad \text{... (Percentage slip)}$$

In terms of slip, the actual speed of motor (N) can be expressed as,

$$\boxed{N = N_s (1 - s)} \quad \text{... (From the expression of slip)}$$

At start, motor is at rest and hence its speed N is zero.

$$\therefore \quad \boxed{s = 1 \text{ at start}}$$

This is maximum value of slip s possible for induction motor which occurs at start. While s = 0 gives us $N = N_s$ which is not possible for an induction motor. So **slip of induction motor cannot be zero** under any circumstances.

Practically motor operates in the slip range of 0.01 to 0.05 i.e. 1 % to 5 %. The slip corresponding to full load speed of the motor is called **full load slip.**

➡ **Example 5.1 :** *A 4 pole, 3 phase induction motor is supplied from 50 Hz supply. Determine its synchronous speed. On full load, its speed is observed to be 1410 r.p.m. Calculate its full load slip.*

Solution : Given values are,

$$P = 4, \quad f = 50 \text{ Hz}, \quad N = 1410 \text{ r.p.m.}$$

$$N_s = \frac{120 f}{P} = \frac{120 \times 50}{4} = 1500 \text{ r.p.m.}$$

Full load absolute slip is given by,

$$s = \frac{N_s - N}{N_s} = \frac{1500-1410}{1500} = 0.06$$

$$\therefore \quad \% s = 0.06 \times 100 = \mathbf{6\ \%}$$

Example 5.2 : *A 4 pole, 3 phase, 50 Hz, star connected induction motor has a full load slip of 4 %. Calculate full load speed of the motor.*

Solution : Given values are,

$$P = 4, \quad f = 50 \text{ Hz}, \quad \% s_{fl} = 4 \%$$

$$s_{fl} = \text{Full load absolute slip} = 0.04$$

$$N_s = \frac{120 f}{P} = \frac{120 \times 50}{4} = 1500 \text{ r.p.m}$$

$$s_{fl} = \frac{N_s - N_{fl}}{N_s} \qquad \text{where } N_{fl} = \text{Full load speed of motor}$$

$$\therefore \quad 0.04 = \frac{1500 - N_{fl}}{1500}$$

$$\therefore \quad N_{fl} = \mathbf{1440 \text{ r.p.m.}}$$

This is the full load speed of the motor.

5.7 Types of Induction Motor

The induction motors which work on three phase supply are called three phase induction motors. From construction point of view, three phase induction motors are classified as squirrel cage and slip ring induction motors.

The induction motors which work on single phase supply are called single phase induction motors. A single phase a.c. supply cannot produce rotating magnetic field. Thus these motors are not self starting. It is necessary to produce rotating magnetic field in single phase induction motors. According to the methods used to make single phase induction motors self starting, these are classified as,

1. Split phase induction motors.
2. Capacitor start induction motors.
3. Capacitor start capacitor run induction motors.
4. Shaded pole induction motors.

Some induction motors use special rotor constructions to obtain higher starting torque and lower starting line currents. Such motors are classified as,

1. Deep bar rotor induction motors.
2. Double cage rotor induction motors.

5.8 Effect of Slip on Rotor Parameters

In case of a transformer, frequency of the induced e.m.f. in the secondary is same as the voltage applied to primary. Now in case of induction motor at start N = 0 and slip s = 1. Under this condition as long as s = 1, the frequency of induced e.m.f. in rotor is same as the voltage applied to the stator. But as motor gathers speed, induction motor has some slip corresponding to speed N. In such case, the frequency of induced e.m.f. in rotor is no longer same as that of stator voltage. Slip affects the frequency of rotor induced e.m.f. Due to this some other rotor parameters also get affected. Let us study the effect of slip on the following rotor parameters.

1. Rotor frequency 2. Magnitude of rotor induced e.m.f. 3. Rotor reactance

4. Rotor power factor and 5. Rotor current

5.8.1 Effect on Rotor Frequency

In case of induction motor, the speed of rotating magnetic field is,

$$N_s = \frac{120\,f}{P} \qquad \dots (1)$$

where f = Frequency of supply in Hz.

At start when N = 0, s = 1 and stationary rotor has maximum relative motion with respect to R.M.F. Hence maximum e.m.f. gets induced in the rotor at start. The frequency of this induced e.m.f. at start is same as that of supply frequency.

As motor actually rotates with speed N, the relative speed of rotor with respect R.M.F. decreases and becomes equal to slip speed of $N_s - N$. The induced e.m.f. in rotor depends on rate of cutting flux i.e. relative speed $N_s - N$. Hence in running condition magnitude of induced e.m.f. decreases so as its frequency. **The rotor is wound for same number of poles as that of stator i.e. P.** If f_r is the frequency of rotor induced e.m.f. in running condition at slip speed $N_s - N$ then there exists a fixed relation between $(N_s - N)$, f_r and P similar to equation (1). So we can write for rotor in running condition,

$$(N_s - N) = \frac{120\,f_r}{P}\text{, rotor poles = Stator poles = P} \qquad \dots (2)$$

Dividing (2) by (1) we get,

$$\frac{N_s - N}{N_s} = \frac{(120\,f_r / P)}{(120\,f / P)} \qquad \text{but } \frac{N_s - N}{N_s} = \text{Slip s}$$

$$\therefore \qquad s = \frac{f_r}{f}$$

$$\therefore \qquad \boxed{f_r = s\,f}$$

5.8.3 Effect on Rotor Resistance and Reactance

The rotor winding has its own resistance and the inductance. In a case of squirrel cage rotor, the rotor resistance is very very small and generally neglected but slip ring rotor has its own resistance which can be controlled by adding external resistance through slip rings. In general let,

R_2 = Rotor resistance per phase on standstill.

X_2 = Rotor reactance per phase on standstill.

Now at standstill,

$f_r = f$ hence if L_2 is the inductance of rotor per phase,

$$X_2 = 2\pi f_r\, L_2 = 2\pi f L_2 \ \Omega/\text{ph.}$$

While R_2 = Rotor resistance in Ω/ph.

Now in running condition,

$f_r = sf$ hence,

$$X_{2r} = 2\pi f_r\, L_2 = 2\pi f s L_2 = s\,.\,(2\pi f L_2)$$

$$\therefore \quad \boxed{X_{2r} = s\, X_2}$$

Where X_{2r} = Rotor reactance in running condition.

Thus resistance as independent of frequency remains same at standstill and in running condition. While the rotor reactance decreases by slip times the rotor reactance at standstill.

Hence we can write rotor impedance per phase as :

Z_2 = Rotor impedance on standstill (N = 0) condition

$$= R_2 + j\, X_2 \quad \Omega/\text{ph}$$

$$\therefore \quad Z_2 = \sqrt{R_2^{\,2} + X_2^{\,2}} \quad \Omega/\text{ph} \qquad \text{... Magnitude}$$

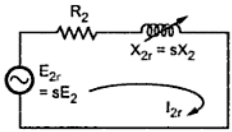
While Z_{2r} = Rotor impedance in running condition.

$$= R_2 + j\, X_{2r} = R_2 + j\,(s\, X_2) \quad \Omega/\text{ph}$$

$$\therefore \quad \boxed{Z_{2r} = \sqrt{R_2^{\,2} + (s\, X_2)^2} \quad \Omega/\text{ph}} \qquad \text{... Magnitude}$$

In the running condition, Z_2 changes to Z_{2r} while the induced e.m.f. changes to E_{2r}. Hence the magnitude of current in the running condition is also different than I_2 on standstill. The equivalent rotor circuit on running condition is shown in the Fig. 5.13.

I_{2r} = Rotor current per phase in running condition.



All values are phase

Fig. 5.13

The value of slip depends on speed which inturn depends on load on motor hence X_{2r} is shown variable in the equivalent circuit. From the equivalent circuit we can write,

$$\boxed{I_{2r} = \frac{E_{2r}}{Z_{2r}} = \frac{s\,E_2}{\sqrt{R_2^{\,2} + (s\,X_2)^2}}}$$

ϕ_{2r} is the angle between E_{2r} and I_{2r} which decides p.f. in running condition.

Key Point : *Putting s = 1 in the expressions obtained in running condition, the values at standstill can be obtained.*

5.9 Induction Motor as a Transformer

We know that, transformer is a device in which two windings are magnetically coupled and when one winding is excited by a.c. supply of certain frequency, the e.m.f. gets induced in the second winding having same frequency as that of supply given to the first winding. The winding to which supply is given is called primary winding while winding in which e.m.f. gets induced is called secondary winding. The induction motor can be regarded as the transformer.

The difference is that the normal transformer is an alternating flux transformer while induction motor is rotating flux transformer. The normal transformer has no air gap as against this an induction motor has distinct air gap between its stator and rotor.

In an alternating flux transformer the frequency of induced e.m.f. and current in primary and secondary is always same. However in the induction motor frequency of e.m.f. and current on the stator side remains same but frequency of rotor e.m.f. and current depends on the slip and slip depends on load on the motor. So we have a variable frequency on the rotor side. But it is important to remember that at start when N = 0 the value of slip is unity (s = 1), then frequency of supply to the stator and of induced e.m.f. in the rotor is same. The effect of slip on the rotor parameters is already discussed in the previous section.

And last difference is that in case of the alternating flux transformer the entire energy present in its secondary circuit, is in the electrical form. As against this, in an induction motor part of its energy in the rotor circuit is in electrical form and the remaining part is converted into mechanical form.

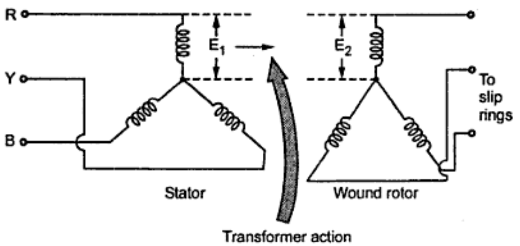


Fig. 5.14 Induction motor as a transformer

In general, an induction motor can be treated as a generalised transformer as shown in the Fig. 5.14. In this, the slip ring induction motor with star connected stator and rotor is shown.

So if E_1 = Stator e.m.f. per phase in volts.

E_2 = Rotor induced e.m.f. per phase in volts at start when motor is at standstill.

Then according to general transformer there exists a fixed relation between E_1 and E_2 called **transformation ratio.**

$\therefore$ At start when $\boxed{N = 0, \ s = 1}$

and we get,
$$\boxed{\frac{E_2}{E_1} = K = \frac{\text{Rotor turns / phase}}{\text{Stator turns / phase}}}$$

Key Point : *So if stator supply voltage is known and ratio of stator to rotor turns per phase is known then the rotor induced e.m.f. on standstill can be obtained.*

Example 5.4 : *A 8 pole, three phase induction motor is supplied from 50 Hz, a.c. supply. On full load, the frequency of induced e.m.f. in rotor is 2 Hz. Find the full load slip and the corresponding speed.*

Solution : The given values are,

$$P = 8, \ f = 50 \text{ Hz}, \ f_r = 2 \text{ Hz}$$

Now $f_r = sf$

$\therefore$ $2 = s \times 50$

$\therefore$ $s = \frac{2}{50} = 0.04$

$\therefore$ $\% s = 0.04 \times 100 = 4 \%$ ... Full load slip.

The corresponding speed is given by,

$$N = N_s (1 - s) \qquad \text{... From } s = N_s - N/N_s$$

where $$N_s = \frac{120 f}{P} = \frac{120 \times 50}{8} = 750 \text{ r.p.m.}$$

$$\therefore \quad N = 750 (1 - 0.04)$$

$$= \mathbf{720 \text{ r.p.m.}} \qquad \text{... Full load speed.}$$

Example 5.5 : *For a 4 pole, 3 phase, 50 Hz induction motor ratio of stator to rotor turns is 2. On a certain load, its speed is observed to be 1455 r.p.m. when connected to 415 V supply. Calculate,*

i) Frequency of rotor e.m.f. in running condition.

ii) Magnitude of induced e.m.f. in the rotor at standstill.

iii) Magnitude of induced e.m.f. in the rotor under running condition.

Assume star connected stator.

Solution : The given values are, K = Rotor turns/Stator turns = 1/2 = 0.5 and

$P = 4$, $f = 50$ Hz, $N = 1455$ r.p.m., $E_{1line} = 415$ V

$$N_s = \frac{120 f}{P} = \frac{120 \times 50}{4} = 1500 \text{ r.p.m.}$$

For a given load, N = 1455 r.p.m.

$$\therefore \quad s = \frac{N_s - N}{N_s} = \frac{1500 - 1455}{1500} = 0.03 \text{ i.e. } 3 \%$$

i) $$f_r = s f = 0.03 \times 50 = \mathbf{1.5 \text{ Hz}}$$

ii) At standstill, induction motor acts as a transformer so,

$$\frac{E_{2ph}}{E_{1ph}} = \frac{\text{Rotor turns}}{\text{Stator turns}} = K$$

But ratio of stator to rotor turns is given as 2, i.e.

$$\frac{N_1}{N_2} = 2 \qquad \therefore \frac{N_2}{N_1} = \frac{1}{2} = K$$

and $$E_{1\,line} = 415 \text{ V}$$

The given values are always line values unless and until specifically stated as per phase.

$$\therefore \quad E_{1ph} = \frac{E_1}{\sqrt{3}} = \frac{415}{\sqrt{3}} \qquad \text{... As star connection } E_{line} = \sqrt{3}\, E_{ph}$$

$\therefore \quad E_{1ph} = 239.6 \text{ V}$

$\therefore \quad \dfrac{E_{2ph}}{E_{1ph}} = \dfrac{1}{2}$

$\therefore \quad E_{2ph} = \dfrac{1}{2} \times 239.6 = \mathbf{119.8 \text{ V}}$ ... Rotor induced e.m.f. on standstill

iii) In running condition,

$$E_{2r} = s\,E_2 = 0.03 \times 119.8 = \mathbf{3.594 \text{ V}}$$

The value of rotor induced e.m.f. in the running condition is also very very small.

Example 5.6 : *A 3 phase, 4 pole, 50 Hz, induction motor has slip ring rotor. The rotor winding is star connected with 0.2 Ω of resistance per phase and standstill reactance of 1 Ω per phase. Its open circuit e.m.f. between the slip rings is 120 V, when stator is excited by a rated voltage. Its full load speed is 1440 r.p.m. Find the rotor current and rotor power factor.*

i) At start and ii) On full load condition.

Solution : The given values are,

$P = 4$, $f = 50$ Hz, $R_2 = 0.2\ \Omega$, $X_2 = 1\ \Omega$

Now open circuit e.m.f. between slip rings means rotor induced e.m.f. on standstill. As long as rotor is open, there cannot be rotor current and rotation of rotor. And between the slip rings means it's a line value of E_2, for a star connected rotor. The open circuit e.m.f. is shown in the Fig. 5.15.

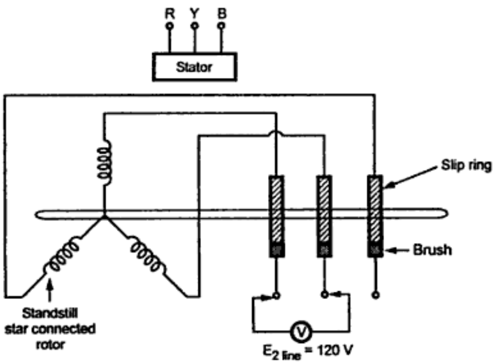


Fig. 5.15

$$\therefore \quad E_{2\,line} = 120 \text{ V, for star } E_{2\,line} = \sqrt{3}\, E_{2ph}$$

$$\therefore \quad E_{2ph} = \frac{E_{2\,line}}{\sqrt{3}} = \frac{120}{\sqrt{3}} = 69.28 \text{ V}$$

$$N_s = \frac{120\,f}{P} = \frac{120 \times 50}{4} = 1500 \text{ r.p.m.}$$

i) At start,

$$\cos\phi_2 = \frac{R_2}{Z_2} = \frac{R_2}{\sqrt{R_2^2 + X_2^2}} = \frac{0.2}{\sqrt{(0.2)^2 + (1)^2}}$$

$$= 0.196 \text{ lagging}$$

and

$$I_2 = \frac{E_2}{Z_2} = \frac{E_2}{\sqrt{{R_2}^2 + {X_2}^2}} = \frac{69.28}{\sqrt{(0.2)^2 + (1)^2}}$$

$$= \mathbf{67.93\ A\ per\ phase}$$

ii) On full load, N = 1440 r.p.m.

$$\therefore \quad s = \frac{N_s - N}{N_s} = \frac{1500 - 1440}{1500}$$

$$= 0.04$$

$$\therefore \quad \cos\phi_{2r} = \frac{R_2}{Z_{2r}} = \frac{R_2}{\sqrt{{R_2}^2 + (sX_2)^2}} = \frac{0.2}{\sqrt{(0.2)^2 + (0.04 \times 1)^2}}$$

$$= 0.9805 \text{ lagging}$$

and

$$I_{2r} = \frac{E_{2r}}{Z_{2r}} = \frac{s\,E_2}{\sqrt{{R_2}^2 + (sX_2)^2}} = \frac{0.04 \times 69.28}{\sqrt{(0.2)^2 + (0.04 \times 1)^2}}$$

$$= \mathbf{13.586\ A}$$

It can be observed that current is drastically reduced from its value at start. In the running condition, slip controls and limits the magnitude of the rotor current.

5.10 Torque Equation

The torque produced in the induction motor depends on the following factors :

1. The part of rotating magnetic field which reacts with rotor and is responsible to produce induced e.m.f. in rotor.
2. The magnitude of rotor current in running condition.
3. The power factor of the rotor circuit in running condition.

Mathematically the relationship can be expressed as,

$$T \propto \phi I_{2r} \cos \phi_{2r} \quad \text{... (1)}$$

where ϕ = Flux responsible to produce induced e.m.f.

I_{2r} = Rotor running current.

$\cos \phi_{2r}$ = Running p.f. of rotor.

The flux ϕ produced by stator is proportional to E_1 i.e. stator voltage.

$$\therefore \quad \phi \propto E_1 \quad \text{... (2)}$$

While E_1 and E_2 are related to each other through ratio of stator turns to rotor turns i.e. k.

$$\therefore \quad \frac{E_2}{E_1} = k \quad \text{... (3)}$$

Using equation (3) in (2) we can write,

$$\therefore \quad E_2 \propto \phi \quad \text{... (4)}$$

Thus in equation (1), ϕ can be replaced by E_2.

While
$$I_{2r} = \frac{E_{2r}}{Z_{2r}} = \frac{s E_2}{\sqrt{R_2^2 + (sX_2)^2}} \quad \text{... (5)}$$

and
$$\cos \phi_{2r} = \frac{R_2}{Z_{2r}} = \frac{R_2}{\sqrt{R_2^2 + (sX_2)^2}} \quad \text{... (6)}$$

Using equation (4), (5), (6) in equation (1),

$$T \propto E_2 \cdot \frac{s E_2}{\sqrt{R_2^2 + (sX_2)^2}} \cdot \frac{R_2}{\sqrt{R_2^2 + (sX_2)^2}}$$

$$\therefore \quad \boxed{T \propto \frac{s E_2^2 R_2}{R_2^2 + (sX_2)^2} \text{ N-m}}$$

$$\therefore \quad T = \frac{k s E_2^2 R_2}{R_2^2 + (sX_2)^2} \quad \text{...(7)}$$

where k = Constant of proportionality

The constant k is proved to be $3/2\pi n_s$ for the three phase induction motor.

$$\therefore \quad k = \frac{3}{2\pi n_s} \quad \text{... (8)}$$

From the expression of T_m, it can be observed that

1. It is inversely proportional to the **rotor reactance.**
2. It is directly proportional to the square of the rotor induced e.m.f. at standstill.
3. The most interesting observation is, **the maximum torque is not dependent on the rotor resistance R_2**. But the slip at which it occurs i.e. speed at which it occurs depends on the value of rotor resistance R_2.

Example 5.8 : *A 400 V, 4 pole, 3 phase, 50 Hz star connected induction motor has a rotor resistance and reactance per phase equal to 0.01 Ω and 0.1 Ω respectively. Determine i) Starting torque ii) Slip at which maximum torque will occur iii) Speed at which maximum torque will occur iv) Maximum torque v) Full load torque if full load slip is 4 %. Assume ratio of stator to rotor turns as 4.*

Solution : The given values are,

P = 4, f = 50 Hz, Stator turns/Rotor turns = 4, R_2 = 0.01 Ω, X_2 = 0.1 Ω

$$E_{1line} = \text{stator line voltage} = 400 \text{ V}$$

$$E_{1ph} = \frac{E_{1\,line}}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230.94 \text{ V} \quad \text{... Star connection}$$

$$K = \frac{E_{2ph}}{E_{1ph}} = \frac{\text{Rotor turns}}{\text{Stator turns}} = \frac{1}{4}$$

$$\therefore \quad E_2 = \frac{1}{4} \times E_{1ph} = \frac{230.94}{4} = 57.735 \text{ V}$$

$$N_s = \frac{120\,f}{P} = \frac{120 \times 50}{4} = 1500 \text{ r.p.m.}$$

i) At start, $s = 1$

$$\therefore \quad T_{st} = \frac{k\,E_2^2\,R_2}{R_2^2 + X_2^2} \qquad \text{where } k = \frac{3}{2\pi n_s}$$

$$n_s = \frac{N_s}{60} = \frac{1500}{60} = 25 \text{ r.p.s.}$$

$$\therefore \quad k = \frac{3}{2\pi \times 25} = 0.01909$$

$$\therefore \quad T_{st} = \frac{0.01909 \times (57.735)^2 \times 0.01}{(0.01)^2 + (0.1)^2} = \mathbf{63.031\ N\text{-}m}$$

ii) Slip at which maximum torque occurs is,

$$s_m = \frac{R_2}{X_2} = \frac{0.01}{0.1} = 0.1$$

$$\%\, s_m = 0.1 \times 100 = \mathbf{10\ \%}$$

At $N = N_s$, $s = 0$ hence $T = 0$. As no torque is generated at $N = N_s$, motor stops if it tries to achieve the synchronous speed. Torque increases linearly in this region, of low slip values.

ii) High slip region :

In this region, slip is high i.e. slip value is approaching to 1. Here it can be assumed that the term R_2^2 is very very small as compared to $(s X_2)^2$. Hence neglecting R_2^2 from the denominator, we get

$$\boxed{T \propto \frac{s R_2}{(sX_2)^2} \propto \frac{1}{s}}$$ where R_2 and X_2 are constants.

So in high slip region torque is inversely proportional to the slip. Hence its nature is like rectangular hyperbola.

Now when load increases, load demand increases but speed decreases. As speed decreases, slip increases. In high slip region as $T \propto 1/s$, torque decreases as slip increases. But torque must increase to satisfy the load demand. As torque decreases, due to extra loading effect, speed further decreases and slip further increases. Again torque decreases as $T \propto 1/s$ hence same load acts as an extra load due to reduction in torque produced. Hence speed further drops. Eventually motor comes to standstill condition. The **motor cannot continue to rotate at any point in this high slip region.** Hence this region is called **unstable region** of operation.

So torque - slip characteristics has two parts,

1. Straight line called **stable region of operation**
2. Rectangular hyperbola called **unstable region of operation.**

Now the obvious question is upto which value of slip, torque-slip characteristic represents stable operation ?

In low slip region, as load increases, slip increases and torque also increases linearly. Every motor has its own limit to produce a torque. The maximum torque, the motor can produce as load increases is T_m which occurs at $s = s_m$. So linear behaviour continues till $s = s_m$.

If load is increased beyond this limit, motor slip acts dominantly pushing motor into high slip region. Due to unstable conditions, motor comes to standstill condition at such a load. Hence T_m i.e. maximum torque which motor can produce is also called **breakdown torque or pull out torque.** So range $s = 0$ to $s = s_m$ is called low slip region, known as stable region of operation. Motor always operates at a point in this region. And range $s = s_m$ to $s = 1$ is called high slip region which is rectangular hyperbola, called unstable region of operation. Motor cannot continue to rotate at any point in this region.

At $s = 1$, $N = 0$ i.e. at start, motor produces a torque called **starting torque** denoted as T_{st}.

The entire torque-slip characteristics is shown in the Fig. 5.16.

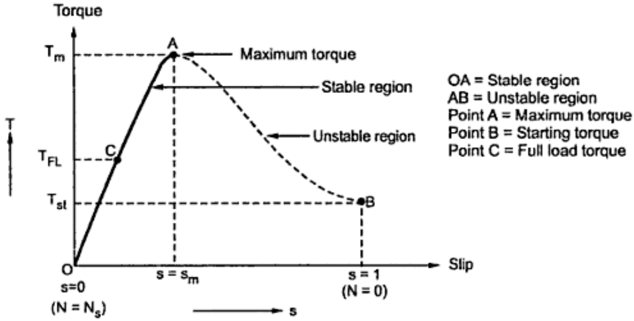


Fig. 5.16 Torque-slip characteristics

5.12.1 Full Load Torque

When the load on the motor increases, the torque produced increases as speed decreases and slip increases. The increased torque demand is satisfied by drawing more current from the supply.

The load which motor can drive safely while operating continuously and due to such load, the current drawn is also within safe limits is called **full load condition** of motor. When current increases, due to heat produced the temperature rises. The safe limit of current is that which when drawn for continuous operation of motor, produces a temperature rise well within the limits. Such a full load point is shown on the torque-slip characteristics as point C in the Fig. 5.16 and corresponding torque as $T_{F.L.}$.

The interesting thing is that the load on the motor can be increased beyond point C till maximum torque condition. But due to high current and hence high temperature rise there is possibility of damage of winding insulation, if **motor is operated for longer time** duration in this region i.e. from point C to B. But motor can be used to drive loads more than full load, producing torque upto maximum torque for short duration of time. **Generally full load torque is less than the maximum torque.**

So region OC upto full load condition allow motor operation continuously and safely from the temperature point of view. While region CB is possible to achieve in practice but only for short duration of time and not for continuous operation of motor. This is the difference between full load torque and the maximum or breakdown torque. The breakdown torque is also called stalling torque.

$$\therefore \qquad \boxed{T_{\text{Full load}} < T_m}$$

5.13.2 Starting Torque and Maximum Torque Ratio

Again starting with torque equation as,

$$T \propto \frac{s E_2^2 R_2}{R_2^2 + (sX_2)^2}$$

Now for T_{st}, $s = 1$

$$\therefore \quad T_{st} \propto \frac{E_2^2 R_2}{R_2^2 + X_2^2}$$

While for T_m, $s = s_m$

$$\therefore \quad T_m \propto \frac{s_m E_2^2 R_2}{R_2^2 + (s_m X_2)^2}$$

$$\therefore \quad \frac{T_{st}}{T_m} = \frac{E_2^2 R_2}{[R_2^2 + X_2^2]} \times \frac{[R_2^2 + (s_m X_2)^2]}{s_m E_2^2 R_2}$$

$$\therefore \quad \frac{T_{st}}{T_m} = \frac{[R_2^2 + (s_m X_2)^2]}{s_m [R_2^2 + X_2^2]}$$

Dividing both numerator and denominator by X_2^2 we get,

$$\therefore \quad \frac{T_{st}}{T_m} = \frac{\left[\frac{R_2^2}{X_2^2} + s_m^2\right]}{s_m \left[\frac{R_2^2}{X_2^2} + 1\right]}$$

Substituting $\frac{R_2}{X_2} = s_m$

$$\therefore \quad \boxed{\frac{T_{st}}{T_m} = \frac{2 s_m^2}{s_m (1 + s_m^2)} = \frac{2 s_m}{1 + s_m^2}}$$

Infact using the same method, ratio of any two torques at two different slip values can be obtained.

Sometimes using the relation, $R_2 = aX_2$ the torque ratios are expressed interms of constant a as,

$$\boxed{\frac{T_{F.L.}}{T_m} = \frac{2 a s_f}{a^2 + s_f^2}}$$

and $$\frac{T_{st}}{T_m} = \frac{2a}{1+a^2}$$

where $$a = \frac{R_2}{X_2} = s_m$$

Example 5.9 : *A 24 pole, 50 Hz, star connected induction motor has rotor resistance of 0.016 Ω per phase and rotor reactance of 0.265 Ω per phase at standstill. It is achieving its full load torque at a speed of 247 r.p.m. Calculate the ratio of*

i) Full load torque to maximum torque ii) Starting torque to maximum torque

Solution : Given values are,

$P = 24,\ f = 50\text{ Hz},\ R_2 = 0.016\ \Omega,\ X_2 = 0.265\ \Omega,\ N = 247\text{ r.p.m.}$

$$N_s = \frac{120f}{P} = \frac{120\times 50}{24} = 250\text{ r.p.m.}$$

$$s_f = \frac{N_s - N}{N_s} = \frac{250-247}{250} = 0.012 = \text{Full load slip}$$

$$s_m = \frac{R_2}{X_2} = \frac{0.016}{0.265} = 0.06037$$

i) $$\frac{T_{F.L.}}{T_m} = \frac{2\,s_m\,s_f}{s_m^2+s_f^2} = \frac{2\times 0.06037\times 0.012}{(0.06037)^2+(0.012)^2} = \mathbf{0.3824}$$

ii) $$\frac{T_{st}}{T_m} = \frac{2\,s_m}{1+s_m^2} = \frac{2\times 0.06037}{1+(0.06037)^2} = \mathbf{0.1203}$$

5.14 Losses in Induction Motor

The various power losses in an induction motor can be classified as,

i) Constant losses ii) Variable losses

i) Constant losses :

These can be further classified as core losses and mechanical losses

Core losses occur in stator core and rotor core. These are also called iron losses. These losses include eddy current losses and hysterisis losses. The eddy current losses are minimised by using laminated construction while hysterisis losses are minimised by selecting high grade silicon steel as the material for stator and rotor.

The iron losses depends on the frequency. The stator frequency is always supply frequency hence stator iron losses are dominant. As against this in rotor circuit, the frequency is very very small which is slip times the supply frequency. Hence rotor iron losses are very small and hence generally neglected, in the running condition.

The mechanical losses include frictional losses at the bearings and windage losses. The friction changes with speed but practically the drop in speed is very small hence these losses are assumed to be the part of constant losses.

ii) Variable losses : This include the copper losses in stator and rotor winding due to current flowing in the winding. As current changes as load changes, these losses are said to be variable losses.

Generally stator iron losses are combined with stator copper losses at a particular load to specify total stator losses at particular load condition.

Rotor copper loss $= 3\, I_{2r}^2\, R_2$ ... analysed separately

where I_{2r} = Rotor current per phase at a particular load

R_2 = Rotor resistance per phase

5.15 Power Flow in an Induction Motor

Induction motor converts an electrical power supplied to it into mechanical power. The various stages in this conversion is called **power flow** in an induction motor.

The three phase supply given to the stator is the **net electrical input** to the motor. If motor power factor is cos ϕ and V_L , I_L are line values of supply voltage and current drawn, then net input electrical power supplied to the motor can be calculated as,

$$\boxed{P_{in} = \sqrt{3}\, V_L\, I_L \cos\phi}$$

where $\boxed{P_{in} = \text{Net input electrical power.}}$

This is nothing but the **stator input**.

The part of this power is utilised to supply the losses in the stator which are stator core as well as copper losses.

The remaining power is delivered to the rotor magnetically through the air gap with the help of rotating magnetic field. This is called **rotor input** denoted as P_2.

So $\boxed{P_2 = P_{in} - \text{stator losses (core + copper)}}$

The rotor is not able to convert its entire input to the mechanical as it has to supply rotor losses. The rotor losses are dominantly copper losses as rotor iron losses are very small and hence generally neglected. So rotor losses are rotor copper losses denoted as P_c.

So $\boxed{P_c = 3 \times I_{2r}^2 \times R_2}$

where I_{2r} = Rotor current per phase in running condition

R_2 = Rotor resistance per phase.

After supplying these losses, the remaining part of P_2 is converted into mechanical which is called **gross mechanical power** developed by the motor denoted as $\mathbf{P_m}$.

$$\therefore \quad \boxed{P_m = P_2 - P_c}$$

Now this power, motor tries to deliver to the load connected to the shaft. But during this mechanical transmission, part of P_m is utilised to provide **mechanical losses** like friction and windage.

And finally the power is available to the load at the shaft. This is called **net output** of the motor denoted as $\mathbf{P_{out}}$. This is also called shaft power.

$$\therefore \quad \boxed{P_{out} = P_m - \text{Mechanical losses.}}$$

The rating of the motor is specified in terms of value of P_{out} when load condition is full load condition.

The above stages can be shown diagrammatically called **power flow diagram** of an induction motor.

This is shown in the Fig. 5.17.

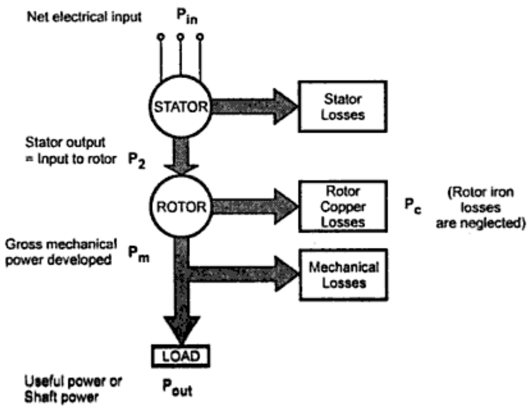


Fig. 5.17 Power flow diagram

From the power flow diagram we can define,

$$\boxed{\text{Rotor efficiency} = \frac{\text{rotor output}}{\text{rotor input}} = \frac{\text{gross mechanical power developed}}{\text{rotor input}}}$$

$$= \frac{P_m}{P_2}$$

$$\boxed{\text{Net motor efficiency} = \frac{\text{net output at shaft}}{\text{net electrical input to motor}} = \frac{P_{out}}{P_{in}}}$$

5.16 Relationship between P_2, P_c, and P_m

The rotor input P_2, rotor copper loss P_c and gross mechanical power developed P_m are related through the slip s. Let us derive this relationship.

Let T = Gross torque developed by motor in N-m.

We know that the torque and power are related by the relation,

$$P = T \times \omega$$

where P = power

and ω = angular speed

$$= \frac{2\pi N}{60}, \; N = \text{speed in r.p.m.}$$

Now input to the rotor P_2 is from stator side through rotating magnetic field which is rotating at synchronous speed N_s.

So torque developed by the rotor can be expressed interms of power input P_2 and angular speed at which power is inputted i.e. ω_s as,

$$P_2 = T \times \omega_s \qquad \text{where } \omega_s = \frac{2\pi N_s}{60} \text{ rad/sec}$$

$$\therefore \quad P_2 = T \times \frac{2\pi N_s}{60} \quad \text{where } N_s \text{ is in r.p.m.} \qquad \dots (1)$$

The rotor tries to deliver this torque to the load. So rotor output is gross mechanical power developed P_m and torque T. But rotor gives output at speed N and not N_s. So from output side P_m and T can be related through angular speed ω and not ω_s.

$$\therefore \quad P_m = T \times \omega \qquad \text{where } \omega = \frac{2\pi N}{60}$$

$$\therefore \quad P_m = T \times \frac{2\pi N}{60} \qquad \dots (2)$$

The difference between P_2 and P_m is rotor copper loss P_c.

$$\therefore \quad P_c = P_2 - P_m = T \times \frac{2\pi N_s}{60} - T \times \frac{2\pi N}{60}$$

$$\therefore \quad P_c = T \times \frac{2\pi}{60}(N_s - N) = \text{rotor copper loss} \qquad \dots (3)$$

$\therefore \quad N = N_s(1 - s) = 1500(1 - 0.04) = 440$ r.p.m.

$\therefore \quad P_{out} = 300 \times \frac{2\pi \times 1440}{60} = \mathbf{45.2389\ kW}$

Remember that T_{sh} is net output torque available to load at shaft.

iii) $\quad T_{lost} = 50$ Nm in friction

$\therefore \quad$ Frictional loss $= T_{lost} \times \omega = T_{lost} \times \frac{2\pi N}{60} = 50 \times \frac{2\pi \times 1440}{60} = 7539.822$ W

Now $\quad P_{out} = P_m -$ frictional loss

$\therefore \quad P_m = P_{out} +$ frictional loss $= 45.2389 \times 10^3 + 7539.822$

$= 52.77872$ kW

We know, $P_2 : P_c : P_m$ is $1 : s : 1 - s$

$$\therefore \quad \frac{P_c}{P_m} = \frac{s}{1-s}$$

$$\therefore \quad P_c = P_m\left(\frac{s}{1-s}\right) = 52.77872 \times 10^3 \times \left(\frac{0.04}{1-0.04}\right) = 2199.1134 \text{ W}$$

These are total rotor copper losses.

$\therefore$ Rotor copper loss per phase $= \frac{P_c}{3} = \frac{2199.1134}{3} = \mathbf{733.0378\ W}$

iv) $$\text{Rotor efficiency} = \frac{\text{Rotor output } P_m}{\text{Rotor input } P_2} \times 100$$

Now $$P_2 = \frac{P_c}{s} = \frac{2199.1134}{0.04} = 54977.8358 \text{ W}$$

$$\therefore \quad \% \text{ Rotor } \eta = \frac{52778.72}{54977.83} \times 100 = \mathbf{96\ \%}$$

v) $\quad I_{2r} = 60$ A $\quad$ given per phase

Now $\quad$ Rotor copper loss/ph $= I_{2r}^2 \times R_2$

$\therefore \quad 733.0378 = (60)^2 \times R_2$

$\therefore \quad R_2 = \mathbf{0.2036\ \Omega/ph}$

Example 5.11 : *While delivering an useful power of 24 kW to the full load, a 3 phase, 50 Hz, 8 pole induction motor draws a line current of 57 A. It runs at a speed of 720 r.p.m. and is connected to 415 supply. The p.f. of the motor is observed to be 0.707 lagging. Stator resistance per phase is 0.1 Ω. Mechanical losses are 1000 watts. Calculate, i) shaft torque ii) gross torque developed iii) rotor copper losses iv) stator copper losses v) stator iron losses vi) overall efficiency.*

Assume star connected stator winding.

Solution : $P_{out} = 24$ kW, $I_L = 57$ A

$$P = 8, \; N = 720 \text{ r.p.m.}, \; f = 50 \text{ Hz}$$

$$I_L = 415 \text{ V}$$

$$\cos\phi = 0.707$$

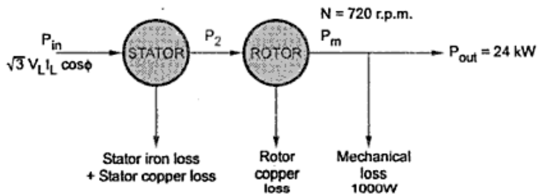


Fig. 5.19

$$N_s = \frac{120\,f}{P} = \frac{120 \times 50}{8} = 750 \text{ r.p.m.}$$

$$\therefore \quad s = \frac{N_s - N}{N_s} = \frac{750 - 720}{750} = 0.04$$

$$P_m - \text{mechanical loss} = P_{out}$$

$$\therefore \quad P_m = P_{out} + \text{mechanical loss} = 24 \times 10^3 + 1000 = 25000 \text{ W}$$

For rotor $P_2 : P_c : P_m$ is $1 : s : 1 - s$

$$\therefore \quad \frac{P_c}{P_m} = \frac{s}{1-s}$$

$$\therefore \quad P_c = P_m \times \left(\frac{s}{1-s}\right) = 25000 \times \frac{0.04}{(1 - 0.04)} = 1041.66 \text{ W}$$

i) Shaft torque $T_{sh} = \dfrac{P_{out}}{\omega} = \dfrac{P_{out}}{\left(\dfrac{2\pi N}{60}\right)} = \dfrac{24 \times 10^3}{\left(\dfrac{2\pi \times 720}{60}\right)} = 318.309$ Nm

ii) Gross torque $T = \frac{P_m}{\omega} = \frac{P_m}{\left(\frac{2\pi N}{60}\right)} = \frac{25\times10^3}{\left(\frac{2\pi\times720}{60}\right)}$ = **331.572 Nm**

iii) Rotor copper losses = **1041.66 W**

Now $\frac{P_2}{P_c} = \frac{1}{s}$

$\therefore$ $P_2 = \frac{1}{s}\times P_c = \frac{1}{0.04}\times1041.66 = 26041.5$ W

And net input $P_{in} = \sqrt{3}\ V_L\ I_L\ \cos\phi = \sqrt{3}\ \times 415\times57\times0.707 = 28966.96$ W

Stator current per phase = $I_L = 57$ A (as star connected)

R_s = Stator resistance per phase = 0.1 Ω

$\therefore$ Stator copper losses $= 3\times I_s^2\times R_s$

$= 3\times(57)^2\times0.1 =$ **974.7 W**

Now P_{in} – Stator losses = P_2

$\therefore$ 28966.96 – Stator losses = 26041.5

$\therefore$ Stator losses = 2925.46 W

But Stator losses = Stator iron loss + Stator copper loss

$\therefore$ 2925.46 = Stator iron losses + 974.7

$\therefore$ Stator iron losses = 1950.76 W

iv) Stator copper losses = 974.7 W

v) Stator iron losses = **1950.76 W**

vi) $\%\eta = \frac{P_{out}}{P_{in}}\times100 = \frac{24\times10^3}{28966.96}\times100 = 82.85\ \%$

Example 5.12 : *The power input to the rotor of a 440 V, 50 Hz, 3 phase, 12 pole induction motor is 75 kW. The rotor e.m.f. has a frequency of 2 Hz. Calculate : i) Slip ii) Rotor speed iii) Rotor copper loss. iv) Mechanical power developed.*

[April-2008 (Set-2)]

Solution : $V_L = 440$ V, f = 50 Hz P = 12, $f_r = 2$ Hz , $P_2 = 75$ kW

$$N_s = \frac{120f}{P} = \frac{120\times50}{12} = 500 \text{ r.p.m.}$$

i) $f_r = s\,f$ i.e. $2 = s\times50$

$\therefore$ s = **0.04** i.e. 4 % ...Slip

$$\therefore \quad I_{2r} = \frac{sE_2}{R_2 + j(sX_2)} = \frac{0.04 \times 31.7542}{0.7 + j(0.04 \times 5)}$$

$$= \frac{1.2702}{0.7 + j\,0.2} = \frac{1.2702}{0.728 \angle 15.94^\circ} = \mathbf{1.745 \angle{-15.94^\circ} A}$$

Example 5.15 : *A 12 pole, 50 Hz, 3-phase induction motor has rotor resistance of 0.15 Ω and standstill reactance of 0.25 Ω per phase. On full load it is running at a speed of 480 r.p.m. The rotor induced e.m.f. per phase at standstill is observed to be 32 V.*

Calculate : i) starting torque, ii) full load torque,

iii) maximum torque, iv) speed at maximum torque

Solution : P = 12, f = 50 Hz, $R_2 = 0.15\ \Omega$, $X_2 = 0.25\ \Omega$, $E_2 = 32$ V per phase given.

Now $$T = \frac{k\,s\,E_2^2\,R_2}{R_2^2 + (s\,X_2)^2} \quad \text{where} \quad k = \frac{3}{2\pi n_s}$$

$$N_s = \frac{120\,f}{P} = \frac{120 \times 50}{12} = 500 \text{ r.p.m.}$$

$$\therefore \quad n_s = \frac{N_s}{60} = \frac{500}{60} = 8.33 \text{ r.p.s.}$$

i) $$T_{st} = \frac{k\,E_2^2\,R_2}{R_2^2 + X_2^2} = \frac{3}{2\pi n_s} \times \frac{E_2^2\,R_2}{R_2^2 + X_2^2} = \frac{3}{2\pi\,8.33} \times \frac{(32)^2 \times 0.15}{[(0.15)^2 + (0.25)^2]}$$

$$= \mathbf{103.57\ Nm}$$

ii) At $N = 480$ r.p.m.

$$s = \frac{N_s - N}{N_s} = \frac{500 - 480}{500} = 0.04$$

$$\therefore \quad T_{F.L.} = \frac{3}{2\pi n_s} \times \frac{s\,E_2^2\,R_2}{R_2^2 + (s\,X_2)^2} = \frac{3}{2\pi \times 8.33} \times \frac{0.04 \times (32)^2 \times 0.15}{[(0.15)^2 + (0.04 \times 0.25)^2]}$$

$$= \mathbf{15.576\ Nm}$$

iii) $$T_m = \frac{3}{2\pi n_s} \times \frac{E_2^2}{2\,X_2} \quad \text{substituting } s_m = \frac{R_2}{X_2}$$

$$= \frac{3}{2\pi \times 8.33} \times \frac{(32)^2}{2 \times 0.25} = \mathbf{117.346\ Nm}$$

iv) Slip at T_m is, $$s_m = \frac{R_2}{X_2} = \frac{0.15}{0.25} = 0.6$$

$$\therefore \quad N = N_s\,(1 - s_m) = 500\,(1 - 0.6) = \mathbf{200\ r.p.m.}$$

Example 5.16 : *A 3 phase, 4 pole, 50 Hz induction motor is supplied by 400 V supply. Its full load slip is 4 %. At full load, stator copper losses are same as rotor copper losses. Stator iron losses are 25 % more than stator copper losses. Mechanical losses are one third of no load losses. Full load output is 50 h.p. Calculate the efficiency on full load.*

Solution : P_{out} = 50 h.p. = 50 × 735.5 W = 36775 W

On no load, the losses are constant losses including mechanical losses and stator iron losses. As no load current is small stator and rotor copper losses are neglected on no load. Rotor iron losses are negligible as its frequency is very small.

Let stator copper loss on full load = X watts

∴ Rotor copper loss on full load = X (given)

and stator iron loss = 1.25 X watts (25% more)

Now on no load,

$$\text{losses} = \text{mechanical} + \text{stator iron}$$

Let mechanical losses be Y watts.

$$\text{Mechanical loss} = \frac{1}{3} \times \text{No load loss}$$

$$\therefore \quad Y = \frac{1}{3} \times (Y + 1.25\ X)$$

$$\therefore \quad Y = 0.625\ X$$

On full load, total losses are

$$= \text{Stator iron} + \text{stator copper} + \text{rotor copper} + \text{mechanical loss}$$

$$= 1.25\ X + X + X + Y = 1.25\ X + X + X + 0.625\ X$$

$$= 3.875\ X$$

Now $\quad P_m - \text{mechanical losses} = P_{out}$

$$\therefore \quad P_m - Y = 36775$$

$$\therefore \quad P_m = 36775 + Y = 36775 + 0.625\ X$$

Now $P_2 : P_c : P_m$ is $1 : s : 1 - s$

$$\therefore \quad \frac{P_c}{P_m} = \frac{s}{1-s}$$

On full load $\quad P_c = X$

$$N_s = \frac{120\,f}{P_M} \quad \text{i.e.} \quad 1500 = \frac{120 \times 50}{P_M}$$

$\therefore \quad P_M = \mathbf{4}$ ... Poles of the induction motor

Example 5.18 : *If the electromotive force in the stator of 8 pole induction motor has a frequency of 50 Hz and that in the rotor is 1.5 Hz. At what speed is the motor running and what is the slip ?* **[JNTU : March-2006 (Set-2)]**

Solution : $P = 8$, $f = 50$ Hz, $f_r = 1.5$ Hz

$$f_r = s\,f \quad \text{i.e.} \quad 1.5 = s \times 50$$

$\therefore \quad s = \frac{1.5}{50} = \mathbf{0.03} \quad$ i.e. $\quad \mathbf{3\ \%}$ ... Slip

$$N_s = \frac{120\,f}{P} = \frac{120 \times 50}{8} = 750 \text{ r.p.m.}$$

$\therefore \quad N = N_s(1-s) = 750\,(1-0.03) = \mathbf{727.5\ r.p.m.}$ ... Speed of the motor

Example 5.19 : *A 3 phase, 4 pole, 50 Hz induction motor supplies a useful torque of 159 Nm. Calculate at 5 % slip,*

a) The rotor input b) The motor input c) The motor efficiency if friction and windage losses is 500 W and the stator losses equal to 1000 W. **[JNTU : Nov.-2008 (Set-3)]**

Solution : $P = 4$, $f = 50$ Hz, $T_{sh} = 159$ Nm, $s = 5\ \% = 0.05$

$$N_s = \frac{120 f}{P} = \frac{120 \times 50}{4} = 1500 \text{ r.p.m.}$$

$\therefore \quad N = N_s(1-s) = 1500 \times (1-0.05) = 1425$ r.p.m.

$\therefore \quad P_{out} = T_{sh} \times \omega = 159 \times \left(\frac{2\pi \times 1425}{60}\right) = 23726.8785$ W

$\therefore \quad P_m = P_{out} + \begin{bmatrix}\text{Friction and} \\ \text{windage loss}\end{bmatrix} = 23726.8785 + 500 = 24226.8785$ W

a) $P_2 : P_c : P_m$ is $1 : s : 1-s$

$\therefore \quad \frac{P_2}{P_m} = \frac{1}{1-s} \quad$ i.e. $\quad \frac{P_2}{24226.8785} = \frac{1}{(1-0.05)}$

$\therefore \quad P_2 = \mathbf{25501.9774\ W}$ ... Rotor input

b) $P_{in} = P_2 + \text{Stator losses} = 25501.9774 + 1000 = \mathbf{26501.9774\ W}$ ... Motor input

c) $\%\eta = \frac{P_{out}}{P_{in}} \times 100 = \frac{23726.8785}{26501.9774} \times 100 = \mathbf{89.528\ \%}$

Example 5.22 : *A 3 phase, 6 pole, 50 Hz induction motor has a slip of 1 % at no load and 3 % at full load. Find,*

a) Synchronous speed b) No load speed c) Full load speed

d) Frequency of rotor current at standstill

e) Frequency of rotor current at full load **[JNTU : March-2006 (Set-3)]**

Solution : P = 6, f = 50 Hz, s_0 = 1 %, s_{fl} = 3 %

a) $$N_s = \frac{120\,f}{P} = \frac{120\times 50}{6} = \mathbf{1000\ r.p.m.}$$ ... Synchronous speed

b) $$N_0 = N_s(1-s_0) = 1000\,(1-0.01) = \mathbf{990\ r.p.m.}$$ ... No load speed

c) $$N_{fl} = N_s(1-s_{fl}) = 1000\,(1-0.03) = \mathbf{970\ r.p.m.}$$ ... Full load speed

d) Frequency of rotor current at standstill = f = **50 Hz**

e) Frequency of rotor current at full load = $s_{fl} \times f = 0.03\times 50 =$ **1.5 Hz**

Example 5.23 : *A 50 H.P. 6 pole, 50 Hz, slip ring induction motor runs at 960 r.p.m. on full load with a rotor current of 40 A allowing 300 W for copper loss in the short-circuiting gear and 1200 W for mechanical losses. Find the resistance R_2 per phase of the 3 phase rotor winding.* **[JNTU : Nov.-2004 (Set-3)]**

Solution : P = 6, f = 50 Hz, I_{2r} = 40 A, N = 960 r.p.m.

$$\therefore \quad N_s = \frac{120\,f}{P} = \frac{120\times 50}{6} = 1000\ \text{r.p.m.}$$

$$\therefore \quad s = \frac{N_s - N}{N_s} = \frac{1000-960}{1000} = 0.04 \quad \text{i.e.} \quad 4\ \%$$

$$P_{out} = 50\ \text{H.P.} = 50\times 735.5 = 36775\ \text{W}$$ ... 1 H. P. = 735.5 W

$$P_m = P_{out} + \text{Mechanical losses} + \text{Gear loss}$$

$$= 36775 + 1200 + 300 = 38275\ \text{W}$$

Now $P_2 : P_c : P_m$ is $1 : s : 1-s$

$$\therefore \quad \frac{P_c}{P_m} = \frac{s}{1-s} \quad \text{i.e.} \quad \frac{P_c}{38275} = \frac{0.04}{(1-0.04)}$$

$$\therefore \quad P_c = \mathbf{1594.7916\ W}$$ ... **Total rotor copper loss**

Now $$P_c = 3\times I_{2r}^2 \times R_2 \quad \text{i.e.} \quad 1594.7916 = 3\times(40)^2\times R_2$$

$$\therefore \quad R_2 = \mathbf{0.3322\ \Omega/ph}$$

Review Questions

1. *What is rotating magnetic field ? Explain in brief.*
2. *Explain the construction of a three phase induction motor.*
3. *List the differences between squirrel cage rotor and slip ring rotor.*
4. *Define the term slip of the induction motor.*
5. *Explain the operating principle of a three phase induction motor.*
6. *How the direction of rotation of a three phase induction motor can be reversed ?*
7. *Explain the effect of slip on the following rotor parameters*

   i) frequency ii) induced e.m.f. iii) current iv) power factor v) reactance vi) impedance
8. *Derive the torque equation of a three phase induction motor.*
9. *Sketch and explain the typical torque-slip characteristics of a three phase induction motor.*
10. *Derive the condition for the maximum torque.*
11. *Derive the following torque ratios in terms of slip and rotor parameters*

    i) $\frac{T_{st}}{T_m}$ *ii)* $\frac{T_{F.L.}}{T_m}$ *iii)* $\frac{T_{st}}{T_{F.L.}}$
12. *List the various losses those take place in an induction motor.*
13. *Draw a power flow diagram of a three phase induction motor and explain all the stages.*
14. *Derive the relationship between rotor output, rotor input and slip in case of three phase induction motor.*
15. *List the various applications of three phase squirrel cage and slip ring induction motor.*
16. *A 50 Hz, 4 pole induction motor has an induced e.m.f. in the rotor with a frequency of 2 Hz. Calculate i) synchronous speed ii) slip iii) speed of the motor.*

    (Ans. : i) 1500 r.p.m. ii) 0.04 or 4 % iii) 1440 r.p.m.)
17. *A 4 pole, three phase, 50 Hz, induction motor has a star connected rotor. The rotor has a resistance of 0.1 Ω per phase and standstill reactance of 2 Ω per phase. The induced e.m.f. between the slip rings is 100 V. If full load speed is 1460 r.p.m. calculate i) slip ii) rotor frequency iii) rotor current iv) rotor power factor on full load condition. Assume slip rings shorted.*

    (Ans. : i) 2.67 % ii) 1.335 Hz iii) 13.57 A iv) 0.883 lagging)
18. *A star connected rotor of 3-phase, 4 pole, 50 Hz induction motor has standstill impedance of (0.35 + j2) Ω per phase. When stator is given a supply, the rotor induced e.m.f. between the slip rings is 160 V.*

    i) Calculate rotor current and rotor p.f. at start

    ii) Calculate rotor current at start if an external rheostat of 3 Ω per phase is introduced in rotor circuit.

    iii) Calculate rotor current and rotor p.f. when running at 1410 r.p.m. speed with external rheostat completely removed and slip rings are shorted.

    (Ans. : i) 45.5 A, 0.17 lag ii) 23.68 A iii) 14.98 A, 0.95 lag)

19. *A 3 phase, 4 pole, 415 V, 50 Hz induction motor has a star connected stator. The rotor impedance at standstill is 0.1 + j 0.9 Ω. The stator to rotor turns ratio is 1.75. Calculate*
i) rotor current at start ii) rotor current if slip of the motor is 5 %
iii) external resistance per phase in the rotor to limit starting rotor current to 60 A.
(Ans. : i) 151.19 A ii) 62.425 A iii) 1.996 Ω)

20. *A 3 kV, 50 Hz, 24 pole, 3 phase induction motor develops a full load torque at 240 r.p.m. The slip ring rotor has a resistance of 0.02 Ω and 0.27 Ω of standstill reactance per phase. Calculate*
i) ratio of maximum torque to full load torque ii) ratio of starting torque to full load torque iii) speed at maximum torque. **(Ans. : i) 1.196 ii) 0.1762 iii) 231.5 r.p.m.)**

21. *A 3 phase, 440 V, 50 Hz, 10 pole induction motor has stator to rotor turns ratio as 4 : 1. The stator is delta connected and the rotor is star connected. Rotor resistance is 0.018 Ω per phase. The motor develops a maximum torque at 540 r.p.m.*
Calculate i) rotor current and rotor p.f. at a full load slip of 4 % and shorted slip rings. ii) full load h.p. output assuming mechanical losses are 2 % of the shaft output.
(Ans. : i) 226.96 A, 0.925 lag ii) 89 h.p.)

22. *A 50 Hz, 8 pole, 3 phase induction has a full load slip of 4 %. The rotor resistance is 0.001 Ω per phase and standstill reactance is 0.005 Ω per phase. Find the ratio of maximum torque to full load torque and the speed at which the maximum torque occurs.* **(Ans. : 2.6, 600 r.p.m.)**

23. *A 6 pole, 3 phase, 50 Hz, 3000 V induction motor develops full load torque at 980 r.p.m. The rotor resistance and standstill reactance per phase are 0.01 Ω and 0.3 Ω respectively. Calculate ratio of maximum torque to full load torque and starting torque to full load torque.* **(Ans. : 1.1333, 0.075)**

24. *A 500 V, 50 Hz, 6 pole, 3 phase induction motor develops 20 h.p. inclusive of mechanical losses, when running at 995 r.p.m. The power factor of the motor 0.87 lag. Calculate i) slip ii) rotor copper loss iii) total input if stator losses are 1500 W iv) line current.*
(Ans. : 0.005, 74.97 W, 16.495 kW, 21.89 A)

□□□

6

Instruments

6.1 Introduction

The measurement of a given quantity is the result of comparison between the quantity to be measured and a definite standard. The instruments which are used for such measurements are called **measuring instruments.** The three basic quantities in the electrical measurement are current, voltage and power. The measurement of these quantities is important as it is used for obtaining measurement of s ome other quantity or used to test the performance of some electronic circuits or components etc.

The **necessary requirements** for any measuring instruments are :

1) With the introduction of the instrument in the circuit, the circuit conditions should not be altered. Thus the quantity to be measured should not get affected due to the instrument used.
2) The power consumed by the instruments for their operation should be as small as possible.

The instrument which measures the current flowing in the circuit is called **ammeter** while the instrument which measures the voltage across any two points of a circuit is called **voltmeter.** But there is no fundamental difference in the operating principle of analog voltmeter and ammeter. The action of almost all the analog ammeters and voltmeters depends on the deflecting torque produced by an electric current. In ammeters such a torque is proportional to the current to be measured. In voltmeters this torque is decided by a current which is proportional to the voltage to be measured. Thus all the analog ammeters and voltmeters are basically current measuring devices. The instruments which are used to measure the power are called **power meters or wattmeters.**

6.2 Classification of Measuring Instruments

Electrical measuring instruments are mainly classified as:

a) Indicating instruments   b) Recording instruments   c) Integrating instruments

a) Indicating instruments : These instruments make use of a dial and pointer for showing or indicating magnitude of unknown quantity. The examples are ammeters, voltmeter etc.

b) Recording instruments : These instruments give a continuous record of the given electrical quantity which is being measured over a specific period.

The examples are various types of recorders. In such recording instruments, the readings are recorded by drawing the graph. The pointer of such instruments is provided with a marker i.e. pen or pencil, which moves on graph paper as per the reading. The X-Y plotter is the best example of such an instrument.

c) Integrating instruments : These instruments measure the total quantity of electricity delivered over period of time. For example a household energy meter registers number of revolutions made by the disc to give the total energy delivered, with the help of counting mechanism consisting of dials and pointers.

6.3 Essential Requirements of an Instrument

In case of measuring instruments, the effect of unknown quantity is converted into a mechanical force which is transmitted to the pointer which moves over a calibrated scale. The moving system of such instrument is mounted on a pivoted spindle. For satisfactory operation of any **indicating instrument,** following systems must be present in an instrument.

1) Deflecting system producing deflecting torque T_d

2) Controlling system producing controlling torque T_c

3) Damping system producing damping torque.

Let us see the various ways in which these torques are obtained in an indicating instrument.

6.4 Deflecting System

In most of the indicating instruments the mechanical force proportional to the quantity to be measured is generated. This force or torque deflects the pointer. The system which produces such a deflecting torque is called **deflecting system** and the torque is denoted as T_d. The deflecting torque overcomes,

1) The inertia of the moving system

2) The controlling torque provided by controlling system

3) The damping torque provided by damping system.

The deflecting system uses one of the following effects produced by current or voltage, to produce deflecting torque.

1) Magnetic effect : When a current carrying conductor is placed in uniform magnetic field, it experiences a force which causes to move it. This effect is mostly used in many instruments like moving iron attraction and repulsion type, permanent magnet moving coil instruments etc.

2) Thermal effect : The current to be measured is passed through a small element which heats it to cause rise in temperature which is converted to an e.m.f. by a thermocouple attached to it.

When two dissimilar metals are connected end to end to form a closed loop and the two junctions formed are maintained at different temperatures, then e.m.f. is induced which causes the flow of current through the closed circuit which is called a **thermocouple.**

3) Electrostatic effects : When two plates are charged, there is a force exerted between them, which moves one of the plates. This effect is used in electrostatic instruments which are normally voltmeters.

4) Induction effects : When a non-magnetic conducting disc is placed in a magnetic field produced by electromagnets which are excited by alternating currents, an e.m.f. is induced in it.

If a closed path is provided, there is a flow of current in the disc. The interaction between induced currents and the alternating magnetic fields exerts a force on the disc which causes to move it. This interaction is called an **induction effect.** This principle is mainly used in energy meters.

5) Hall effect : If a bar of semiconducting material is placed in uniform magnetic field and if the bar carries current, then an e.m.f. is produced between two edges of conductor. The magnitude of this e.m.f. depends on flux density of magnetic field, current passing through the conducting bar and hall effect co-efficient which is constant for a given semiconductor. This effect is mainly used in flux-meters.

Thus the deflecting system provides the deflecting torque or operating torque for movement of pointer from its zero position. It acts as the prime mover for the deflection of pointer.

6.5 Controlling System

This system should provide a force so that current or any other electrical quantity will produce deflection of the pointer proportional to its magnitude. The important functions of this system are,

1) It produces a force equal and opposite to the deflecting force in order to make the deflection of pointer at a definite magnitude. If this system is absent, then the pointer will swing beyond its final steady position for the given magnitude and deflection will become indefinite.

Now generally all meters are current sensing meters where,

Deflecting torque $\boxed{T_d = K_t I}$

where K_t = Another constant

In equilibrium position, $\boxed{T_d = T_c}$

$\therefore$ $K_t I = K \sin\theta$

$\therefore$ $\boxed{I \propto \sin\theta}$

Thus the deflection is proportional to current i.e. quantity to be measured.

Key Point: *But as it is a function of sin θ, the scale for the instrument using gravity control is not uniform.*

Its **advantages** are :

1) Its performance is not time dependent.
2) It is simple and cheap.
3) The controlling torque can be varied by adjusting the position of the control weight.
4) Its performance is not temperature dependent.

Its **disadvantages** are :

1) The scale is nonuniform causing problems to record accurate readings.
2) The system must be used in vertical position only and must be properly levelled. Otherwise it may cause serious errors in the measurement.
3) As delicate and proper levelling required, in general it is not used for indicating instruments and portable instruments.

6.5.2 Spring Control

Two hair springs are attached to the moving system which exerts controlling torque. To employ spring control to an instrument, following requirements are essential.

1) The spring should be nonmagnetic.
2) The spring should be free from mechanical stress.
3) The spring should have a small resistance, sufficient cross sectional area.
4) It should have low resistance temperature co efficient.

The arrangement of the springs is shown in the Fig. 6.3.

The springs are made up of nonmagnetic materials like silicon bronze, hard rolled silver or copper, platinum silver and german silver. For most of the instruments, phosphor bronze spiral springs are provided. Flat spiral springs are used in almost all indicating instruments.

At equilibrium, $T_d = T_c$

$\therefore$ $\boxed{I \propto \theta}$

Key Point: *Thus the deflection is proportional to the current. Hence the scale of the instrument using spring control is uniform.*

When the current is removed, due to spring force the pointer comes back to initial position. The spring control is very popular and is used in almost all indicating instruments.

6.5.3 Comparison of Controlling Systems

Sr. No.	Gravity Control	Spring Control
1.	Adjustable small weight is used which produces the controlling torque.	Two hair springs are used which exert controlling torque.
2.	Controlling torque can be varied.	Controlling torque is fixed.
3.	The performance is not temperature dependent.	The performance is temperature dependent.
4.	The scale is nonuniform.	The scale is uniform.
5.	The controlling torque is proportional to sin θ.	The controlling torque is proportional to θ.
6.	The readings can not be taken accurately.	The readings can be taken very accurately.
7.	The system must be used in vertical position only.	The system need not be necessarily in vertical position.
8.	Proper levelling is required as gravity control.	The levelling is not required.
9.	Simple, cheap but delicate.	Simple, rigid but costlier compared to gravity control.
10.	Rarely used for indicating and portable instruments.	Very popularly used in most of the instruments.

6.6 Damping System

The deflecting torque provides some deflection and controlling torque acts in the opposite direction to that of deflecting torque. So before coming to the rest, pointer always oscillates due to inertia, about the equilibrium position. Unless pointer rests, final reading can not be obtained. So to bring the pointer to rest within short time, damping system is required. The system should provide a damping torque only when the moving system is in motion. Damping torque is proportional to velocity of the moving system but it does not depend on operating current. It must not affect controlling torque or increase the friction.

The quickness with which the moving system settles to the final steady position depends on relative damping. If the moving system reaches to its final position rapidly but smoothly without oscillations, the instrument is said to be **critically damped.** If the instrument is **under damped,** the moving system will oscillate about the final steady position with a decreasing amplitude and will take sometime to come to rest. While the instrument is said to be **over damped** if the moving system moves slowly to its final steady position. In over damped case the response of the system is very slow and sluggish. In practice slightly under damped systems are preferred. The time response of damping system for various types of damping conditions is shown in the Fig. 6.4.

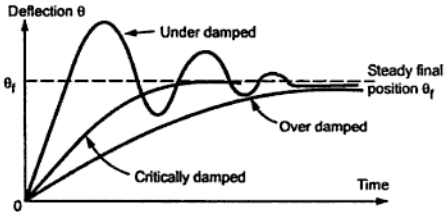


Fig. 6.4

The following methods are used to produce damping torque.

1) Air friction damping 2) Fluid friction damping 3) Eddy current damping.

6.6.1 Air Friction Damping

This arrangement consists of a light aluminium piston which is attached to the moving system, as shown in the Fig. 6.5.

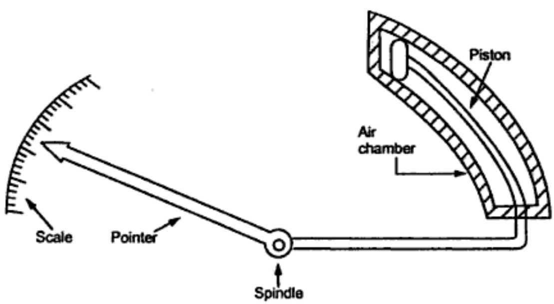


Fig. 6.5 Air friction damping

The piston moves in a fixed air chamber. It is close to one end. The clearance between piston and wall chambers is uniform and small. The piston reciprocates in the chamber when there are oscillations. When piston moves into the chamber, air inside is compressed and pressure of air developed due to friction opposes the motion of pointer. There is also opposition to motion of moving system when piston moves out of the chamber. Thus the oscillations and the overshoot gets reduced due to to and fro motion of the piston in the chamber, providing necessary damping torque. This helps in settling down the pointer to its final steady position very quickly.

6.6.2 Fluid Friction Damping

Fluid friction damping may be used in some instruments. The method is similar to air friction damping, only air is replaced by working fluid. The friction between the disc and fluid is used for opposing motion. Damping force due to fluid is greater than that of air due to more viscosity. The disc is also called vane.

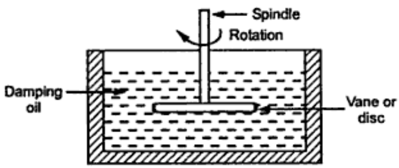


Fig. 6.6 Fluid friction damping

The arrangement is shown in the Fig. 6.6. It consists of a vane attached to the spindle which is completely dipped in the oil. The frictional force between oil and the vane is used to produce the damping torque, which opposes the oscillating behaviour of the pointer.

The advantages of this method are :

1) Due to more viscosity of fluid, more damping is provided.
2) The oil can also be used for insulation purposes.
3) Due to up thrust of oil, the load on the bearings is reduced, thus reducing the frictional errors.

The disadvantages of this method are :

1) This can be only used for the instruments which are in vertical position.
2) Due to oil leakage, the instruments can not be kept clean.

6.6.3 Eddy Current Damping

This is the most effective way of providing damping. It is based on the **Faraday's law** and **Lenz's law**. When a conductor moves in a magnetic field cutting the flux, e.m.f. gets induced in it. And direction of this e.m.f. is so as to oppose the cause producing it.

In this method, an aluminium disc is connected to the spindle. The arrangement of disc is such that when it rotates, it cuts the magnetic flux lines of a permanent magnet. The arrangement is shown in the Fig. 6.7.

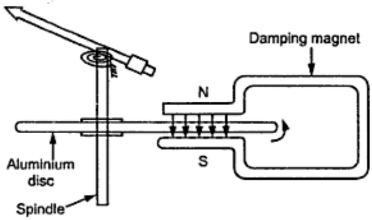


Fig. 6.7 Eddy current damping

When the pointer oscillates, aluminium disc rotates under the influence of magnetic field of damping magnet. So disc cuts the flux which causes an induced e.m.f. in the disc. The disc is a closed path hence induced e.m.f. circulates current through the disc called **eddy current**. The direction of such eddy current is so as oppose the cause producing it. The cause is relative motion between disc and field. Thus it produces an opposing torque so as to reduce the oscillations of pointer. This brings pointer to rest quickly. This is most effective and efficient method of damping.

6.7 Permanent Magnet Moving Coil Instruments (PMMC)

The permanent magnet moving coil instruments are most accurate type for d.c. measurements.The action of these instruments is based on the motoring principle. When a current carrying coil is placed in the magnetic field produced by permanent magnet, the coil experiences a force and moves.As the coil is moving and the magnet is permanent, the instrument is called permanent magnet moving coil instrument. This basic principle is called **D'Arsonval principle.** The amount of force experienced by the coil is proportional to the current passing through the coil.

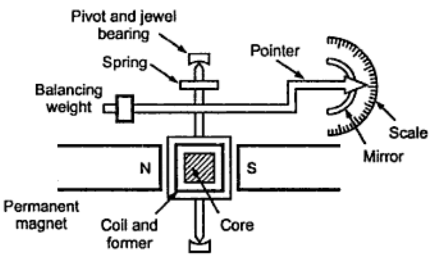


Fig. 6.8 Construction of PMMC instrument

The PMMC instrument is shown in the Fig. 6.8.

The moving coil is either rectangular or circular in shape. It has number of turns of fine wire. The coil is suspended so that it is free to turn about its vertical axis. The coil is placed in uniform, horizontal and radial magnetic field of a permanent magnet in the shape of a horse-shoe. The iron core is spherical if coil is circular and is cylindrical if the coil is rectangular. Due to iron core, the deflecting torque increases, increasing the sensitivity of the instrument.

The controlling torque is provided by two phosphor bronze hair springs.

The damping torque is provided by eddy current damping. It is obtained by movement of the aluminium former, moving in the magnetic field of the permanent magnet.

The pointer is carried by the spindle and it moves over a graduated scale. The pointer has light weight so that it can deflect rapidly. The mirror is placed below the pointer to get the accurate reading by removing the parallax. The weight of the instrument is normally counter balanced by the weights situated diametrically opposite and rigidly connected to it. The scale markings of the basic d.c. PMMC instruments are usually linearly spaced as the deflecting torque and hence the pointer deflection are directly proportional to the current passing through the coil.

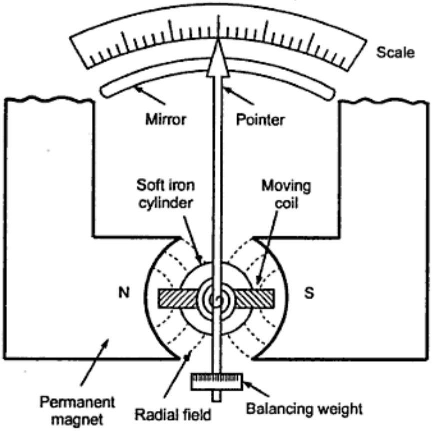


Fig. 6.9 PMMC instrument

The top view of PMMC instrument is shown in the Fig. 6.9.

In a practical PMMC instrument, a Y shaped member is attached to the fixed end of the front control spring. An eccentric pin through the instrument case engages the Y shaped member so that the zero position of the pointer can be adjusted from outside.

6.7.1 Torque Equation

The equation for the developed torque can be obtained from the basic law of the electromagnetic torque. The deflecting toque is given by,

$$\boxed{T_d = NBAI}$$

where T_d = Deflecting torque in N-m

B = Flux density in air gap, Wb/m^2

N = Number of turns of the coil

A = Effective coil area m^2

I = Current in the moving coil, amperes

$\therefore \quad T_d = GI$

where $G = NBA$ = Constant

The controlling torque is provided by the springs and is proportional to the angular deflection of the pointer.

$$T_c = K\theta$$

where T_c = Controlling torque

K = Spring constant, Nm/rad or Nm/deg

θ = Angular deflection

For the final steady state position,

$$T_d = T_c$$

$$\therefore \quad G\,I = K\theta$$

$$\therefore \quad \theta = \left(\frac{G}{K}\right) I$$

or

$$\boxed{I = \left(\frac{K}{G}\right)\theta}$$

Key Point: *Thus the deflection is directly proportional to the current passing through the coil.*

The pointer deflection can therefore be used to measure current.

As the direction of the current through to the coil changes, the direction of the deflection of the pointer also changes. Hence such instruments are well suited for the d.c. measurements.

In the microammeters and milliammeters upto about 20 mA, the entire current to be measured is passed through the coil. The springs carry current to the coil. Thus the current carrying capacity of the springs, limits the current which can be safely carried. For higher currents, the moving coil is shunted by sufficient resistance. While the voltmeters having high ranges use a moving coil together with sufficient series resistance, to limit the instrument current. Most d.c. voltmeters are designed to produce full scale deflection with a current of 20, 10, 5 or 1 mA.

The power requirement of PMMC instrument is very small, typically of the order of 25 μW to 200 μW. Accuracy is generally of the order of 2 to 5 % of the full scale reading.

➡ **Example 6.1 :** *A PMMC instrument has a coil of dimensions 10 mm × 8 mm. The flux density in the air gap is 0.15 Wb/m². If the coil is wound for 100 turns, carrying a current of 5 mA then calculate the deflecting torque. Calculate the deflection if the spring constant is 0.2 ×10⁻⁶ Nm/degree.*

Solution : The deflecting torque is given by,

$$T_d = NBAI = 100 \times 0.15 \times (A) \times 5 \times 10^{-3} \text{ Nm}$$

Now $$A = \text{Area} = 10 \times 8 = 80 \text{ mm}^2 = 80 \times 10^{-6} \text{m}^2$$

$\therefore$ $$T_d = 100 \times 0.15 \times 80 \times 10^{-6} \times 5 \times 10^{-3} = 6 \times 10^{-6} \text{ Nm}$$

Now $$T_d = T_c = K\theta$$

$\therefore$ $$6 \times 10^{-6} = 0.2 \times 10^{-6} \times \theta$$

$\therefore$ $$\theta = \frac{6 \times 10^{-6}}{0.2 \times 10^{-6}} = \textbf{30 degrees}$$

6.7.2 Advantages

The various advantages of PMMC instruments are,

1) It has uniform scale.
2) With a powerful magnet, its torque to weight ratio is very high. So operating current is small.
3) The sensitivity is high.
4) The eddy currents induced in the metallic former over which coil is wound, provide effective damping.
5) It consumes low power, of the order of 25 W to 200 μW.
6) It has high accuracy.
7) Instrument is free from hysteresis error.
8) Extension of instrument range is possible.
9) Not affected by external magnetic fields called stray magnetic fields.

6.7.3 Disadvantages

The various disadvantages of PMMC instruments are,

1) Suitable for d.c. measurements only.
2) Ageing of permanent magnet and the control springs introduces the errors.
3) The cost is high due to delicate construction and accurate machining.
4) The friction due to jewel-pivot suspension.

6.7.4 Taut Band Instrument

The friction due to jewel-pivot suspension can be eliminated by using taut band movement. The working principle of taut band instrument is same based on D'Arsonval's principle. The main difference is the method of mounting the coil.

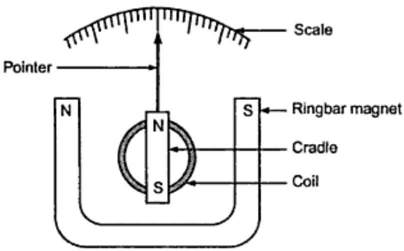


Fig. 6.10 Taut band instrument

In the taut band instrument the movable coil is suspended by means of two torsion ribbons. The ribbons are placed under sufficient tension to eliminate any sag. This tension is provided by the tension string. The coil is mounted in a cradle and surrounded by ring bar magnet. The construction is shown in the Fig 6.10.

The taut band instrument can be used in any position while jewel-pivot instrument should be used vertically. The sensitivity of the taut band instruments is higher than jewel-pivot instruments. The taut band instruments are relatively insensitive to shocks and temperature and are capable of with standing overloads.

6.7.5 Temperature Compensation

The basic PMMC instrument is sensitive to the temperature.The magnetic field strength and spring tension decrease with increase in temperature. The coil resistance increases with increase in the temperature. Thus pointer reads low for a given current. The meter tends to read low by approximately 0.2 % per °C rise in the temperature. Hence the temperature compensation is provided by appropriate use of series and shunt resistances of copper and manganin.

The simple compensation circuit uses a resistance in series with the movable coil, as shown in the Fig. 6.11. The resistor is called a **swamping resistor**. It is made up of manganin having practically zero temperature coefficient, combined with copper in the ratio of 20/1 or 30/1.

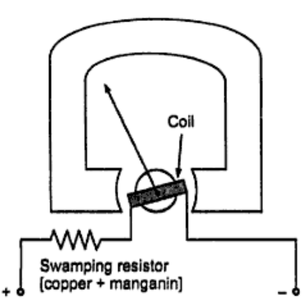


Fig. 6.11 Simple temperature compensation

The resultant resistance of coil and the swamping resistor increases slightly as temperature increases, just enough to compensate the change in springs and magnet due to temperature. Thus the effect of temperature is compensated.

More complicated but complete cancellation of temperature effects can be obtained by using the swamping resistors in series and parallel combination as shown in the Fig. 6.12.

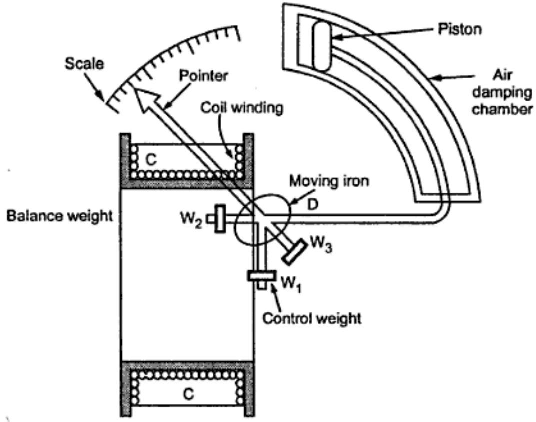


Fig. 6.13 Moving iron attraction type instrument

on the range of the instrument. For passing large current through the coil only few turns are required.

The controlling torque is provided by the springs but gravity control may also be used for vertically mounted panel type instruments.

The damping torque is provided by the air friction. A light aluminium piston is attached to the moving system. It moves in a fixed chamber. The chamber is closed at one end. It can also be provided with the help of vane attached to the moving system.

The operating magnetic field in moving iron instruments is very weak. Hence eddy current damping is not used since it requires a permanent magnet which would affect or distort the operating field.

6.8.2 Moving Iron Repulsion Type Instrument

These instruments have two vanes inside the coil, the one is fixed and other is movable. When the current flows in the coil, both the vanes are magnetised with like polarities induced on the same side. Hence due to the repulsion of like polarities, there is a force of repulsion between the two vanes causing the movement of the moving vane. The repulsion type instruments are the most commonly used instruments.

The two different designs of repulsion type instruments are :

i) Radial vane type and

ii) Coaxial vane type

6.8.2.1 Radial Vane Repulsion Type Instrument

The Fig. 6.14 shows the radial vane repulsion type instrument. Out of the other moving iron mechanisms, this is the most sensitive and has most linear scale.

The two vanes are radial strips of iron. The fixed vane is attached to the coil. The movable vane is attached to the spindle and suspended in the induction field of the coil. The needle of the instrument is attached to this vane.

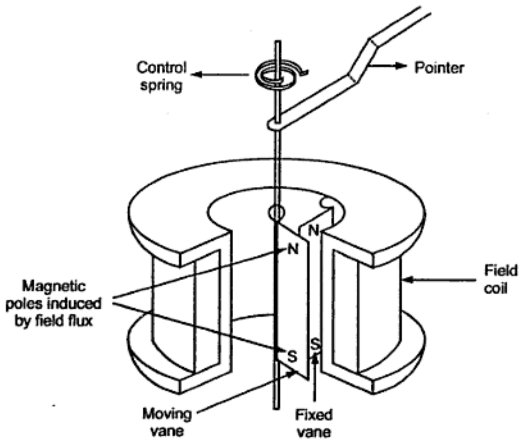


Fig. 6.14 Radial vane repulsion type instrument

Eventhough the current through the coil is alternating, there is always repulsion between the like poles of the fixed and the movable vane. Hence the deflection of the pointer is always in the same direction.The deflection is effectively proportional to the actual current and hence the scale is calibrated directly to read amperes or volts. The calibration is accurate only for the frequency for which it is designed because the impedance is different for different frequencies.

6.8.2.2 Concentric Vane Repulsion Type Instrument

Fig. 6.15 shows the concentric vane repulsion type instrument. The instrument has two concentric vanes. One is attached to the coil frame rigidly while the other can rotate coaxially inside the stationary vane.

Both the vanes are magnetised to the same polarity due to the current in the coil. Thus the movable vane rotates under the repulsive force. As the movable vane is attached to the pivoted shaft, the repulsion results in a rotation of the shaft. The pointer deflection is proportional to the current in the coil. The concentric vane type instrument is moderately sensitive and the deflection is proportional to the square of the current through coil. Thus the instrument is said to have square law response. Thus the scale of the instrument is nonuniform in nature. Thus whatever may be the direction of the current in the coil, the deflection in the moving iron instruments is in the same direction. Hence moving iron instruments can be used for both a.c. and d.c. measurements. Due to square law response, the scale of the moving iron instrument is non-uniform.

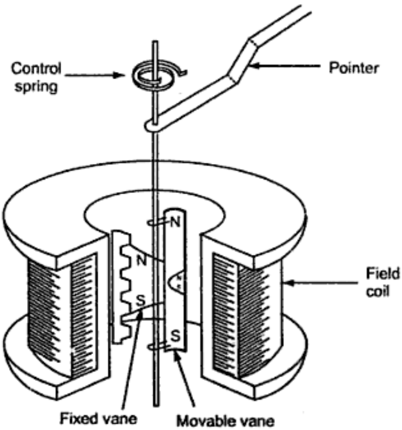


Fig. 6.15 Concentric vane repulsion type instrument

6.8.3 Torque Equation of Moving Iron Instruments

Consider a small increment in current supplied to the coil of the instrument. Due to this current let $d\theta$ be the deflection under the deflecting torque T_d. Due to such deflection, some mechanical work will be done.

$\therefore$ Mechanical work = $T_d d\theta$

There will be a change in the energy stored in the magnetic field due to the change in inductance. This is because the vane tries to occupy the position of minimum reluctance hence the force is always in such a direction so as to increase the inductance of coil. The inductance is inversely proportional to the reluctance of the magnetic circuit of coil.

Let I = Initial current

L = Instrument inductance

θ = Deflection

dI = Increase in current

$d\theta$ = Change in deflection

dL = Change in inductance

In order to effect an increment dI in the current, there must be an increase in the applied voltage given by,

$$e = \frac{d(LI)}{dt}$$

$$= I\frac{dL}{dt} + L\frac{dI}{dt} \qquad \text{as both I and L are changing.}$$

The electrical energy supplied is given by,

$$eIdt = \left(I\frac{dL}{dt} + L\frac{dI}{dt}\right) Idt = I^2\ dL + IL\ dI$$

The stored energy increases from $\frac{1}{2} L I^2$ to $\frac{1}{2}(L + dL)(I + dI)^2$

Hence the change in the stored energy is given by,

$$= \frac{1}{2}(L + dL)(I + dI)^2 - \frac{1}{2}LI^2$$

Neglecting higher order terms, this becomes, $IL\ dI + \frac{1}{2} I^2\ dL$

The energy supplied is nothing but increase in stored energy plus the energy required for mechanical work done.

$$\therefore \quad I^2 dL + IL\ dI = IL\ dI + \frac{1}{2} I^2\ dL + T_d \cdot d\theta$$

$$\therefore \quad T_d \cdot d\theta = \frac{1}{2} I^2\ dL$$

$$\therefore \quad \boxed{T_d = \frac{1}{2} I^2 \frac{dL}{d\theta}}$$

While the controlling torque is given by,

$$\boxed{T_c = K\,\theta}$$

where K = Spring constant

$$\therefore \quad K\,\theta = \frac{1}{2} I^2 \frac{dL}{d\theta} \qquad \text{under equilibrium}$$

$$\therefore \quad \boxed{\theta = \frac{1}{2}\frac{I^2}{K}\frac{dL}{d\theta}}$$

Thus the deflection is proportional to the square of the current through the coil. And the instrument gives square law response.

6.8.4 Advantages

The various advantages of moving iron instruments are,

1) The instruments can be used for both a.c. and d.c. measurements.
2) As the torque to weight ratio is high, errors due to the friction are very less.
3) A single type of moving element can cover the wide range hence these instruments are cheaper than other types of instruments.
4) There are no current carrying parts in the moving system hence these meters are extremely rugged and reliable.
5) These are capable of giving good accuracy. Modern moving iron instruments have a d.c. error of 2% or less.
6) These can withstand large loads and are not damaged even under severe overload conditions.
7) The range of instruments can be extended.

6.8.5 Disadvantages

The various disadvantages of moving iron instruments are,

1) The scale of the moving iron instruments is not uniform and is cramped at the lower end. Hence accurate readings are not possible at this end.
2) There are serious errors due to hysteresis, frequency changes and stray magnetic fields.
3) The increase in temperature increases the resistance of coil, decreases stiffness of the springs, decreases the permeability and hence affect the reading severely.
4) Due to the non linearity of B-H curve, the deflecting torque is not exactly proportional to the square of the current.
5) There is a difference between a.c. and d.c. calibrations on account of the effect of inductance of the meter. Hence these meters must always be calibrated at the frequency at which they are to be used. The usual commercial moving iron instrument may be used within its specified accuracy from 25 to 125 Hz frequency range.
6) Power consumption is on higher side.

6.8.6 Errors in Moving Iron Instruments

The various errors in the moving iron instruments are,

1) Hysteresis errors : Due to hysteresis effect, the flux density for the same current while ascending and descending values is different. While descending, the flux density is higher and while ascending it is lesser. So meter reads higher for descending values of current or voltage. So remedy for this is to use smaller iron parts which can demagnetise quickly or to work with lower flux densities.

6.9 Basic D.C. Ammeter

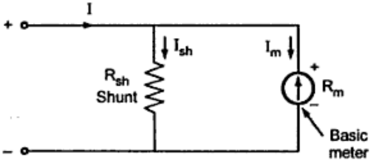


Fig. 6.16 Basic d.c. ammeter

The basic d.c. ammeter is nothing but a D'Arsonval galvanometer. The coil winding of a basic movement is very small and light and hence it can carry very small currents. So as mentioned earlier, for large currents, the major part of current is required to be bypassed using a resistance called **shunt**. It is shown in the Fig. 6.16.

The shunt resistance can be calculated as :

Let R_m = Internal resistance of coil

R_{sh} = Shunt resistance

I_m = Full scale deflection current

I_{sh} = Shunt current

I = Total current

Now $I = I_{sh} + I_m$

As the two resistances R_{sh} and R_m are in parallel, the voltage drop across them is same.

$$\therefore \quad I_{sh}R_{sh} = I_m R_m$$

$$\therefore \quad R_{sh} = \frac{I_m R_m}{I_{sh}}$$

but $I_{sh} = I - I_m$

$$\therefore \quad R_{sh} = \frac{I_m R_m}{(I - I_m)}$$

$$\therefore \quad \boxed{R_{sh} = \frac{R_m}{m-1}} \quad \text{where} \quad m = \frac{I}{I_m}$$

The m is called **multiplying power of the shunt** and defined as the ratio of total current to the current through the coil. It can be expressed as,

$$\boxed{m = \frac{I}{I_m} = 1 + \frac{R_m}{R_{sh}}}$$

The shunt resistance may consist of a constant temperature resistance wire within the case of the meter or it may be external shunt having low resistance.

Thus to increase the range of ammeter 'm' times, the shunt resistance required is 1/(m–1) times the basic meter resistance. This is nothing but **extension of ranges of an ammeter.**

➡ **Example 6.3 :** *A 2 mA meter with an internal resistance of 100 Ω is to be converted to 0-150 mA ammeter. Calculate the value of the shunt resistance required.*

Solution : Given values are,

$$R_m = 100\ \Omega,\ I_m = 2\text{ mA},\ I = 150\text{ mA}$$

$$R_{sh} = \frac{I_m R_m}{I - I_m}$$

$$\therefore \quad R_{sh} = \frac{2\times10^{-3}\times100}{[150\times10^{-3}-2\times10^{-3}]}$$

$$= \mathbf{1.351\ \Omega}$$

➡ **Example 6.4 :** *A moving coil ammeter has fixed shunt of 0.01 Ω. With a coil resistance of 750 Ω and a voltage drop of 400 mV across it, the full scale deflection is obtained.*

a) Calculate the current through shunt.

b) Calculate the resistance of meter to give full scale deflection if the shunted current is 50 A.

Solution : a) The drop across the shunt is same as drop across the coil.

$$\therefore \quad I_{sh}R_{sh} = 400\text{ mV}$$

$$\therefore \quad I_{sh} = \frac{400\times10^{-3}}{0.01} = \mathbf{40\ A}$$

b) The voltage across shunt for shunted current of 50 A is,

$$V_{sh} = I_{sh}R_{sh} = 50\times0.01$$

$$= 0.5\text{ V}$$

For this voltage the meter should give full scale deflection. In first case, the current through meter for full scale deflection was,

$$I_m = \frac{400\text{ mV}}{R_m} = \frac{400\times10^{-3}}{750}$$

$$= 5.33\times10^{-4}\text{A}$$

The same I_m must flow for new voltage across the meter of 0.5 V

$$\therefore \quad I_m R'_m = 0.5$$

$$\therefore \quad 5.33 \times 10^{-4} R'_m = 0.5$$

$$\therefore \quad R'_m = 937.5\ \Omega$$

This is the resistance of the meter required for 50 A shunted current to give full scale deflection.

6.10 Requirements of a Shunt

1) The temperature coefficient of shunt and the meter should be low and should be as equal as possible.
2) The shunt resistances should be stable and constant with time.
3) The shunt resistances should not carry currents which will cause excessive temperature rise.
4) The type of material used to join the shunts should have low thermo dielectric voltage drop i.e. the soldering of joints should not cause a voltage drop.
5) Due to the soldering, the values of resistance should not be change.
6) The resistances should have low thermal electromotive force with copper.

The manganin is usually used for the shunts of d.c. instruments while the constantan is useful for the shunts of a.c. instruments.

6.11 Basic D.C. Voltmeter

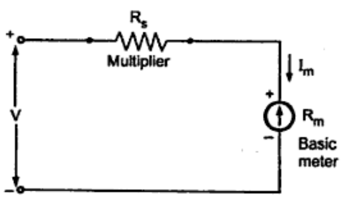


Fig. 6.17 Basic d.c. voltmeter

The basic d.c. voltmeter is nothing but a PMMC D'Arsonoval galvanometer. The resistance is required to be connected in series with the basic meter to use it as a voltmeter. This series resistance is called a **multiplier.** The main function of the multiplier is to limit the current through the basic meter so that the meter current does not exceed the full scale deflection value. The voltmeter measures the voltage across the two points of a circuit or a voltage across a circuit component. The basic d.c. voltmeter is shown in the Fig. 6.17.

The voltmeter must be connected across the two points or a component, to measure the potential difference, with the proper polarity.

The multiplier resistance can be calculated as :

Let R_m = Internal resistance of coil i.e. meter

R_s = Series multiplier resistance

I_m = Full scale deflection current

V = Full range voltage to be measured

From Fig. 6.17, $\therefore V = I_m(R_m + R_s)$

$$\therefore \quad V = I_m R_m + I_m R_s$$

$$\therefore \quad I_m R_s = V - I_m R_m$$

$$\therefore \quad \boxed{R_s = \frac{V}{I_m} - R_m}$$

The **multiplying factor** for multiplier is the ratio of full range voltage to be measured and the drop across the basic meter.

Let v = Drop across the basic meter = $I_m R_m$

$$\therefore \quad m = \text{Multiplying factor} = \frac{V}{v}$$

$$= \frac{I_m(R_m + R_s)}{I_m R_m}$$

$$\therefore \quad \boxed{m = 1 + \frac{R_s}{R_m}}$$

Hence multiplier resistance can also be expressed as,

$$R_s = (m-1) R_m$$

Thus to increase the range of voltmeter 'm' times, the series resistance required is (m–1) times the basic meter resistance. This is nothing but **extension of ranges of a voltmeter.**

➠ **Example 6.5 :** *A moving coil instrument gives a full scale deflection with a current of 40 μA, while the internal resistance of the meter is 500 Ω. It is to be used as a voltmeter to measure a voltage range of 0 - 10 V. Calculate the multiplier resistance needed.*

Solution : Given values are, $R_m = 500\ \Omega$, $I_m = 40\ \mu A$ and $V = 10\ V$

Now $$R_s = \frac{V}{I_m} - R_m = \frac{10}{40 \times 10^{-6}} - 500 = \mathbf{249.5\ k\Omega}$$

This is the required multiplier resistance.

Example 6.6 : *A moving coil instrument gives a full scale deflection for a current of 20 mA with a potential difference of 200 mV across it. Calculate : i) Shunt required to use it as an ammeter to get a range of 0 - 200 A ii) Multiplier required to use it as a voltmeter of range 0 - 500 V*

Solution : The meter current $I_m = 20 \text{ mA}$

Drop across meter, $V_m = 200 \text{ mV}$

Now $V_m = I_m R_m$

$\therefore$ $200 \text{ mV} = (20 \text{ mA}) R_m$

$\therefore$ $R_m = 10\ \Omega$

i) For using it as an ammeter, I = 200 A

$$R_{sh} = \frac{I_m R_m}{I - I_m} = \frac{20 \times 10^{-3} \times 10}{200 - 20 \times 10^{-3}}$$

$$= \mathbf{0.001\ \Omega}$$

This is the required shunt.

ii) For using it as a voltmeter,

$$V = 500 \text{ V}$$

$\therefore$
$$R_s = \frac{V}{I_m} - R_m$$

$$= \frac{500}{20 \times 10^{-3}} - 10$$

$$= \mathbf{24.99\ k\Omega}$$

This is the required multiplier.

Examples with Solutions

Example 6.7 : *A voltage of 80 V is applied to a circuit consisting of two resistors of 105 Ω and 55 Ω in series. The voltage across the 55 Ω resistor is measured by a voltmeter of internal resistance of 100 Ω/V. Given that meter is set to a scale of 0-50 V. Determine the voltage indicated.* **[JNTU : March - 2006 (Set-2)]**

Solution : The arrangement is shown in the Fig. 6.18 (a).

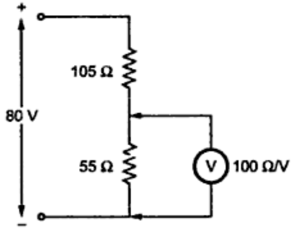


Fig. 6.18 (a)

The voltmeter range = 50 V

$\therefore \; R_m$ = Voltmeter resistance

$= 100 \; \Omega/V \times 50 = 5000 \; \Omega$

This appears in parallel with 55 Ω resistor as shown in the Fig. 6.18 (b).

$$\therefore \quad R' = 55 \,||\, 5000 = \frac{55 \times 5000}{55 + 5000}$$

$$= 54.40158 \; \Omega$$

$$\therefore \quad I = \frac{80}{105 + R'} = \frac{80}{105 + 54.40158}$$

$$= 0.50187 \text{ A}$$

$\therefore$ Voltage across R' = I R'

$$= 0.501877 \times 54.40158 = 27.3029 \text{ V}$$

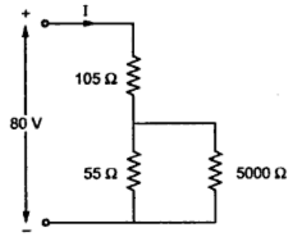


Fig. 6.18 (b)

The voltmeter will sense this voltage.

$\therefore$ Voltage indicated = **27.3029 V**

➡ **Example 6.8 :** *A moving coil instrument gives full scale deflection with 15 mA and has a resistance of 5 Ω. Calculate the resistance of the necessary components in order that the instrument may be used as,*

i) 2 A ammeter ii) 100 V voltmeter.

[JNTU : Nov.-2004 (Set-1); Nov.-2005 (Set-2); March-2006, (Set-4)]

Solution : $I_m = 15 \text{ mA}, \quad R_m = 5 \Omega$

i) $I = 2 \text{ A}$

$$R_{sh} = \frac{I_m R_m}{I - I_m} = \frac{15 \times 10^{-3} \times 5}{2 - 15 \times 10^{-3}} = \mathbf{0.03778 \; \Omega} \qquad \text{... In shunt}$$

ii) $V = 100 \text{ V}$

$$R_s = \frac{V}{I_m} - R_m = \frac{100}{15 \times 10^{-3}} - 5 = \mathbf{6661.6667 \; \Omega} \qquad \text{... In series}$$

Example 6.9 : *The coil of moving coil meter has a resistance of 5 Ω and gives a full scale deflection when a current of 15 mA passes through it. What modifications must be made to the instrument to convert it into i) Ammeter reading 15 A and ii) Voltmeter reading 15 V.*

[JNTU : Nov.-2004 (Set-2); Nov.-2005 (Set-1)]

Solution : $R_m = 5\ \Omega,\ \ I_m = 15\text{ mA}$

i) $I = 15\text{ A}$

$$\therefore \quad R_{sh} = \frac{I_m R_m}{I - I_m} = \frac{15\times 10^{-3}\times 5}{15 - 15\times 10^{-3}}$$

$$= \mathbf{5.005\ m\Omega} \qquad \text{... In shunt with coil}$$

ii) $V = 15\text{ V}$

$$\therefore \quad R_s = \frac{V}{I_m} - R_m = \frac{15}{15\times 10^{-3}} - 5 = \mathbf{995\ \Omega} \qquad \text{... In series}$$

Example 6.10 : *A moving iron voltmeter in which full scale deflection is given by 100 V, has a coil of 10,000 turns and a resistance of 2000 Ω. Calculate the number of turns required on the coil of the instrument converted for use as an ammeter reading 20 A full scale deflection.*

[JNTU : May-2005 (Set-4)]

Solution : $R_m = 2000\ \Omega,\ \ V_m = 100\text{ V},\ \ N = 10000$

$$\therefore \quad I_m = \frac{V_m}{R_m} = \frac{100}{2000} = 0.05\text{ A}$$

$$\therefore \text{AT for full scale deflection} = I_m \times N = 0.05\times 10000$$

$$= 500\text{ AT}$$

For $I = 20\text{ A},\ \ AT = I\times N'$

$$\therefore \quad 500 = 20\times N'$$

$$\therefore \quad N' = \mathbf{25} \qquad \text{... Turns required on coil}$$

Example 6.11 : *A moving coil instrument gives full scale deflection with 15 mA, has a copper coil having a resistance of 1.5 Ω at 15 °C and a temperature coefficient of $\frac{1}{234.5}$ / °C at 0 °C in series with a resistor of 3.5 Ω having a negligible temperature coefficient. Determine : i) The resistance of shunt required for a full scale deflection of 20 A and ii) The resistance required for a full scale deflection of 250 V.*

If the instrument reads correctly at 15 °C, determine the percentage error in each case when the temperature is 25 °C.

[JNTU : Nov.-2003 (Set-4); Nov.-2004 (Set-4); March-2006 (Set-1)]

Solution : $I_m = 15\ mA,\ R_m = 1.5\ \Omega$ at $15\ °C,\ R = 3.5\ \Omega$

$\therefore\quad R_{mT}$ = Total meter resistance $= 1.5 + 3.5 = 5\ \Omega$

i) $I = 20\ A$

$$\therefore\quad R_{sh} = \frac{I_m R_{mT}}{I - I_m} = \frac{15\times10^{-3}\times5}{20-15\times10^{-3}} = \mathbf{0.0037528\ \Omega}$$

ii) $V = 250\ V$

$$\therefore\quad R_s = \frac{V}{I_m} - R_{mT} = \frac{250}{15\times10^{-3}} - 5 = \mathbf{16661.667\ \Omega}$$

Now at 25 °C, R'_m is the new meter resistance.

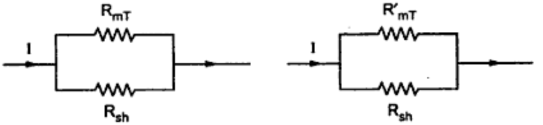
$$R'_m = R_m\,[1+\alpha_1(t_2-t_1)] \text{ where } t_1 = 15\ °C,\ t_2 = 25\ °C$$

$$\alpha_1 = \frac{\alpha_0}{1+\alpha_0 t_1} = \frac{\frac{1}{234.5}}{1+\frac{1}{234.5}\times15} = 0.004/°C$$

$$\therefore\quad R'_m = 1.5\,[1 + 0.004\times(25-15)] = 1.56012\ \Omega$$

$$\therefore\quad R'_{mT} = 1.56012 + 3.5 = 5.06012\ \Omega$$

Error in part (i) : The Fig. 6.19 shows two cases.



(a) At 15 °C (b) At 25 °C

Fig. 6.19

$$\therefore\quad I_{m1} \text{ at } 15\ °C = I\times\frac{R_{sh}}{R_{sh}+R_{mT}} = 7.4999\times10^{-4}\ I$$

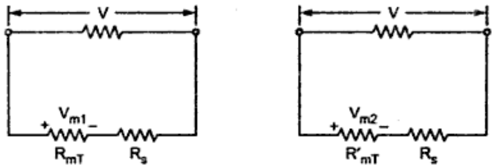
$$\therefore\quad I_{m2} \text{ at } 25\ °C = I\times\frac{R_{sh}}{R_{sh}+R'_{mT}} = 7.41092\times10^{-4}\ I$$

$$\%\ \text{error} = \frac{I_{m2}-I_{m1}}{I_{m1}}\times100 = \frac{7.41092\times10^{-4}\ I - 7.4999\times10^{-4}\ I}{7.4999\times10^{-4}\ I}\times100$$

$$= \mathbf{-1.1863\ \%}$$

Negative sign indicates that the reading is less than the actual reading.

Error in part (ii) : The Fig. 6.19 shows the two cases.



(c) At 15 °C (d) At 25 °C

Fig. 6.19

$$\therefore \quad V_{m1} = \frac{V}{R_{mT}+R_s} \times R_{mT} = V\left[\frac{5}{5+16661.67}\right] = 2.9999 \times 10^{-4} \text{ V}$$

$$\therefore \quad V_{m2} = \frac{V}{R'_{mT}+R_s} \times R'_{mT} = V\left[\frac{5.06012}{5.06012+16661.67}\right] = 3.03606 \times 10^{-4} \text{ V}$$

$$\% \text{ error} = \frac{V_{m2}-V_{m1}}{V_{m1}} \times 100 = \left[\frac{3.03606 \times 10^{-4}\text{ V} - 2.9999 \times 10^{-4} \text{ V}}{2.9999 \times 10^{-4} \text{ V}}\right] \times 100$$

$$= \mathbf{+\ 1.2023\ \%}$$

Review Questions

1. *State the basic requirement of any measuring instrument. How the various measuring instruments are classified ?*
2. *Which torques are necessary for the successful operation if any indicating instrument ? Explain briefly how these torques are produced in various instruments.*
3. *Which are the various effects with which deflecting torque is produced ?*
4. *Differentiate between spring control and gravity control methods used to produce the controlling torque.*
5. *Explain the various methods of providing damping torque in an indicating instrument.*
6. *Describe the construction and working of PMMC instrument.*
7. *Derive the equation for deflection in spring controlled PMMC instrument.*
8. *How is the current range of a PMMC instrument extended with the help of shunts ?*
9. *State the advantages, disadvantages and errors in PMMC instruments.*
10. *Explain the working of attraction type and repulsion type moving iron instruments with neat diagrams.*
11. *Derive the torque equation for moving iron instruments.*

There is no difference between an electron of copper and an electron of aluminium or an electron of any other element. Similarly the neutrons and protons of various atoms are characteristicwise identical in nature. Then why do various elements behave differently ? This is because of the difference in the arrangement of electrons, protons and neutrons of which each atom is composed. Let us see the structure of an atom.

7.2.1 Structure of an Atom

The atoms have a planetary type of structure, according to classical **Bohr Model.**

All the protons and neutrons are bound together at the centre of an atom, which is called Nucleus. While all the electrons are moving round the nucleus. So nucleus can be thought of as a central sun, about which electrons revolve in a particular fashion like the planets.

In a normal atom the number of protons is equal to the number of electrons. As neutron is electrically neutral, an atom as a whole is electrically neutral. The number of protons in an atom is called as its **atomic number.** While the atomic weight is approximately equal to the total number of protons and neutrons in the nucleus of an atom.

The electrons which are revolving round the nucleus, do not move in the same orbit. The electrons are arranged in the different **orbits** or **shells** at fixed distances from the nucleus. Each shell can contain a fixed number of electrons. In general, a shell can contain a maximum of $\mathbf{2n^2}$ electrons where n is the number of the shell. The first shell can occupy maximum of two electrons (2×1^2) while the second shell can occupy maximum of eight electrons (2×2^2) and so on.

Each shell has an energy level associated with it. The closer an electron is to the nucleus, the stronger are the forces that bind it to the nucleus. So the first shell which is closest to the nucleus is always under the tremendous force of attraction. While the shell which is farthest from the nucleus is under very weak force of attraction. The electrons revolving in the last shell i.e. farthest from the nucleus are very loosely bound to the nucleus. Such electrons in the outermost shell are responsible for the electrical and chemical characteristics of an atom.

Key Point: *The outermost shell is called the **valence shell** and the electrons in this shell are called **valence electrons.***

The exception to the '$2n^2$' rule stated above is that the outermost shell in an atom cannot accommodate more than eight electrons. The valence electrons revolving in the outermost shell are said to be having highest energy level. **The amount of energy required to extract the valence electron from the outer shell is very less.**

Key Point: *Each shell has energy level associated with it. Closer the shell to the nucleus, more tightly it is bound to the nucleus and possesses lower energy level.*

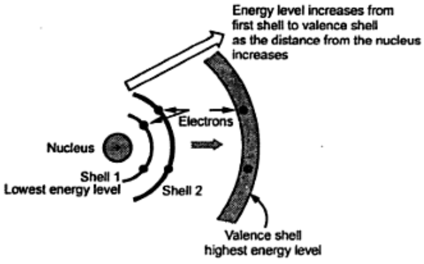


Fig. 7.1 Concept of energy level

Thus energy level of shell one is lowermost while the energy level of valence shell is highest. More energy level indicates that the electrons of that shell are loosely bound to the nucleus. Hence valence electrons are loosely bound to the nucleus as having highest energy level. The concept of energy level is shown in the Fig. 7.1

When an atom absorbs energy from a heat source or from light or due to high atmospheric temperature, the energy levels of the electrons are raised. When such an additional energy is imparted to the electrons, the electrons move to the next orbit which is farther from the nucleus. If such an energy is imparted to a valence electron, it tries to jump to the next orbit. But as a valence electron is in the outermost orbit, actually it gets completely removed from the force of attraction of the nucleus.

Key Points:

1) *An electron which is not subjected to the force of attraction of the nucleus is called a free electron. Such free electrons are basically responsible to the flow of current.*

2) *More the number of free electrons, better is the conductivity of the metal.*

7.2.2 Structure of Semiconductor Materials

The semiconductor materials such as Ge and Si have four electrons in their valence shell i.e. outermost shell. The Fig. 7.2 shows atomic structure of the semiconductor materials, germanium and silicon.

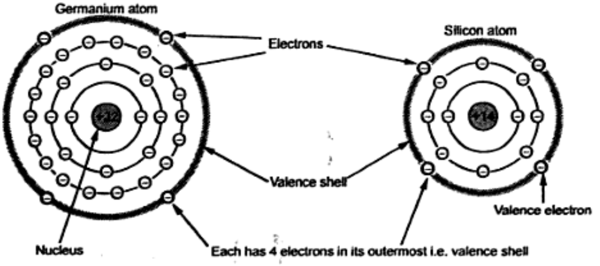


Fig. 7.2 Atomic structure of germanium and silicon atoms

So instead of the presence of widely separated energy levels as that of the isolated atoms, the closely spaced energy levels are present in a solid, which are called energy bands.

Out of all the energy bands, three bands are most important to understand the behaviour of solids. These bands are,

1] Valence band 2] Conduction band 3] Forbidden band or gap

Key Point: *The energy band formed due to merging of energy levels associated with the valence electrons i.e. electrons in the last shell, is called* ***valence band.***

As mentioned earlier in normal condition, valence electrons form the covalent bonds and are not free. But when certain energy is imparted to them, they become free.

Key Point: *The energy band formed due to merging of energy levels associated with the free electrons is called* ***conduction band.***

Under normal condition, the conduction band is empty and once energy is imparted, the valence electrons jump from valence band to conduction band and become free.

While jumping from valence band to conduction band, the electrons have to cross an energy gap.

Key Point: *The energy gap which is present separating the conduction band and the valence band is called* ***forbidden band*** *or* ***forbidden gap.***

The energy imparted to the electrons must be greater than the energy associated with the forbidden gap, to extract the electrons from valence band and transfer them to conduction band. The energy associated to forbidden band is denoted as E_G.

Key Point: *The electrons cannot exist in the forbidden gap.*

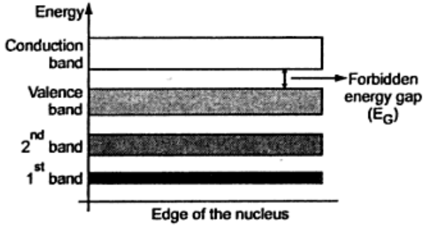


Fig. 7.3 Energy band diagram

The graphical representation of the energy bands in a solid is called **energy band diagram.** Such an energy band diagram for a solid silicon is shown in the Fig. 7.3.

The electrons in the various orbits revolving around the nucleus occupy the various bands including fully or partly occupied valence band. The conduction band which is normally empty carries the electrons which get drifted from the valence band. These electrons present in the conduction band are free electrons and they drift about in the spaces between the atoms.

For any given type of material the forbidden energy gap may be large, small or nonexistent. The classification of materials as insulators, conductors or semiconductors is mainly dependent on the widths of the forbidden energy gap. Let us see the classification of materials as insulators, conductors and semiconductors based on energy band diagram. Before that let us see the unit in which energy associated with the bands is measured.

7.3.1 The eV, Unit of Energy

The energy is measured in joules (J) in the M.K.S. system. As mentioned earlier, each electron has an energy level associated with it. The unit joule is very large for the discussion of such energies associated with electrons. Hence such energies associated with the electrons are measured in electron volts denoted as eV.

The charge on a single electron is 1.6×10^{-19} coulomb. So **one electron volt is defined as the energy required by an electron to fall through a potential of one volt.**

$$\therefore \quad 1 \text{ eV} = 1.6\times10^{-19} \text{ (C)} \times 1 \text{ (V)} = 1.6\times10^{-19} \text{ J}$$

$$\therefore \quad \boxed{\mathbf{1\ eV = 1.6\times10^{-19}\ J}}$$

7.4 Classification of Materials Based on Energy Band Theory

Based on the ability of various materials to conduct current, the materials are classified as conductors, insulators and the semiconductors.

A metal which is very good carrier of electricity is called conductor. A very poor conductor of electricity is termed as insulator. A metal having conductivity which is between conductor and an insulator is called semiconductor. The copper and aluminium are good examples of a conductor. The glass, wood, mica, diamond are the examples of an insulator which does not conduct current. The silicon and germanium are the examples of a semiconductor which does not conduct current at low temperatures but as temperature increases these materials behave as good conductors. Let us see the energy band diagrams for these three types of metals.

7.4.1 Conductors

It has been mentioned earlier that a material having large number of free electrons can conduct very easily. For example, copper has 8.5×10^{28} free electrons per cubic metre which is a very large number. Hence copper is called good conductor. In fact, in the metals like copper, aluminium there is no forbidden gap between valence band and conduction band. **The two bands overlap.** Hence even at room temperature, a large number of electrons are available for conduction. So without any additional energy, such metals contain a large number of free electrons and hence called good **conductors.** An energy band diagram for a conductor is shown in the Fig. 7.4 (a).

7.4.2 Insulators

An insulator has an energy band diagram as shown in the Fig. 7.4 (b). In case of such insulating material, there exists a large forbidden gap in between the conduction band and the valence band. Practically it is impossible for an electron to jump from the valence band to the conduction band. Hence such materials cannot conduct and called **insulators.** The forbidden gap is very wide, approximately of about 7 eV is present in insulators. For a diamond, which is an insulator, the forbidden gap is about 6 eV. Such materials may conduct only at very high temperatures or if they are subjected to high voltage. Such a conduction is rare and is called breakdown of an insulator. The other insulating materials are glass, wood, mica, paper etc.

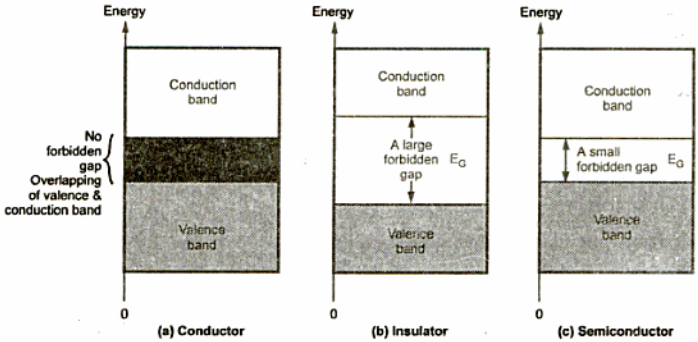


Fig. 7.4 Energy band diagrams

7.4.3 Semiconductors

Now let us come to an important category of materials, which are neither insulators nor conductors. The forbidden gap in such materials is very narrow as shown in Fig. 7.4 (c). Such materials are called **semiconductors.** The forbidden gap is about 1 eV. In such materials, the energy provided by the heat at room temperature is sufficient to lift the electrons from the valence band to the conduction band. Therefore at room temperature, semiconductors are capable of conduction. But at 0 °K or absolute zero (– 273 °C), all the electrons of semiconductor materials find themselves locked in the valence band. Hence at 0° k, the semiconductor materials behave as perfect insulators. In case of semiconductors, forbidden gap energy depends on the temperature. For silicon and germanium, this energy is given by,

$$\boxed{\begin{array}{l} E_G = 1.21 - 3.6 \times 10^{-4} \times T \quad \text{eV (for Silicon)} \\ E_G = 0.785 - 2.23 \times 10^{-4} \times T \text{ eV} \quad \text{(for Germanium)} \end{array}}$$

where T = Absolute temperature in °K

Assuming room temperature to be 27 °C i.e. 300 °K, the forbidden gap energy for Si and Ge can be calculated from the above equations. The forbidden gap for the germanium is 0.72 eV while for the silicon it is 1.12 eV at room temperature. The silicon and germanium are the two widely used semiconductor materials in electronic devices, as mentioned earlier.

Key Point : *While calculating* E_G, *substitute T in* °K.

Why Silicon is most widely used ?

Looking at the structure of silicon and germanium atom, it can be seen that valence shell of silicion is 3^{rd} shell while valence shell of germanium is 4^{th} shell. Hence valence electrons of germanium are at larger distance from nucleus than valence electrons of silicon. Hence valence electrons of germanium are more loosely bound to the nucleus than those of silicon. Thus valence electrons of germanium can easily escape from the atom, due to very small additional energy imparted to them. So at high temperature, germanium becomes unstable than silicon and and hence silicon is widely used semiconductor material.

➠ **Example 7.1 :** *Calculate the value of forbidden gap for silicon and germanium at the temperature of 35 °C.*

Solution : Forbidden gap for silicon is given by,

$$E_G = 1.21 - 3.6 \times 10^{-4} \times T$$

Now $$T = 35 + 273 = 308 \text{ °K}$$

$\therefore$ $$E_G = 1.21 - 3.6 \times 10^{-4} \times 308 = \mathbf{1.099 \text{ eV}}$$

While forbidden gap for germanium is given by,

$$E_G = 0.785 - 2.23 \times 10^{-4} \times T = 0.785 - 2.23 \times 10^{-4} \times 308$$

$$= \mathbf{0.7163 \text{ eV}}$$

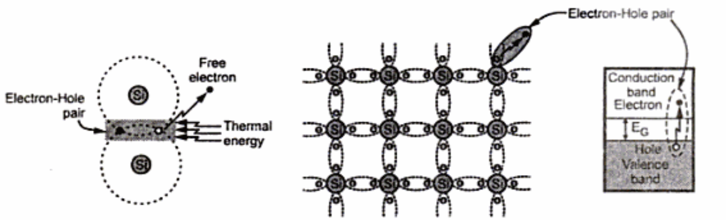
7.5 Intrinsic Semiconductors

A sample of semiconductor in its purest form is called an **intrinsic semiconductor.** The impurity content in intrinsic semiconductor is very very small, of the order of one part in 100 million parts of semiconductor. For achieving such a pure form, the semiconductor materials are carefully refined. To understand the conduction in an intrinsic semiconductor let us study the crystalline structure of an intrinsic semiconductor.

7.5.1 Crystal Structure of Intrinsic Semiconductor

Consider an atomic structure of an intrinsic semiconductor material like silicon. An outermost shell of an atom is capable of holding eight electrons. It is said to be completely filled and stable, if it contains eight electrons. But the outermost shell of an intrinsic semiconductor like silicon has only four electrons. Each of these four electrons form a bond with another valence electron of the neighbouring atoms. This is nothing but sharing of electrons. Such bonds are called **covalent bonds.** The atoms align themselves to form a three dimensional uniform pattern called **a crystal.**

The crystal structure of germanium and silicon materials consists of repetitive occurrence in three dimensions of a unit cell. This unit cell is in the form of a tetrahedron with an atom at each vertex. But such a three dimensional structure is very difficult to



(a) Breaking of covalent bond (b) Electron-hole pair in a silicon crystal (c) Energy band diagram

Fig. 7.6 Thermal generation

The concentration of free electrons and holes is always equal in an intrinsic semiconductor. The hole also serves as a carrier of electricity similar to that of free electron. **An electron is negatively charged particle. Thus a hole getting created due to electron drift is said to be positively charged.**

Key Point: *Thus in an intrinsic semiconductors both holes as well as free electrons are the charge carriers.*

7.5.3 Conduction by Electrons and Holes

The electrons and holes generated due to thermal generation move randomly and hence cannot constitute any current. Now consider that battery is connected across the intrinsic semiconductor.

Under the influence of applied voltage there is electron as well as hole motion in one particular direction, causing the flow of current.

The free electrons which are available in the conduction band are moved under the influence of applied voltage. The electrons as negatively charged get repelled from the negative terminal of battery and attracted towards the positive terminal. Thus there is an electric current due to the movement of electrons in conduction band. This is called **electron current** .

There are electrons present in the valence band which are involved in forming the covalent bonds. Some holes are also present in the valence band due to escape of electrons from valence to conduction band. Under the influence of applied voltage, the electrons involved in covalent bonds break the covalent bonds and try to fill the holes present. The electron breaking the covalent bond jumps to the hole of neighbouring atom, leaving a hole behind. This is illustrated in the Fig. 7.7 (a), (b) and (c).

The atom x has a hole due to escape of an electron to the conduction band. The electron from atom y breaks its covalent bond and fill the hole of atom x. Now the hole is

7.5.6 Recombination of Electrons and Holes

The movement of holes in the valence band is always random and similarly the movement of free electrons in the conduction band is also random. Thermal agitation continues to produce new hole-electron pairs. Occasionally, a free electron approaches a hole and falls into it. This merging of a free electron and a hole is called **recombination.** After the recombination, an electron-hole pair gets disappeared. Due to recombination the number of charge carriers decreases. **The amount of time between the creation and disappearence of a free electron or hole is called the mean life time of the charge carrier.**

At any temperature, at any instant, the free electrons and holes, the two types of charge carriers are present in equal numbers. This concentration is called **intrinsic concentration.** Mathematically this is indicated as,

$$\boxed{n = p = n_i} \quad \dots (2)$$

where n = Number of free electrons per unit volume

p = Number of holes per unit volume

and n_i = Intrinsic concentration.

The concentration is measured in the units **number per m^3 or per cm^3.**

7.6 Drift Current

When a voltage is applied to a semiconductor, the free electrons try to move in a straight line towards the positive terminal of the battery. The electrons, moving towards positive terminal collide with the atoms of semiconductor and connecting wires, along its way. Each time the electron strikes an atom, it rebounds in a random direction. But still the applied voltage make the electrons drift towards the positive terminal. This drift causes current to flow in a semiconductor, under the influence of the applied voltage. This current produced due to drifting of free electrons is called **drift current** and the velocity with which electrons drift is called **drift velocity.** Thus drift current means the flow of current due to bouncing of electrons from one atom to another, travelling from negative terminal to positive terminal of the applied voltage.

> **Key Point** : *The direction of conventional current is always opposite to the direction of drifting electrons.*

This is shown in the Fig. 7.10.

The conventional current direction is always from positive terminal to the negative terminal of the battery. But the operating principle of many semiconductor devices is generally explained considering the direction of flow of electrons rather than the conventional current.

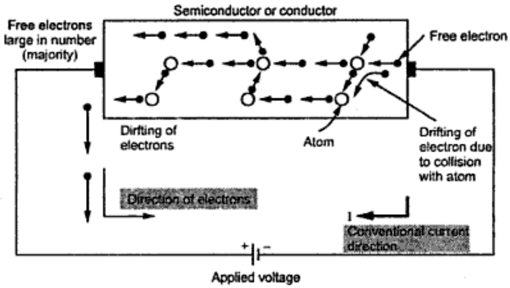


Fig. 7.10 Drift mechanism causing drift current

7.7 Mobility of Charged Particle

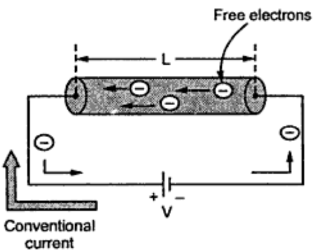


Fig. 7.11 Concept of mobility

Consider a material having a large number of free electrons. It is electrically neutral i.e. number of electrons is equal to number of protons. All the free electrons move around randomly inside the material. As the motion is random, on an average, the number of electrons passing through unit area in a given time in any direction inside the material is same as the number of electrons passing in opposite direction. Hence net movement of the electrons is cancelled. And hence **without electric field, there cannot be any drift current in the material.**

Consider a material of length L, subjected to the voltage V as shown in the Fig. 7.11. The electric field to which it is subjected is given by,

$$\boxed{E = \frac{V}{L}\ V/m} \quad \text{... (1)}$$

The free electrons drift inside the material constituting the drift current.

Finally a steady state condition is reached where electrons continue to move with a finite steady velocity. Such a speed attained by the electrons under the influence of applied electric field in steady state condition is called as **drift speed** or **drift velocity.** It is denoted by v measured in metres per second. **The magnitude of the drift velocity is proportional to the electric field** E, so mathematically we can write

$$v \propto E$$

$$\therefore \quad \boxed{v = \mu E} \quad \text{... (2)}$$

where v = Drift velocity in m/sec.

E = Applied electric field in V/m

where μ is constant of proportionality and is called **mobility** of the electrons. This is applicable to the free electrons as well as the holes whichever are the majority carriers.

So in general,

$$\mu = \text{Mobility of a charged particle} = \frac{v}{E}$$

$$\therefore \quad \boxed{\text{Units of } \mu = \frac{m/sec}{V/m} = \frac{m^2}{V\text{-}sec}} \quad \text{... (3)}$$

So it is measured in square metres per volt-second. Such steady movement of majority charge carriers with drift velocity constitutes a current. This current is called **drift current**.

7.8 General Expression for Conductivity

Consider a tube of metal with large number of free elecrons as shown in the Fig. 7.12.

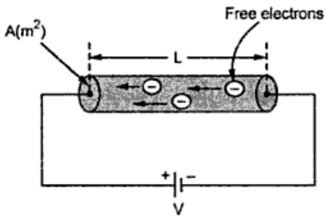


Fig. 7.12 Tube of metal subjected to voltage

Let, A = Cross-sectional area in m^2

L = Length in m

V = Voltage applied in volts

T = Time required by an electron to travel distance of 'L' m

$$\therefore \quad v = \text{Drift velocity of electron} = \frac{L}{T} \quad \text{...(1)}$$

$$E = \frac{V}{L} = \text{Electric field} \quad \text{...(2)}$$

$$v = \mu E \quad \text{where } \mu = \text{Mobility of electrons} \quad \text{...(3)}$$

Consider any cross-section as shown in the Fig. 7.13.

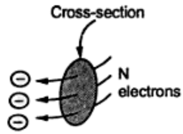


Fig. 7.13 N electrons crossing in T

Let N be the number of electrons passing through area A in time T. So number of electrons crossing the area A in **unit time** is $\frac{N}{T}$.

If $\boxed{e = \text{Charge on each electron} = 1.6 \times 10^{-19}\ C}$

then the total charge crossing the cross-section area A in **unit time** is,

$$dq = \text{Charge on each electron} \times \frac{N}{T} = \frac{Ne}{T}\ C$$

But charge passing per unit time through a cross-section is the **current**.

$$\therefore \quad I = \frac{\text{Charge passing}}{\text{Unit time}} = \frac{\frac{Ne}{T}}{(1\text{sec.})}$$

$$\therefore \quad \boxed{I = \frac{Ne}{T}\ A} \qquad \text{... (4)}$$

The **current density** J for the bar is current per unit cross-sectional area of the conducting material.

$$\therefore \quad \boxed{J = \frac{I}{A}\ A/m^2} \qquad \text{... (5)}$$

$$\therefore \quad J = \frac{Ne}{TA} \quad \text{but } T = \frac{L}{v} \qquad \text{... from (1)}$$

$$\therefore \quad J = \frac{Ne}{\frac{L}{v} \times A} = \frac{Nev}{LA}$$

But $\quad LA = \text{Volume of the tube}$

$\therefore \quad n = \text{Concenration of free electrons}$

$\quad = \text{Number of elecrons per unit volume}$

$$\therefore \quad n = \frac{N}{LA}\ /m^3$$

$$\therefore \quad J = nev \quad \text{but } v = \mu E$$

$$\therefore \quad \boxed{J = ne\mu E \quad A/m^2} \quad \text{... (6)}$$

This is the general expression for current density in a given material.

The current density is related to electric field E by the relation,

$$\therefore \quad \boxed{J = \sigma E} \quad \text{... (7)}$$

where σ = Conductivity of the material in $(\Omega\text{-m})^{-1}$

Key Point: *The conductivity indicates the ease with which current can flow through the given material.*

Comparing (6) and (7)

$$\therefore \quad \boxed{\sigma = ne\mu \ (\Omega - m)^{-1}} \quad \text{... (8)}$$

This is the general expression for the conductivity of the given material

$$\therefore \quad \boxed{\rho = \text{resistivity} = \frac{1}{\sigma} \ (\Omega - m)} \quad \text{... (9)}$$

Key Point: *The resistivity ρ is the reciprocal of the conductivity.*

7.9 Conductivity of an Intrinsic Semiconductor

In a semiconductor, there are two charged particles. One is negatively charged free electrons while the other is positively charged holes. These particles move in opposite direction, under the influence of an electric field but as both are of opposite sign, **they constitute current in the same direction.**

For the semiconductor,

n = Concentration of free electrons/m^3

p = Concentration of holes/m^3

μ_n = Mobility of electrons in m^2/V-s

μ_p = Mobility holes in m^2/V-s

then the current density is given by,

$$\therefore \quad \boxed{J = (n\mu_n + p\mu_p) \ e \ E \ A/m^2} \quad \text{... (1)}$$

This can be obtained from the general expression for J derived in last section, equation (6).

Hence the conductivity for a semiconductor is given by,

$$\therefore \quad \boxed{\sigma = (n\mu_n + p\mu_p) \ e \ (\Omega - m^{-1})} \quad \text{... (2)}$$

7.9.2 Effect of Light on Semiconductor

The effect of light on a semiconductor is exactly similar to the effect of heat on a semiconductor. Just as thermal energy causes electrons to break their covalent bonds, similarly the light energy also causes electrons to break their covalent bonds. Under the influence of light energy, electron-hole pairs get generated in a semiconductor, increasing its conductivity.

When not illuminated there are few free electrons in a semiconductor and its resistance is high called **dark resistance**. As the light is incident on a semiconductor and it is illuminated it imparts light energy to the electrons. The electrons breaking their bonds move from valence band to conduction band and the conduction can take place readily. Thus there is decrease in resistance of a semiconductor. When illumination increases, a semiconductor may behave comparable to a conductor.

Thus effect of light on a semiconductor is to cause increase in the conductivity of a semiconductor.

> **Key Point**: *Both heat and light are responsible to generate electron-hole pairs and hence to increase the conductivity of a semiconductor.*

➠ **Example 7.2 :** *Find the resistivity of an intrinsic silicon at 300 °K if intrinsic concentration of silicon at 300 °K is* 1.5×10^{10} *per* cm^3 *while* $\mu_n = 1300\ cm^2/V\text{-sec}$ *and* $\mu_p = 500\ cm^2/V\text{-sec}$. *Assume* $e = 1.6\times 10^{-19}$ *C.*

Solution : The given values are, $n_i = 1.5\times10^{10}/cm^3$

$$\therefore \quad n_i = \frac{1.5\times 10^{10}}{10^{-6}}\ /m^3$$

$$= 1.5\times 10^{16}\ /m^3$$

And $$\mu_n = 1300\times10^{-4}\ m^2/V\text{-sec}$$

$$\mu_p = 500\times10^{-4}\ m^2/V\text{-sec}$$

Now $$\sigma_i = n_i(\mu_n + \mu_p)\,e$$

$$= 1.5\times 10^{16}[1300 + 500]\times 10^{-4}\times 1.6\times 10^{-19}$$

$$= 0.000432\ (\Omega\text{ - m})^{-1} \quad \text{... conductivity}$$

$$\therefore \quad \rho = \frac{1}{\sigma_i}$$

$$= \frac{1}{0.000432} = \mathbf{2314.8148\ \Omega\text{ - m}}$$

This is the required resistivity.

Example 7.3 : *A bar of intrinsic silicon has a cross-sectional area of 2.5×10^{-4} m^2. The electron density is 1.5×10^{16} per m^3. How long the bar be in order that the current in the bar will be 1.2 mA when 9 volts are applied across it ?*

Assume : μ_n = 0.14 m^2/V-s, μ_p = 0.05 m^2/V-s.

Solution : Electron density = n_i = Carrier intrinsic concentration

$$\therefore \quad n_i = 1.5\times10^{16}/m^3,$$

For intrinsic semiconductor,

$$\sigma_i = n_i(\mu_n+\mu_p)\,e$$

where e = Charge on one electron = 1.6×10^{-19} C

$$\therefore \quad \sigma = 1.5\times10^{16}\,(0.14+0.05)\,1.6\times10^{-19}$$

$$= 4.56\times10^{-4} = \text{Conductivity in } (\Omega\text{-m})^{-1}$$

$$\therefore \quad \rho = \text{Resistivity} = \frac{1}{\sigma}$$

$$= 2192.982\ \Omega\text{ - m}$$

Now $$R = \frac{\rho l}{A}$$

$$\therefore \quad \frac{V}{I} = \frac{\rho l}{A}$$

$$\therefore \quad \frac{9}{1.2\times10^{-3}} = \frac{2192.982\times l}{2.5\times10^{-4}}$$

$$\therefore \quad l = 8.55\times10^{-4}\text{m} = \mathbf{0.855\ mm}$$

This is the length of the bar.

Example 7.4 : *Estimate the value of resistivity of intrinsic Germanium at 300 °K given : Intrinsic concentration = 2.5 × 10^{13} cm^3*

Electron mobility = 3800 cm^2/V-s

Hole mobility = 1800 cm^2/V-s

Electron charge = 1.6 × 10^{-19} C

Solution : Given values are

$$n_i = 2.5 \times 10^{13} / cm^3$$

$$\therefore \quad n_i = \frac{2.5 \times 10^{13}}{10^{-6}} = 2.5 \times 10^{19} / m^3$$

$$\mu_n = 3800 \ cm^2/V\text{-}s = 3800 \times 10^{-4} \ m^2 / V\text{-}s$$

$$\mu_p = 1800 \ cm^2/V\text{-}s = 1800 \times 10^{-4} \ m^2 / V\text{-}s$$

$$\sigma_i = (\mu_n + \mu_p) \, n_i \, e$$

$$= (3800 + 1800) \times 1.8 \times 10^{-19} \times 10^{-4} \times 2.5 \times 10^{19}$$

$$= 2.24 \ (\Omega - m)^{-1}$$

$$\rho_i = \frac{1}{\sigma_i} = \frac{1}{2.24} = \mathbf{0.4464 \ \Omega \text{ - } cm}$$

$$= \mathbf{0.4464 \ \Omega \text{ - } cm}$$

7.10 Law of Mass Action

If n is the concentration of free electrons and p is the concentration of holes then the law of mass action states that **the product of concentrations of electrons and holes is always constant, at a fixed temperature.**

Mathematically it is expressed as,

$$\mathbf{np = n_i^2} \qquad \text{... (1)}$$

where n_i is intrinsic concentration.

Important Observations

1. The law can be applied to both intrinsic and extrinsic semiconductors.
2. As n_i depends on temperature, the law is applicable at a fixed temperature.
3. In case of extrinsic semiconductors, n_i is the intrinsic concentration of the basic semiconductor material used.

Key Point *: The law can be used to find the electron and hole densities in intrinsic semiconductors.*

7.11 Extrinsic Semiconductors

In order to change the properties of intrinsic semiconductors a small amount of some other material is added to it. The process of adding other material to the crystal of intrinsic semiconductors to improve its conductivity is called **doping**. The impurity added is called **dopant**. Doped semiconductor material is called **extrinsic semiconductor.** The doping increases the conductivity of the basic intrinsic semiconductors hence the extrinsic semiconductors are used in practice for manufacturing of various electronic devices such as diodes, transistors etc.

Depending upon the type of impurities, the two types of extrinsic semiconductors are,

1. n-type and 2. p-type

7.11.1 Types of Impurities

The impurity material having five valence electrons is called **pentavalent atom.** When this is added to an intrinsic semicondcutor, it is called **donor doping** as each impurity atom donates one free electron to an intrinsic material. Such an impurity is called **donor impurity**. The examples of such impurity are arsenic, bismuth, phosphorous etc. This creates an extrinsic semiconductor with large number of free electrons, called **n-type semiconductor.**

Another type of impurity used is **trivalent atom** which has only three valence electrons. Such an impurity is called **acceptor impurity**. When this is added to an intrinsic semiconductor, it creates more holes and ready to accept an electron hence the doping is called **acceptor doping**. The examples of such impurity are gallium, indium and boron. The resulting extrinsic semiconductor with large number of holes is called **p-type semiconductor.**

7.12 n-Type Semiconductor

When a small amount of pentavalent impurity is added to a pure semiconductor, it is called **n-type semiconductor**. The pentavalent impurity has five valence electrons. These elements are such as arsenic, bismuth, phosphorous and antimony. Such an impurity is called **donor impurity.**

Consider the formation of n-type material by adding arsenic (As) into silicon (Si). The arsenic atom has five valence electrons. An arsenic atom fits in the silicon crystal in such a way that its four valence electrons form covalent bonds with four adjacent silicon atoms. The fifth electron has no chance of forming a covalent bond. This spare electron enters the conduction band as a free electron.

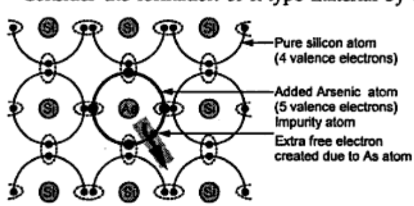


Fig. 7.15 n-type material formation

Such n-type material formation is represented in the Fig. 7.15. This means that **each arsenic atom added into silicon atom gives one free electron.** The number of such free electrons can be controlled by the amount of impurity added to the silicon. Since the free electrons have negative charges, the material is known as n-type material and an impurity donates a free electron hence called donor impurity.

Key Point: *One donor impurity atom donates one free electron in n-type material. The free electrons are majority charge carriers.*

7.12.1 Conduction in n-Type Semiconductor

When the voltage is applied to the n-type semiconductor, the free electrons which are readily available due to added impurity, move in a direction of positive terminal of voltage applied. This constitutes a current. Thus the conduction is predominantly by free electrons. The holes are less in number hence electron current is dominant over the hole current. Hence in n-type semiconductors free electrons are called **majority carriers** while the holes which are small in number are called **minority carriers**. The conduction in n-type material is shown in the Fig. 7.16.

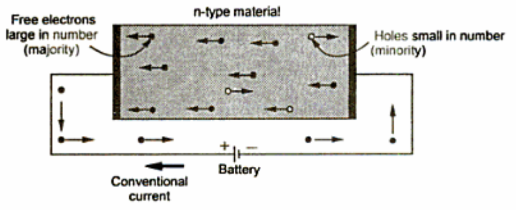


Fig. 7.16 Conduction in n-type material

7.13 p-Type Semiconductor

When a small amount of trivalent impurity is added to a pure semiconductor, it is called **p-type semiconductor**. The trivalent impurity has three valence electrons. These elements are such as gallium, boron or indium. Such an impurity is called **acceptor impurity.**

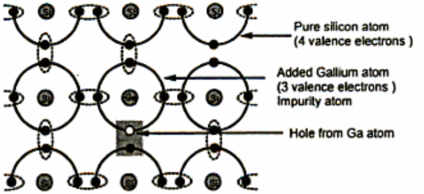


Fig. 7.17 p-type material formation

Consider the formation of p-type material by adding gallium (Ga) into silicon (Si). The gallium atom has three valence electrons. So gallium atom fits in the silicon crystal in such a way that its three valence electrons form covalent bonds with the three adjacent silicon atoms. Being short of one electron, the fourth covalent bond in the valence shell is incomplete. The resulting vacancy is called a hole. Such p-type material formation is represented in the Fig. 7.17. This means that **each gallium atom added into silicon atom gives one hole.** The number of such holes can be controlled by the amount of impurity added to the silicon. As the holes are treated as positively charged, the material is known as p-type material.

At room temperature, the thermal energy is sufficient to extract an electron from the neighbouring atom which fills the vacancy in the incomplete bond around impurity atom. But this creates a vacancy in the adjacent bond from where the electron had jumped, which is nothing but a hole. This indicates that a hole created due to added impurity is ready to accept an electron and hence is called acceptor impurity. Thus even for a small amount of impurity added, large number of holes get created in the p-type material.

Key Point : *One acceptor impurity creats one hole in a p-type material. The holes are majority charge carriers.*

7.13.1 Conduction in p-Type Semiconductor

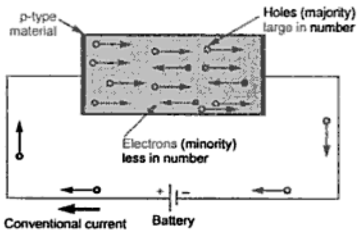


Fig. 7.18 Conduction in p-type material

If now such p-type material is subjected to an electric field by applying a voltage then the holes move in a valence band and are mainly responsible for the conduction. So the current conduction in p-type material is predominantly due to the holes. The free electrons are also present in conduction band but are very less in number. Hence holes are the **majority carriers** while electrons are **minority carriers** in p-type material. The conduction in p-type material is shown in the Fig. 7.18.

7.14 Conductivity of Extrinsic Semiconductor

In an extrinsic semiconductors, there are two types of materials n-type and p-type. Let us obtain the expressions for the conductivity of n-type and p-type materials.

7.14.1 Conductivity of n-Type Material

It is known that in n-type material, the free electrons are majority carriers while the holes are minority carriers.

Let n_n = Concentration of free electrons in n-type

p_n = Concentration of holes in n-type

N_D = Concentration of donor atoms

Key Point: *In the symbol, main letter n or p indicates concentration of type of charge carrier electron or hole while the suffix indicates the type of material i.e. n-type or p-type. Thus* n_n *indicates electron concentration in n-type material while* n_p *indicates electron concentration in p-type material and so on.*

From the basic equation of conductivity, the conductivity of n-type material can be expressed as,

$$\sigma_n = (n_n \mu_n + p_n \mu_p)\, e \qquad \text{... (1)}$$

But $p_n << n_n$ as holes are in minority hence,

$$\therefore \qquad \boxed{\sigma_n \approx n_n \mu_n\, e} \qquad \text{... (2)}$$

The number of free electrons is dominantly controlled by donor atoms added than the thermal generation at room temperature. Hence concentration of donor atoms N_D added can be approximately assumed to be equal to the concentration of free electrons n_n in n type materials.

Thus as $N_D >> n_i$ we can write,

$$n_n \approx N_D \qquad \text{... (3)}$$

$$\therefore \qquad \boxed{\sigma_n \approx N_D \mu_n\, e} \qquad \text{... (4)}$$

7.14.2 Conductivity of p-Type Material

For a p-type material, holes are in majority and electrons are in minority.

Let n_p = Concentration of free electrons in p-type

p_p = Concentration of holes in p-type

N_A = Concentration of acceptor atoms

Thus the conductivity of p-type material can be expressed as,

$$\sigma_p = (n_p \mu_n + p_p \mu_p)\, e \qquad \text{... (5)}$$

But $n_p << p_p$ as free electrons are in minority hence,

$$\boxed{\sigma_p \approx p_p \mu_p\, e} \qquad \text{... (6)}$$

The number of holes is dominantly controlled by added acceptor impurity than the thermal generation. Each added impurity atom creats a hole hence $N_A >> n_i$. Thus all the holes generated p_p can be approximately assumed to be equal to the concentration of acceptor N_A. Thus,

$$p_p \approx N_A \qquad \text{... (7)}$$

$$\therefore \qquad \sigma_p \approx N_A \mu_p\, e \qquad \text{... (8)}$$

7.14.3 Law of Mass Action for Extrinsic Semiconductors

It is seen that, mathematically the law is expressed as,

$$\boxed{np = n_i^2} \quad \text{... (9)}$$

where n_i is intrinsic concentration.

Important Observations

1. The law can be applied to both intrinsic and extrinsic semiconductors.
2. In case of extrinsic semiconductors, n_i is the intrinsic concentration of the basic semiconductor material used.
3. For n-type material, $n = n_n$ while $p = p_n$ hence law can be stated as,

$$n_n p_n = n_i^2 \quad \text{... (10)}$$

4. For p-type material $n = n_p$ while $p = p_p$ hence law can be stated as,

$$n_p p_p = n_i^2 \quad \text{... (11)}$$

5. The law is applicable irrespective of amount of doping.
6. As n_i depends on temperature, the law is applicable at a fixed temperature.
7. The law can be used to find both majority and minority carrier concentrations in an extrinsic semiconductor.

7.14.4 Carrier Concentrations in Extrinsic Semiconductors

Let us obtain the concentrations of minority and majority carriers in n-type and p-type materials using Law of mass action.

n type material :

For n-type material it is seen that, $n_n \approx N_D$.

At any fixed temperature, according to law of mass action,

$$n_n \times p_n = n_i^2$$

where n_n is electrons i.e. majority carrier concentration while p_n is hole i.e. minority carrier concentration. Using $n_n \approx N_D$, we can write minority carrier concentration as,

$$N_D \, p_n = n_i^2$$

$$\therefore \quad \boxed{p_n = \frac{n_i^2}{N_D}} \quad \text{... (12)}$$

Knowing n_i and N_D, the **number of holes in n-type material i.e. minority carrier** concentration can be obtained.

$$\therefore \qquad n = \frac{-1\times10^{20} \pm \sqrt{(1\times10^{20})^2 - 4(-6.25\times10^{38})}}{2}$$

$$= 5.901\times10^{18} \text{ per } m^3 \qquad \text{... neglecting negative value}$$

$$\therefore \qquad p = 1\times10^{20} + n = 1.059\times10^{20} \text{ per } m^3$$

These are the actual concentrations of electrons holes in the sample. As $p > n$, holes are much more than electrons and sample will behave as **p-type** material

Example 7.7 : *If a donor impurity is added to the extent of one atom per 10^8 germanium atoms, calculate its resistivity at 300 °K. If its resistivity without addition of impurity at 300 °K is 44.64 Ω-cm, comparing two values, comment on the result.*
Assume : $\mu_n = 3800\ cm^2/V\text{-sec}$.

Solution : Referring to the Table 7.2 of properties of germanium, germanium has 4.4×10^{22} atoms/cm^3.

For 10^8 germanium atom there is 1 atom impurity added, as given.

Thus, for 4.4×10^{22} germanium atoms, we have,

$$= \frac{4.4\times10^{22}}{10^8} = 4.4\times10^{14} \text{ atoms of impurity/cm}^3$$

This is nothing but concentration of donor atoms i.e. N_D

$$\therefore \qquad N_D = 4.4\times10^{14} \text{ per cm}^3 = \frac{4.4\times10^{14}}{10^{-6}} = 4.4\times10^{20} \text{ per m}^3$$

Now as donor impurity is added, n-type material will form,

$$\therefore \qquad \sigma_n = n_n \mu_n q = N_D \mu_n q$$

where $\qquad n_n \cong N_D$ and $\mu_n = 3800\ cm^2/V\text{-sec} = 3800\times10^{-4} m^2/V\text{-sec}$

$$\therefore \qquad \sigma_n = 4.4\times10^{20}\times3800\times10^{-4}\times1.6\times10^{-19} = 26.752\ (\Omega\text{-m})^{-1}$$

$$\therefore \qquad \text{Resistivity} = \rho_n = \frac{1}{\sigma_n} = \frac{1}{26.752} = 0.0373\ \Omega\text{-m}$$

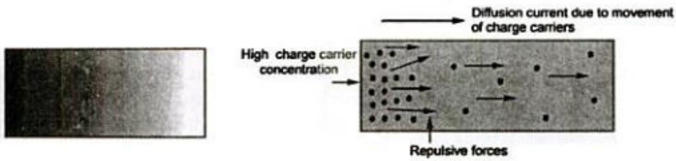
$$= 3.73\ \Omega\text{-cm}$$

Comparing this with resistivity of intrinsic germanium it can be observed that resistivity reduces considerably due to addition of impurity. Hence conductivity of n-type material is much higher and hence it can carry more current as compared to the intrinsic semiconductor. By controlling amount of doping we can control the conductivity.

7.15 Diffusion Current

This is the current which is due to the transport of charges occurring because of nonuniform concentration of charged particles in a semiconductor.

Consider a piece of semiconductor which is nonuniformly doped. Due to such nonuniform doping, one type of charge carriers occur at one end of a piece of semiconductor. The charge carriers are either electrons or holes, of one type depending upon the impurity used. They have the same polarity and hence experience a force of repulsion between them. The result is that there is a tendency of the charge carriers to move gradually i.e. to diffuse from the region of high carrier density to the low carrier density. This process is called **diffusion**. This movement of charge carriers under the process of diffusion constitutes a current called **diffusion current**. This is shown in the Fig. 7.19.



(a) Nonuniform concentration (b) Process of diffusion

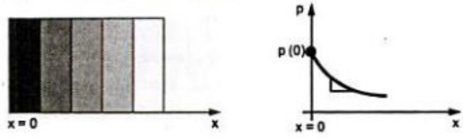
Fig. 7.19 Diffusion current

The diffusion current continues till all the carriers are evenly distributed throughout the material. A diffusion current is possible only in case of nonuniformly doped semiconductors while drift current is possible in semiconductors as well as conductors.

> **Key Point**: *The diffusion current exists without external voltage applied while drift current can not exist without an external voltage applied.*

7.15.1 Concentration Gradient

Consider a p-type semiconductor bar which is nonuniformly doped. Along its length, in the direction of x as shown in Fig. 7.20 (a), there exists a nonuniform doping. As x increases, the doping concentration decreases.



(a) Nonuniform concentration in a semiconductor bar (b) The graph of concentration of holes against x

Fig. 7.20

To form p-type of semiconductor, acceptor impurity is added which creates holes as the majority charged particles. Let p be the concentration of holes. But due to nonuniform doping it is not constant but is changing with respect to x.

Let concentration of holes at x = 0 is p = p(0) and is maximum as bar is heavily doped at x = 0. As x increases, the concentration of holes decreases. The nature of the variation in p against distance x is shown in the Fig. 7.20 (b).

The slope of the graph can be observed from the Fig. 7.20 (b) is the ratio of change in concentration to change in distance. It is called rate of change of concentration or **concentration gradient**.

$$\therefore \textbf{ Slope of graph} = \textbf{Concentration gradient} = \frac{dp}{dx} \qquad \text{... (1)}$$

Hence nonuniform doping produces a concentration gradient in a semiconductor. Due to such concentration gradient, holes move from the higher concentration area to the lower concentration area to adjust the concentration. Such a **movement of holes, due to the concentration gradient in a semiconductor is called diffusion.** Due to the movement of holes, current is constituted in a bar which is called **diffusion current.** There exists such a diffusion current in n-type semiconductor if it is nonuniformly doped, due to movement of electrons which are majority carriers.

Key Point: *Nonuniform doping creats concentration gradient, due to which diffusion of charge carriers exists.*

7.15.2 Diffusion Current Density

Consider a nonuniformly doped p-type semiconductor bar as shown in Fig. 7.21 (a).

The diffusion current density is proportional to the concentration gradient, which is responsible for the diffusion and hence the diffusion current.

$$\therefore \qquad J_p \propto \frac{dp}{dx} \qquad \text{... (2)}$$

where J_p = Diffusion current density due to holes

Hence the diffusion current density is expressed by,

$$J_p = e\, D_p \frac{dp}{dx}$$

where D_p = Diffusion constant for holes expressed in square metres per second. (m^2/sec).

Note : The current due to holes is in the direction of the conventional current and hence treated as positive. But slope of the graph i.e. $\frac{dp}{dx}$ is negative giving the negative diffusion current density for holes. But to get positive sign for holes, an additional

negative sign is used to compensate for negative sign of $\frac{dp}{dx}$. Hence diffusion current density for holes is mathematically expressed as,

$$\boxed{J_p = -e\,D_p\,\frac{dp}{dx}} \qquad \ldots (3)$$

In case of n-type bar, such diffusion current is due to the electrons. Current due to the electrons is in opposite direction to the conventional current and mathematically treated to be negative. The concentration gradient $\frac{dn}{dx}$ is negative where n is concentration of electrons.

Hence diffusion current density to electrons is expressed by,

$$\boxed{J_n = +e\,D_n\,\frac{dn}{dx}} \qquad \ldots (4)$$

where D_n = Diffusion constant for electrons in square metres per second (m^2/sec).

The Fig. 7.21 (a) shows the direction of diffusion of holes and corresponding diffusion current density. The Fig. 7.21 (b) shows the direction of diffusion of electrons and corresponding diffusion current density.

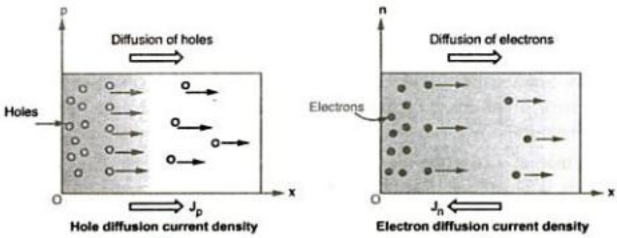


Fig. 7.21 Diffusion current densities

Observe that the charge carriers whether it is hole or electron, always **move from high concentration area towards low concentration area.** Hence direction of diffusion is same in both the cases. But resulting current densities have opposite directions.

7.15.3 Total Current Density Due to Drift and Diffusion

We have seen that the drift current is due to the applied voltage while the diffusion current is due to the concentration gradient. But in semiconductor it is very much possible that both the types of currents may exist simultaneously. In practice in such situation there exists four components of current as the drift current due to electrons and due to holes, while the diffusion current due to electrons and due to holes.

Drift current density for electrons and holes can be expressed as,

$$J_n = n e \mu_n E \text{ and } J_p = p e \mu_p E \qquad \text{(Refer section 7.8)}$$

Diffusion current density due to the electrons and holes can be expressed from equations (3) and (4) as,

$$J_n = +e D_n \frac{dn}{dx} \text{ and } J_p = -e D_p \frac{dp}{dx}$$

∴ **Total current density due to the electrons** can be expressed as,

$$\boxed{J_n = n e \mu_n E + e D_n \frac{dn}{dx}} \qquad \dots (5)$$

and **Total current density due to the holes** can be expressed as,

$$\boxed{J_p = p e \mu_p E - e D_p \frac{dp}{dx}} \qquad \dots (6)$$

And hence the total current density due to the electrons and holes (drift + diffusion) is,

$$\boxed{J = J_n + J_p} \qquad \dots (7)$$

7.15.4 Einstein's Relationship

It is now known that drift current density is proportional to the mobility (μ) while diffusion current density is proportional to the diffusion constant (D). There exists a fixed relation between these two constants which is called **Einstein's relation.**

It states that, at a fixed temperature, the ratio of diffusion constant to the mobility is constant. This is Einstein's relation. Mathematically it is expressed as,

$$\boxed{\frac{D_p}{\mu_p} = \frac{D_n}{\mu_n} = kT = \text{constant at fixed temperature.}} \qquad \dots (8)$$

where T is the temperature in °K

And k is the Boltzmann's constant = 8.62×10^{-5} eV/°K

7.15.5 Voltage Equivalent of Temperature

In the equation (8), the product kT is called **voltage equivalent of temperature.**

The voltage equivalent of temperature is denoted by V_T.

$$\boxed{V_T = kT = \text{Voltage equivalent of temperature}} \qquad \dots (9)$$

At room temperature i.e. at 27 °C,

$$T = 273 + 27 = 300\ ^\circ K$$

$$\therefore \quad V_T = kT = 8.62 \times 10^{-5} \times 300 = 0.02586\ V \approx 26\ mV \text{ at } 300\ ^\circ K$$

Key Point: *The value of V_T = 26 mV at 27 °C i.e. 300 °K is very commonly used while solving the examples.*

Substituting this in equation (8) we get,

$$\frac{D_n}{\mu_n} = \frac{D_p}{\mu_p} = V_T = 0.02586 \text{ at room temperature}$$

$$\therefore \quad \boxed{\mu_n = 39\ D_n} \text{ and } \boxed{\mu_p = 39\ D_p} \text{ at room temperature.} \quad \text{... (10)}$$

In general we can express the relation between mobility and diffusion constant as,

$$\boxed{\mu = 39\ D \quad \text{at room temperature}} \quad \text{... (11)}$$

7.16 The p-n Junction Diode

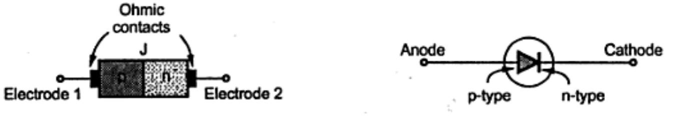
The p-n junction forms a popular semiconductor device called **p-n junction diode.** The p-n junction has two terminals called **electrodes,** one each from p-region and n-region. Due to the two electrodes it is called diode i.e. di + electrode.

To connect the n and p regions to the external terminals, a metal is applied to the heavily doped n and p-type semiconductor regions. Such a contact between a metal and a heavily doped semiconductor is called **ohmic contact.** Such an ohmic contact has two important properties,

1. It conducts current equally in both the directions.
2. The drop across the contact is very small, which do not affect the performance of the device.

Thus ohmic contacts are used to connect n and p-type regions to the electrodes.

The Fig. 7.22 (a) shows schematic arrangement of p-n junction diode while the Fig. 7.22 (b) shows the symbol of p-n junction diode. The **p-region** acts as **anode** while the **n-region** acts as cathode. The arrowhead in the symbol indicates the direction of the conventional current, which can flow when an external voltage is connected in a specific manner across the diode.



(a) Two electrodes

(b) Symbol of a diode

Fig. 7.22

The large number of majority carriers constitute a current called **forward current.** Once the conduction electrons enter the p-region, they become valence electrons. Then they move from hole to hole towards the positive terminal of the battery. The movement of valence electrons is nothing but movement of holes in opposite direction to that of electrons, in the p-region. So current in the p-region is the movement of holes which are majority carriers. This is the **hole current.** While the current in the n-region is the movement of free electrons which are majority carriers. This is the **electron current.** Hence the overall forward current is due to the majority charge carriers. The action is shown in the Fig. 7.24. These majority carriers can then travel around the closed circuit and a relatively large current flows. The direction of flow of electrons is from negative to positive of the battery. While direction of the conventional current is from positive to negative of the battery as shown in the Fig. 7.24.

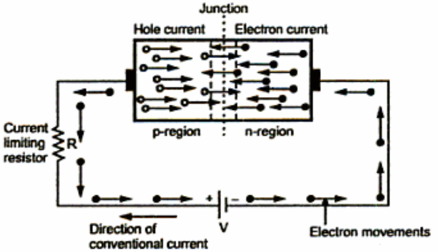
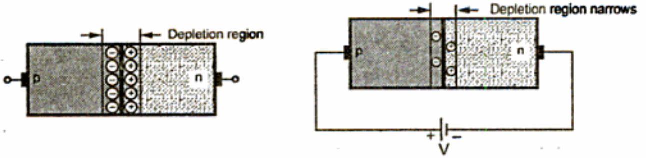


Fig. 7.24 Forward current in a diode

Key Point : *The direction of flow of electrons and conventional current is opposite to each other.*

7.17.2 Effect on the Depletion Region

Due to the forward bias voltage, more electrons flow into the depletion region, which reduces the number of positive ions. Similarly flow of holes reduces the number of negative ions. This reduces the width of the depletion region. This is shown in the Fig. 7.25.



(a) Unbiased diode

(b) Forward biased diode

Fig. 7.25

Key Point : *Depletion region narrows due to forward bias voltage.*

creates more positive ions and hence more positive charge in the n-region and more negative ions and hence more negative charge in the n-region. This is because the applied voltage helps the barrier potential. This is shown in the Fig. 7.27.

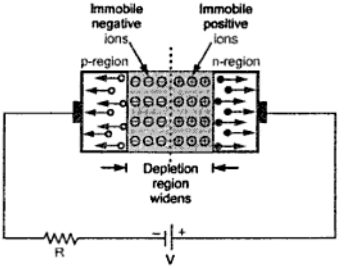


Fig. 7.27 Depletion region widens in reverse bias

Key Point: *Reverse biasing increases the width of the depletion region*

As depletion region widens, barrier potential across the junction also increases. However, this process cannot continue for long time. In the steady state, majority current ceases as holes and electrons stop moving away from the junction.

The polarities of barrier potential are same as that of the applied voltage. Due to increased barrier potential, the positive side drags the electrons from p-region towards the positive of battery. Similarly negative side of barrier potential drags the holes from n-region towards the negative of battery. The electrons on p-side and holes on n-side are minority charge carriers, which constitute the current in reverse biased condition. Thus reverse conduction takes place.

The reverse current flows due to minority charge carriers which are small in number. Hence reverse current is always very small.

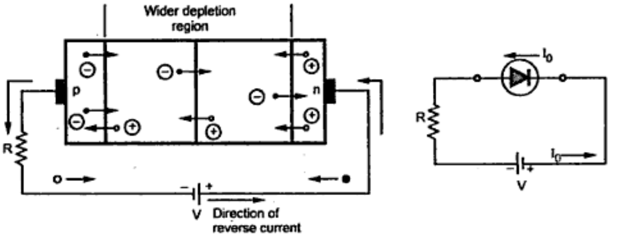
Key Point : *The generation of minority charge carriers depends on the the temperature and not on the applied reverse bias voltage. Thus the reverse current depends on the temperature i.e. thermal generation and not on the reverse voltage applied.*

For a constant temperature, the reverse current is almost constant though reverse voltage is increased upto a certain limit. Hence it is called **reverse saturation current** and denoted as I_0.

Key Point : *Reverse saturation current is very small of the order of few microamperes for germanium and few nanoamperes for silicon p-n junction diodes.*

The reverse current and its direction is shown in the Fig. 7.28.

The reverse biasing produces a voltage drop across the diode denoted as V_R which is almost equal to applied reverse voltage.



(a) Flow of minority charge carriers (b) Direction of reverse current

Fig. 7.28 Reverse biased diode

7.18.2 Breakdown in Reverse Biased

Though the reverse saturation current is not dependent on the applied reverse voltage, if reverse voltage is increased beyond particular value, large reverse current can flow damaging the diode. This is called **reverse breakdown** of a diode. Such a reverse breakdown of a diode can take place due to the following two effects,

1. Avalanche effect and 2. Zener effect.

Breakdown Due to the Avalanche Effect

Though reverse current is not dependent on reverse voltage, if reverse voltage is increased, at a particular value, velocity of minority carriers increases. Due to the kinetic energy associated with the minority carriers, more minority carriers are generated when there is collision of minority carriers with the atoms. The collision make the electrons to break the co-valent bonds. These electrons are available as minority carriers and get accelerated due to high reverse voltage. They again collide with another atoms to generate more minority carriers. This is called **carrier multiplication.** Finally large number of minority carriers move across the junction, breaking the p-n junction. These large number of minority carriers give rise to a very high reverse current. This effect is called **avalanche effect** and the mechanism of destroying the junction is called **reverse breakdown** of a p-n junction. The voltage at which the breakdown of a p-n junction occurs is called **reverse breakdown voltage.** The series resistance must be used to avoid breakdown condition, limiting the reverse current.

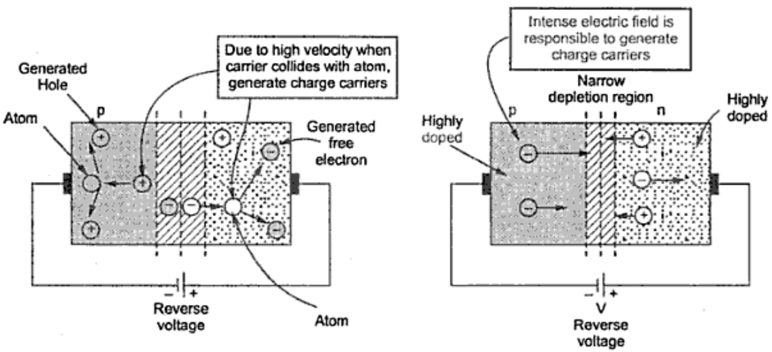
Breakdown Due to the Zener Effect

The breakdown of a p-n junction may occur because of one more effect called **zener effect.** When a p-n junction is heavily doped the depletion region is very narrow. So under reverse bias conditions, the electric field across the depletion layer is very intense. Electric field is voltage per distance and due to narrow depletion region and high reverse voltage, it is intense. Such an intense field is enough to pull the electrons out of the valence bands of the stable atoms. So this is not due to the collision of carriers with atoms. Such a creation of free electrons is called **zener effect** which is different than the avalanche effect. These minority carriers constitute very large current and mechanism is called **zener breakdown.**

> **Key Point**: *The normal p-n junction diode is practically not operated in reverse breakdown region though may be operated in reverse biased condition.*

The breakdown effects are not required to be considered for p-n junction diode. These effects are required to be considered for special diodes such as zener diode as such diodes are always operated in reverse breakdown condition.

The Fig. 7.29(a) shows the avalanche effect while the Fig. 7.29 (b) shows the zener effect.



(a) Avalanche breakdown (b) Zener effect

Fig. 7.29 Breakdown mechanisms

7.19 The Current Components in a p-n Junction Diode

It is indicated earlier that when a p-n junction diode is forward biased a large forward current flows, which is mainly due to majority carriers. The depletion region near the junction is very very small, under forward biased condition. In forward biased condition holes get diffused into n-side from p-side while electrons get diffused into p-side from n-side. So on p-side, the current carried by electrons which is diffusion current due to minority carriers, decreases exponentially with respect to distance measured from the junction. This current due to electrons, on p-side which are minority carriers is denoted as I_{np}. Similarly holes from p-side diffuse into n-side carry current which decreases exponentially with respect to distance measured from the junction. This current due to holes on n-side, which are minority carriers is denoted as I_{pn}. If distance is denoted by x then,

I_{np} (x) = Current due to electrons in p-side as a function of x

I_{pn} (x) = Current due to holes in n-side as a function of x

At the junction i.e. at x = 0, electrons crossing from n-side to p-side constitute a current, $I_{np}(0)$ in the same direction as holes crossing the junction from p-side to n-side constitute a current, $I_{pn}(0)$.

Hence the current at the junction is the total conventional current I flowing through the circuit.

$$\therefore \quad \boxed{I = I_{pn}(0) + I_{np}(0)} \quad \text{... (1)}$$

Now $I_{pn}(x)$ decreases on n-side as we move away from junction on n-side. Similarly $I_{np}(x)$ decreases on p-side as we move away from junction on p-side.

But as the entire circuit is a series circuit, the total current must be maintained at I, independent of x. This indicates that on p-side there exists one more current component which is due to holes on p-side which are the majority carriers. It is denoted by $I_{pp}(x)$ and the addition of the two currents on p-side is total current I.

$$I_{pp}(x) = \text{Current due to holes in p-side.}$$

Similarly on n-side, there exists one more current component which is due to electrons on n-side, which are the majority carriers. It is denoted as $I_{nn}(x)$ and the addition of the two currents on n-side is total current I.

$$I_{nn}(x) = \text{Current due to electrons in n-side.}$$

On p-side, $$\boxed{I = I_{pp}(x) + I_{np}(x)} \quad \text{... (2)}$$

On n-side, $$\boxed{I = I_{nn}(x) + I_{pn}(x)} \quad \text{... (3)}$$

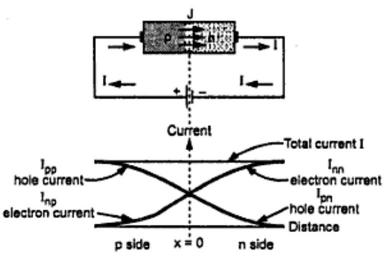


Fig. 7.30 Current components

These current components are plotted as a function of distance in the Fig. 7.30.

The current I_{pp} decreases towards the junction, at the junction enters the n-side and becomes I_{pn} which further decreases exponentially. Similarly the current I_{nn} decreases towards the junction, at the junction enters the p-side and becomes I_{np} which also further decreases exponentially.

Key Point : *In forward bias condition, the current enters the p-side as a hole current and leaves the n-side as an electron current, of the same magnitude.*

So sum of the currents carried by electrons and holes at any point inside the diode is always constant equal to total forward current I. But the proportion due to holes and electrons in constituting the current varies with the distance, from the junction.

7.20 The Volt-Ampere (V-I) Characteristics of a Diode

The response of a diode when connected in an electrical circuit, can be judged from its characteristics known as Volt-Ampere commonly called **V-I characteristics.** The V-I characteristics in the forward biased and reverse biased condition is the graph of voltage across the diode against the diode current.

7.20.1 Forward Characteristics of p-n Junction Diode

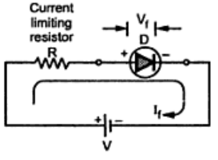


Fig. 7.31 Forward biased diode

The response of p-n junction can be easily indicated with the help of characteristics called **V-I characteristics** of p-n junction. It is the graph of voltage applied across the p-n junction and the current flowing through the p-n junction.

The Fig. 7.31 shows the forward biased diode. The applied voltage is V while the voltage across the diode is V_f. The current flowing in the circuit is the forward current I_f. The graph of forward current I_f against the forward voltage V_f across the diode is called **forward characteristics** of a diode.

The forward characteristics of a diode is shown in the Fig. 7.32.

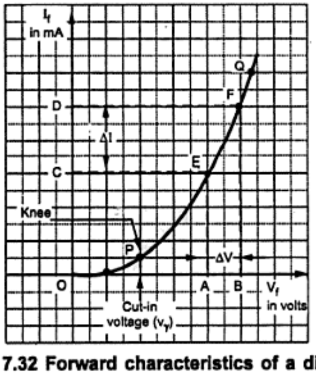


Fig. 7.32 Forward characteristics of a diode

Basically forward characteristics can be divided into two regions :

1. **Region O to P :** As long as V_f is less than cut-in voltage (V_γ) , the current flowing is very small. Practically this current is assumed to be zero.

2. **Region P to Q and onwards :** As V_f increases towards V_γ the width of depletion region goes on reducing. When V_f exceeds V_γ i.e. cut-in voltage, the depletion region becomes very thin and current I_f increases suddenly. This increase in the current is **exponential** as shown in the Fig. 7.32 by the region P to Q.

The point P, after which the forward current starts increasing exponentially is called **knee** of the curve.

Key Point: *The normal forward biased operation of the diode is above the knee point of the curve. i.e. in the region P-Q.*

The forward current is the conventional current, hence it is treated as positive and the forward voltage V_f is also treated positive. Hence the forward characteristics is plotted in the **first quadrant.**

Forward Resistance of a Diode

The resistance offered by the p-n junction diode in forward biased condition is called **forward resistance.** The forward resistance is defined in two ways :

1) Static Forward Resistance :

This is the forward resistance of p-n junction diode when p-n junction is used in d.c. circuit and the applied forward voltage is d.c. This resistance is denoted as R_F and is calculated at a particular point on the forward characteristics.

Thus at a point E shown in the forward characteristics, the static resistance R_F is defined as the ratio of the d.c. voltage applied across the p-n junction to the d.c. current flowing through the p-n junction.

$$\boxed{R_F = \frac{\text{Forward d.c. voltage}}{\text{Forward d.c. current}} = \frac{OA}{OC} \text{ at point E}}$$

2) Dynamic Forward Resistance :

The resistance offered by the p-n junction under a.c. conditions is called **dynamic resistance** denoted as r_f.

Key Point: *The dynamic resistance is reciprocal of the slope of the forward characteristics.*

Consider the change in applied voltage from point A to B shown in the Fig. 1.18. This is denoted as ΔV. The corresponding change in the forward current is from point C to D. It is denoted as ΔI. Thus the slope of the characteristics is $\Delta I/\Delta V$. The reciprocal of the slope is dynamic resistance r_f.

$$\boxed{r_f = \frac{\Delta V}{\Delta I} = \frac{1}{(\Delta I/\Delta V)} = \frac{1}{\text{Slope of forward characteristics}}}$$

Key Point : *Generally the value of r_f is very small of the order of few ohms, in the operating region i.e. above the knee.*

7.20.2 Reverse Characteristics of p-n Junction Diode

The Fig. 7.33 shows the reverse biased diode. The reverse voltage across the diode is V_R while the current flowing is reverse current I_R flowing due to minority charge carriers. The graph of I_R against V_R is called **reverse characteristics** of a diode.

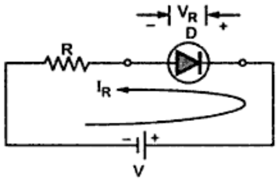


Fig. 7.33 Reverse biased diode

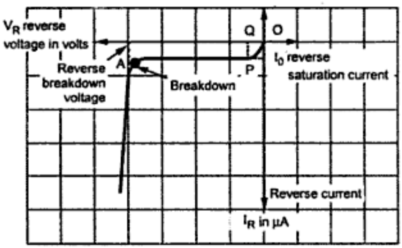


Fig. 7.34

The polarity of reverse voltage applied is opposite to that of forward voltage. Hence in practice reverse voltage is taken as negative. Similarly the reverse saturation current is due to minority carriers and is opposite to the forward current. Hence in practice reverse saturation current is also taken as negative. Hence the reverse characteristics is plotted in the **third quadrant** as shown in the Fig. 7.34.

Key Point : *Typically the reverse breakdown voltage is greater than 50 V for normal p-n junctions.*

As reverse voltage is increased, reverse current increases initially but after a certain voltage, the current remains constant equal to reverse saturation current I_0 though reverse voltage is increased. The point A where breakdown occurs and reverse current increases rapidly is called knee of the reverse characteristics.

Reverse Resistance of a Diode

The p-n junction offers large resistance in the reverse biased condition called **reverse resistance.** This is also defined in two ways.

1. Reverse static resistance :

This is reverse resistance under d.c. conditions, denoted as R_r. It is the ratio of applied reverse voltage to the reverse saturation current I_0.

$$\therefore \quad \boxed{R_r = \frac{OQ}{I_0} = \frac{\text{Applied reverse voltage}}{\text{Reverse saturation current}}}$$

2. Reverse dynamic resistance :

This is the reverse resistance under the a.c. conditions, denoted as r_r. It is the ratio of incremental change in the reverse voltage applied to the corresponding change in the reverse current.

$$\therefore \quad \boxed{r_r = \frac{\Delta V_R}{\Delta I_R} = \frac{\text{Change in reverse voltage}}{\text{Change in reverse current}}}$$

The dynamic resistance is most important in practice whether the junction is forward or reverse biased.

7.20.3 Complete V-I Characteristics of a Diode

The complete V-I characteristics of a diode is the combination of its forward as well as reverse characteristics. This is shown in the Fig. 7.35.

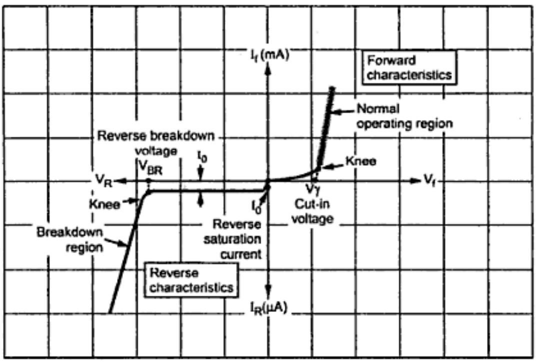


Fig. 7.35 Complete V-I characteristics of a diode

In forward characteristics, it is seen that initially forward current is small as long as the bias voltage is less that the barrier potential. At a certain voltage close to barrier potential, current increases rapidly. The voltage at which diode current starts increasing rapidly is called as cut-in voltage. It is denoted by V_γ. Below this voltage, current is less than 1% of maximum rated value of diode current. The cut-in voltage for germanium is about 0.2 V while for silicon it is 0.6 V.

It is important to note that the breakdown voltage is much higher and practically diodes are not operated in the breakdown condition. **The voltage at which breakdown occurs is called reverse breakdown voltage denoted as V_{BR}.**

Key Point : *Reverse current before the breakdown is very very small and can be practically neglected.*

The factor η is called **emission coefficient** or **ideality factor**. This factor takes into account the effect of recombination taking place in the depletion region. The effect is dominant in silicon diodes and hence for silicon $\eta = 2$. The range of factor is from 1 to 2.

The **voltage equivalent of temperature** indicates dependence of diode current on temperature. The voltage equivalent of temperature V_T for a given diode at temperature T is calculated as,

$$\boxed{V_T = kT \text{ volts}} \quad \dots (2)$$

where k = Boltzmann's constant = 8.62×10^{-5} eV/°K

T = Temperature in °K.

At room temperature of 27 °C i.e. T = 27 + 273 = 300 °K and the value of V_T is 26 mV, as seen earlier.

The value of V_T also can be expressed as,

$$\boxed{V_T = \frac{T}{\left(\frac{1}{k}\right)} = \frac{T}{\left(\frac{1}{8.62 \times 10^{-5}}\right)} = \frac{T}{11600}} \quad \dots (3)$$

Key Point : *The diode current equation is applicable for all the conditions of diode i.e. unbiased, forward biased and reverse biased.*

When unbiased, $V = 0$ hence we get,

$$I = I_0 [e^0 - 1] = 0 \text{ A}$$

Thus there is no current through diode when unbiased.

Key Point : *For forward biased,* ***V must be taken positive*** *and we get current I positive which is forward current. For reverse biased,* ***V must be taken negative*** *and we get negative current I which indicates that it is reverse current.*

If both sides of diode current equation is divided by cross-sectional area A of the junction,

$$\frac{I}{A} = \frac{I_0}{A}\left[e^{V/\eta V_T} - 1\right]$$

i.e.

$$\boxed{J = J_0\left[e^{V/\eta V_T} - 1\right] \text{A/m}^2} \quad \dots (4)$$

where J = Forward current density

J_0 = Reverse saturation current density

Example 7.9 : *The voltage across a silicon diode at room temperature of 300 °K is 0.71 V when 2.5 mA current flows through it. If the voltage increases to 0.8 V, calculate the new diode current.*

Solution : The current equation of a diode is

$$I = I_0 \left(e^{V/\eta V_T} - 1\right)$$

At 300 °K, $V_T = 26 \text{ mV} = 26 \times 10^{-3} \text{ V}$

$V = 0.71 \text{ V for } I = 2.5 \text{ mA}$ and $\eta = 2$ for silicon

$$\therefore \quad 2.5 \times 10^{-3} = I_0 \left[e^{(0.71/2 \times 26 \times 10^{-3})} - 1\right]$$

$$\therefore \quad I_0 = 2.93 \times 10^{-9} \text{ A}$$

Now V = 0.8 V

$$\therefore \quad I = 2.93 \times 10^{-9} \left[e^{(0.8/2 \times 26 \times 10^{-3})} - 1\right]$$

$$= 0.0141 \text{ A} = \mathbf{14.11 \text{ mA}}$$

Example 7.10 : *A germanium diode has a reverse saturation current of 3 μA. Calculate the forward bias voltage at the room temperature of 27 °C and 1% of the rated current is flowing through the forward biased diode. The diode forward rated current is 1 A.*

Solution : The given values are, $I_0 = 3\ \mu A$, $T = 27\ °C = 27 + 273 = 300\ °K$, $\eta = 1$

Now $I_{rated} = 1 \text{ A}$ for diode

and $I = 1\%$ of I_{rated} at 27 °C

$$\therefore \quad I = \frac{1}{100} \times (1) = 0.01 \text{ A}$$

$$V_T = kT = 8.62 \times 10^{-5} \times 300 = 0.026 \text{ V}$$

According to diode equation, $I = I_0 [e^{V/\eta V_T} - 1]$

$$\therefore \quad 0.01 = 3 \times 10^{-6} [e^{V/1 \times 0.026} - 1]$$

$$\therefore \quad 3333.333 = e^{V/0.026} - 1$$

$$\therefore \quad e^{V/0.026} = 3334.3333$$

$$\therefore \quad ln\, [e^{V/0.026}] = ln\, [3334.3333] \qquad \text{... taking natural log}$$

$$\therefore \quad \frac{V}{0.026} = 8.112$$

$$\therefore \quad V = \mathbf{0.2109 \text{ V}}$$

Example 7.11 : *A diode operating at 300 °K at a forward voltage of 0.4 V carries a current of 10 mA. When voltage is changed to 0.42 V, the current becomes twice. Calculate the value of reverse saturation current and η for the diode.*

Solution : At $V_1 = 0.4$ V, $I_1 = 10$ mA and at $V_2 = 0.42$ V, $I_2 = 2\ I_1 = 20$ mA

Now $$I = I_0[e^{V/\eta V_T} - 1]$$

$$\therefore \quad 10\times10^{-3} = I_0\left[e^{0.4/\eta\times26\times10^{3}} - 1\right] \quad ...(1)$$

and $$20\times10^{-3} = I_0\left[e^{0.42/\eta\times26\times10^{3}} - 1\right] \quad ...(2)$$

In forward bias condition $1 << e^{V/\eta V_T}$, ∴ Neglecting 1

$$10\times10^{-3} = I_0\, e^{\frac{15.384}{\eta}} \quad ...(3)$$

and $$20\times10^{-3} = I_0\, e^{\frac{16.153}{\eta}} \quad ...(4)$$

Dividing the two equations (3) and (4) ,

$$\frac{1}{2} = \frac{e^{15.384/\eta}}{e^{16.153/\eta}}$$

$$\therefore \quad e^{16.153/\eta} = 2e^{15.384/\eta}$$

Taking natural logarithm of both sides,

$$\therefore \quad \frac{16.153}{\eta} = ln2 + \frac{15.384}{\eta}$$

$$\therefore \quad \frac{1}{\eta}(16.153-15.384) = 0.6931$$

$$\therefore \quad \eta = 1.109$$

Hence $$I_0 = 9.45 \text{ nA}$$

7.22 Effect of Temperature on Diode

The temperature has following effects on the diode parameters,

1. The cut-in voltage decreases as the temperature increases. The diode conducts at smaller voltages at large temperature.
2. The reverse saturation current increases as temperature increases.

This increase in reverse current I_0 is such that it **doubles at every 10 °C rise in temperature.** Mathematically,

$$\boxed{I_{02} = 2^{(\Delta T/10)}\, I_{01}}$$

where I_{02} = Reverse current at T_2 °C

I_{01} = Reverse current at T_1 °C

$\Delta T = (T_2 - T_1)$

3. The voltage equivalent of temperature V_T also increases as temperature increases.
4. The reverse breakdown voltage increases as temperature increases.

This is shown in the Fig. 7.39.

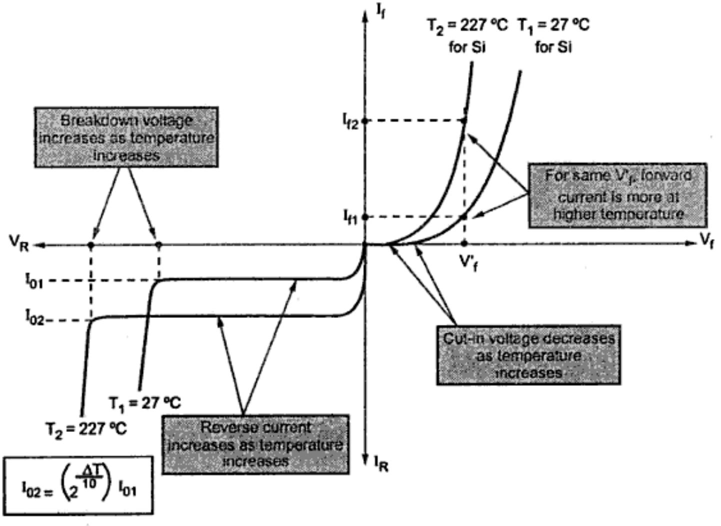


Fig. 7.39 Effect of temperature on diode

Example 7.12 : *The reverse saturation current of a silicon diode is 3 nA at 27 °C. Find,*

i) Reverse saturation current at 82 °C.

ii) Forward current at 82 °C if forward voltage applied is 0.25 V.

Solution : I_{01} = 3 nA at T_1 = 27 °C, T_2 = 82 °C

i) $$\Delta T = T_2 - T_1 = 82 - 27 = 55\ ^\circ C$$

$$\therefore \quad I_{02} = 2^{(\Delta T/10)} \times I_{01} = 2^{\left(\frac{55}{10}\right)} \times 3 = \mathbf{135.764\ nA}$$

ii) $$V = 0.25\ V,\ I_{02} = 135.764\ nA \text{ at } 82\ ^\circ C$$

$$\therefore \quad I_f = I_0\left[e^{V/\eta V_T} - 1\right]$$

$$V_T = \frac{T}{11600} = \frac{(82+273)}{11600} = 0.0306\ V,\ \eta = 2 \text{ for Si}$$

$$\therefore \quad I_f = 135.764 \times 10^{-9}\ [e^{0.25/2\times 0.0306} - 1] = \mathbf{7.934\ \mu A}$$

7.23 Rectifiers

Key Point : *A **rectifier** is a device which converts a.c. voltage to pulsating d.c. voltage, using one or more p-n junction diodes.*

The p-n junction diode conducts only in one direction. It conducts when forward biased while practically it does not conduct when reverse biased. Thus if an alternating voltage is applied across a p-n junction diode, during positive half cycle the diode will be forward biased and will conduct successfully. While during the negative half cycle it will be reversed biased and will not conduct at all. Thus the conduction occurs only during positive half cycle. If the resistance is connected in series with the diode, the output voltage across the resistance will be unidirectional i.e. d.c. Thus p-n junction diode subjected to an a.c. voltage acts as a rectifier converting alternating voltage to a pulsating d.c. voltage.

7.23.1 The Important Characteristics of a Rectifier Circuit

The important points to be studied while analysing the various rectifier circuits are,

a) **Waveform of the load current :** As rectifier converts a.c. to pulsating d.c., it is important to analyze the nature of the current through load which ultimately determines the waveform of the load voltage.

b) **Regulation of the output voltage :** As the load current changes, load voltage changes. Practically load voltage should remain constant. So concept of regulation is to study the effect of change in load current on the load voltage.

c) **Rectifier efficiency :** It signifies, how efficiently the rectifier circuit converts a.c. power into d.c. power.

d) **Peak value of current in the rectifier circuit :** The peak value is the maximum value of an alternating current in the rectifier circuit. This decides the rating of the rectifier circuit element which is diode.

e) **Peak value of voltage across the rectifier element in the reverse direction (PIV) :** When the diode is not conducting, the reverse voltage gets applied across the diode. The peak value of such voltage decides the peak inverse voltage i.e. PIV rating of a diode.

f) **Ripple factor :** The output of the rectifier is of pulsating d.c. type. The amount of a.c. content in the output can be mathematically expressed by a factor called ripple factor. Less is the ripple factor, better is the performance of the circuit.

Using one or more diodes, following rectifier circuits can be designed.

1. Half wave rectifier 2. Full wave rectifier 3. Bridge rectifier

This chapter explains various rectifying circuits using diodes and introduces the concept of filtering alongwith the detail discussion of capacitor input filter.

7.24 Half Wave Rectifier

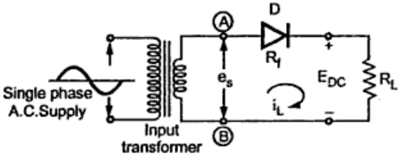


Fig. 7.40 Half wave rectifier

In half wave rectifier, rectifying element conducts only during positive half cycle of input a.c. supply. The negative half cycles of a.c. supply are eliminated from the output.

This rectifier circuit consists of resistive load, rectifying element, i.e. p-n junction diode, and the source of a.c. voltage, all connected in series. The circuit diagram is shown in the Fig. 7.40. Usually, the rectifier circuits are operated from a.c. mains supply. To obtain the desired d.c. voltage across the load, the a.c. voltage is applied to rectifier circuit using suitable step-up or step-down transformer, mostly a step-down one, with necessary turns ratio.

The input voltage to the half wave rectifier circuit shown in the Fig. 7.40 is a sinusoidal a.c. voltage, having a frequency which is the supply frequency, 50 Hz.

The transformer decides the peak value of the secondary voltage. If N_1 are the primary number of turns and N_2 are the secondary number of turns and E_{pm} is the peak value of the primary voltage then,

$$\boxed{\frac{N_2}{N_1} = \frac{E_{sm}}{E_{pm}}}$$

where E_{sm} = The peak value of the secondary a.c. voltage.

As the nature of E_{sm} is sinusoidal the instantaneous value will be,

$$\boxed{e_s = E_{sm} \sin \omega t}$$

$$\omega = 2\pi f$$

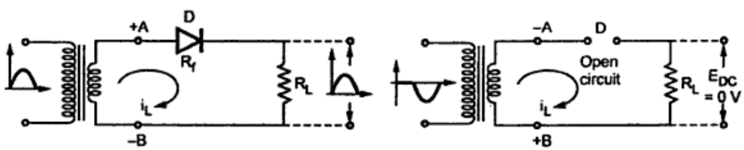
$$f = \text{Supply frequency}$$

Let R_f represents the forward resistance of the diode. Assume that, under reverse biased condition, the diode acts almost as an open circuit, conducting no current.

7.24.1 Operation of the Circuit

During the positive half cycle of secondary a.c voltage, terminal (A) becomes positive with respect to terminal (B). The diode is forward biased and the current flows in the circuit in the clockwise direction, as shown in the Fig. 7.41 (a). The current will flow for almost full positive half cycle. This current is also flowing through load resistance R_L hence denoted as i_L, the load current.

During negative half cycle when terminal (A) is negative with respect to terminal (B), diode becomes reverse biased. Hence no current flows in the circuit as shown in the Fig. 7.41 (b). Thus the circuit current, which is also the load current, is in the form of half sinusoidal pulses.



(a) Diode forward biased (b) Diode reverse biased

Fig. 7.41

The load voltage, being the product of load current and load resistance, will also be in the form of half sinusoidal pulses. The different waveforms are illustrated in Fig. 7.42.

The d.c. output waveform is expected to be a straight line but the half wave rectifier gives output in the form of positive sinusoidal pulses.

> **Key Point:** *Hence the output is called pulsating d.c. It is discontinuous in nature. Hence it is necessary to calculate the average value of load current and average value of output voltage.*

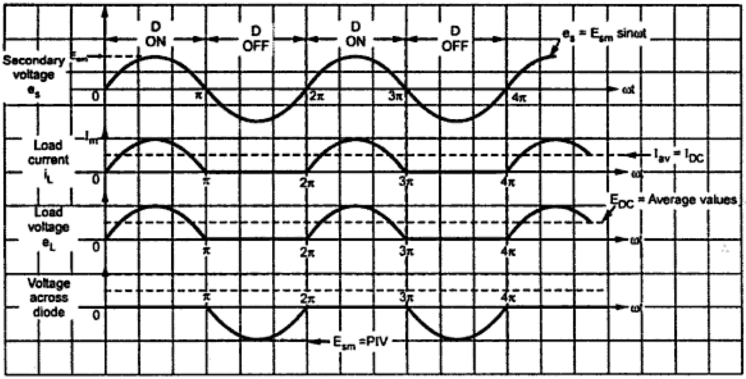


Fig. 7.42 Load current and load voltage waveforms for half wave rectifier

7.24.2 Average D.C. Load Current (I_{DC})

The average or d.c. value of alternating current is obtained by integration.

For finding out the average value of an alternating waveform, we have to determine the area under the curve over one complete cycle i.e. from 0 to 2π and then dividing it by the base i.e. 2π .

Mathematically, current waveform can be described as,

$$i_L = I_m \sin \omega t \qquad \text{for } 0 \le \omega t \le \pi$$

$$i_L = 0 \qquad \text{for } \pi \le \omega t \le 2\pi$$

where I_m = Peak value of load current

$$\therefore \quad I_{DC} = \frac{1}{2\pi}\int_0^{2\pi} i_L \, d(\omega t) = \frac{1}{2\pi}\int_0^{2\pi} I_m \sin(\omega t)\, d(\omega t)$$

As no current flows during negative half cycle of a.c. input voltage, i.e. between $\omega t = \pi$ to $\omega t = 2\pi$, we change the limits of integration.

$$\therefore \quad I_{DC} = \frac{1}{2\pi}\int_0^{\pi} I_m \sin(\omega t)\, d(\omega t) = \frac{I_m}{2\pi}\left[-\cos(\omega t)\right]_0^{\pi}$$

$$= -\frac{I_m}{2\pi}\left[\cos(\pi) - \cos(0)\right] = -\frac{I_m}{2\pi}[-1-1] = \frac{I_m}{\pi}$$

$$= I_m \sqrt{\frac{1}{2\pi}\int_0^{\pi}\frac{[1-\cos(2\omega t)]\,d(\omega t)}{2}} = I_m \sqrt{\frac{1}{2\pi}\left\{\frac{\omega t}{2}-\frac{\sin(2\omega t)}{4}\right\}_0^{\pi}}$$

$$= I_m \sqrt{\frac{1}{2\pi}\left(\frac{\pi}{2}\right)} \qquad \text{as } \sin(2\pi) = \sin(0) = 0$$

$$= \frac{I_m}{2}$$

$$\therefore \qquad \boxed{I_{RMS} = \frac{I_m}{2}}$$

Note : Students must remember that this R.M.S. value is for **half wave** rectified waveform hence it is $I_m/2$. For full sine wave it is $I_m/\sqrt{2}$, which is derived later.

7.24.5 D.C. Power Output (P_{DC})

The d.c. power output can be obtained as,

$$P_{DC} = E_{DC}\, I_{DC} = I_{DC}^2 R_L$$

$$\text{D.C. Power output} = I_{DC}^2 R_L = \left[\frac{I_m}{\pi}\right]^2 R_L = \frac{I_m^2}{\pi^2} R_L$$

$$\therefore \qquad P_{DC} = \frac{I_m^2}{\pi^2} R_L$$

where
$$I_m = \frac{E_{sm}}{R_f + R_L + R_s}$$

$$\therefore \qquad \boxed{P_{DC} = \frac{E_{sm}^2 R_L}{\pi^2 [R_f + R_L + R_s]^2}}$$

7.24.6 A.C. Power Input (P_{AC})

The power input taken from the secondary of transformer is the power supplied to three resistances namely load resistance R_L , the diode resistance R_f and winding resistance R_s. The a.c. power is given by,

$$\boxed{P_{AC} = I_{RMS}^2 [R_L + R_f + R_s]}$$

but
$$I_{RMS} = \frac{I_m}{2} \qquad \text{for half wave,}$$

$$\therefore \qquad \boxed{P_{AC} = \frac{I_m^2}{4} [R_L + R_f + R_s]}$$

7.24.9 Load Current

The load current i_L which is composed of a.c. and d.c. components can be expressed using Fourier series as,

$$i_L = I_m\left[\frac{1}{\pi}+\frac{1}{2}\sin\omega t-\frac{2}{3\pi}\cos 2\omega t-\frac{2}{15\pi}\cos 4\omega t \; ...\right]$$

This expression shows that the current may be considered to be the sum of an infinite number of current components, according to Fourier series.

The first term of the series is the average or d.c. value of the load current. The second term is a varying component having frequency same as that of a.c. supply voltage. This is called fundamental component of the current having frequency same as the supply. The third term is again a varying component having frequency twice the frequency of supply voltage. This is called second harmonic component. Similarly all the other terms represent the a.c. components and are called harmonics.

Thus ripple in the output is due to the fundamental component alongwith the various harmonic components. And the average value of the total pulsating d.c. is the d.c. value of the load current, given by the constant term in the series, I_m/π.

7.24.10 Peak Inverse Voltage (PIV)

The **Peak Inverse Voltage** is the peak voltage across the diode in the reverse direction i.e. when the diode is reverse biased. In half wave rectifier, the load current is ideally zero when the diode is reverse biased and hence the maximum value of the voltage that can exist across the diode is nothing but E_{sm}. This is shown in the Fig. 7.43.

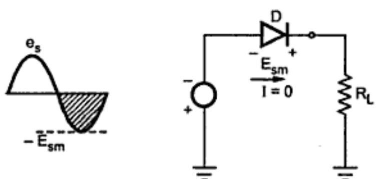


Fig. 7.43 PIV rating of diode

Thus PIV occurs at the peak of each negative half cycle of the input, when diode is reverse biased and not conducting.

$$\therefore \quad \boxed{\text{PIV of diode} = E_{sm} = \text{Maximum value of secondary voltage} = \pi\, E_{DC}\big|_{I_{DC}=0}}$$

This is called **PIV rating** of a diode. So diode must be selected based on this PIV rating and the circuit specifications.

7.24.11 Transformer Utilization Factor (T.U.F.)

The factor which indicates how much is the utilization of the transformer in the circuit is called Transformer Utilization Factor (T.U.F.)

Definition :

The T.U.F. is defined as the ratio of d.c. power delivered to the load to the a.c power rating of the transformer. While calculating the a.c. power rating, it is necessary to consider r.m.s. value of a.c. voltage and current.

The T.U.F. for half wave rectifier can be obtained as,

$$\text{A.C. power rating of transformer} = E_{RMS}\, I_{RMS}$$

$$= \frac{E_{sm}}{\sqrt{2}} \cdot \frac{I_m}{2} = \frac{E_{sm}\, I_m}{2\sqrt{2}}$$

Remember that the secondary voltage is purely sinusoidal hence its r.m.s. value is $1/\sqrt{2}$ times maximum while the current is half sinusoidal hence its r.m.s. value is 1/2 of the maximum, as derived earlier.

$$\text{D.C. power delivered to the load} = I_{DC}^2\, R_L$$

$$= \left(\frac{I_m}{\pi}\right)^2 R_L$$

$$\therefore \quad \boxed{\text{T.U.F.} = \frac{\text{D.C. Power delivered to the load}}{\text{A.C. Power rating of the transformer}}}$$

$$\therefore \quad \text{T.U.F.} = \frac{\left(\frac{I_m}{\pi}\right)^2 R_L}{\left(\frac{E_{sm} I_m}{2\sqrt{2}}\right)}$$

Neglecting the drop across R_f and R_s we can write,

$$E_{sm} = I_m R_L$$

$$\therefore \quad \text{T.U.F} = \frac{I_m^2}{\pi^2} \cdot \frac{R_L \cdot 2\sqrt{2}}{I_m^2\, R_L} = \frac{2\sqrt{2}}{\pi^2} = \mathbf{0.287}$$

Key Point : *The value of T.U.F. is low which shows that in half wave circuit, the transformer is not fully utilized.*

7.24.12 Voltage Regulation

The secondary voltage should not change with respect to the load current. The voltage regulation is the factor which tells us about the change in the d.c. output voltage as load changes from no load to full load condition.

If $(V_{dc})_{NL}$ = D.C. voltage on no load

$(V_{dc})_{FL}$ = D.C. voltage on full load

then voltage regulation is defined as,

$$\boxed{\text{Voltage regulation} = \frac{(V_{dc})_{NL} - (V_{dc})_{FL}}{(V_{dc})_{FL}}} \quad \text{... (1)}$$

Key Point: *Less the value of voltage regulation, better is the performance of rectifier circuit.*

For a half wave circuit,

$$(V_{dc})_{NL} = \frac{E_{sm}}{\pi}$$

While $$(V_{dc})_{FL} = I_{DC}\,R_L = \frac{I_m}{\pi}\,R_L = \frac{E_{sm}}{\pi[R_f + R_s + R_L]} \times R_L$$

$$\therefore \quad \% R = \frac{\frac{E_{sm}}{\pi} - \frac{E_{sm}}{\pi}\cdot\frac{R_L}{[R_f + R_s + R_L]}}{\frac{E_{sm}}{\pi}\cdot\frac{R_L}{R_f + R_s + R_L}} \times 100$$

$$= \frac{1 - \frac{R_L}{R_f + R_s + R_L}}{\frac{R_L}{R_f + R_s + R_L}} \times 100 = \frac{R_f + R_s}{R_L} \times 100$$

Neglecting winding resistance,

$$\boxed{\% R \approx \frac{R_f}{R_L} \times 100}$$

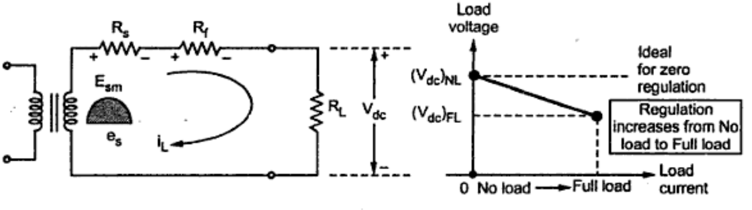
where R_f = Diode forward resistance

Key Point: *Ideally the load regulation is zero as ideal value of R_f is zero in forward biased condition.*

7.24.12.1 Regulation Characteristics

Consider the equivalent circuit of the half wave rectifier, for positive half cycle of the transformer secondary voltage as shown in the Fig. 7.44 (a).

As load current increases (R_L decreases), the drop across R_s and R_f goes on increasing, but the transformer, secondary voltage remains same. Hence the d.c. output voltage decreases. Hence load voltage decreases as load changes from no load to full load. To keep the drop across R_s and R_f minimum, values of R_s and R_f must be as small as possible. The graph of load voltage against load current is called **regulation characteristics** which is drooping in nature as shown in the Fig. 7.44 (b).



(a) Equivalent circuit (b) Regulation characteristics

Fig. 7.44

7.24.13 Disadvantages of Half Wave Rectifier Circuit

The various disadvantages of the half wave rectifier circuit are,

1. The ripple factor of half wave rectifier circuit is 1.21, which is quite high. The output contains lot of varying components.
2. The maximum theoretical rectification efficiency is found to be 40 %. The practical value will be less than this. This indicates that half wave rectifier circuit is quite inefficient.
3. The circuit has low transformer utilization factor, showing that the transformer is not fully utilized.
4. The d.c. current is flowing through the secondary winding of the transformer which may cause d.c. saturation of the core of the transformer. To minimize the saturation, transformer size have to be increased accordingly. This increases the cost.

Because of all these disadvantages, the half wave rectifier circuit is normally not used as a power rectifier circuit.

7.24.14 Effect of Barrier Potential

Consider a half wave rectifier shown in the Fig. 7.45.

Example 7.14 : *a) Assuming ideal diode, calculate the d.c. output voltage for the network shown in the Fig. 7.47.*

b) Repeat part (a) if the ideal diode is replaced by a silicon diode, having a cut-in voltage of 0.7 V. Neglect diode forward resistance.

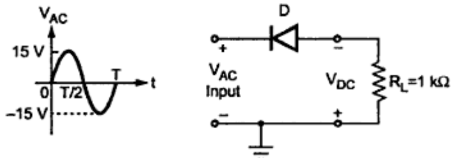


Fig. 7.47

Solution : In the circuit of the Fig. 7.47 (a), the diode will be forward biased during negative half cycle of a.c. input voltage, and d.c. output voltage will be negative w.r.t. common ground terminal, as shown.

a) For an ideal diode, cut-in voltage $V_\gamma = 0$, $R_f = 0$

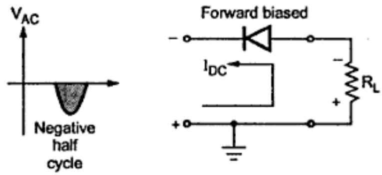


Fig. 7.47 (a)

$$\text{D.C. output voltage} = \frac{-\text{ Maximum value of a.c. input voltage}}{\pi}$$

$$= -\frac{15}{\pi} = \mathbf{-4.77\ V}$$

Negative sign indicates that voltage is negative w.r.t. ground.

b) For a silicon diode, $V_\gamma = 0.7$ V, R_f is assumed to be zero.

$$\therefore \quad \text{D.C. output voltage} = \frac{-[\text{Maximum A.C. voltage} - V_\gamma]}{\pi}$$

$$= \frac{-[15 - 0.7]}{\pi} = \mathbf{-4.55\ V}$$

Example 7.15 : *A half wave rectifier with R_L = 1 kΩ is given an input of 10 V peak from step down transformer. Calculate D.C. voltage and load current for ideal and silicon diode.*

Solution : Given values are R_L = 1 kΩ , V_m = 10 V peak

Case i) Ideal diode

Cut-in voltage V_γ = 0 V, R_f = 0 Ω

$$\therefore \qquad E_{DC} = \frac{V_m}{\pi} = \frac{10}{\pi} = 3.18 \text{ V}$$

$$\therefore \qquad I_{DC} = \frac{E_{DC}}{R_L} = \frac{3.18}{1 \times 10^3} = 3.18 \text{ mA}$$

Case ii) Silicon diode

$$\text{Cut-in voltage } V_\gamma = 0.7 \text{ V}$$

$$\therefore \qquad E_{DC} = \frac{V_m - V_\gamma}{\pi} = \frac{10-0.7}{\pi} = 2.96 \text{ V}$$

$$\therefore \qquad I_{DC} = \frac{E_{DC}}{R_L} = 2.96 \text{ mA}$$

Example 7.16 : *A voltage V = 300 cos 100t is applied to a half wave rectifier, with $R_L = 5\,k\Omega$. The rectifier may be represented by ideal diode in series with a resistance of 1kΩ. Calculate*

i) I_m ii) D.C. power iii) A.C. power iv) rectifier efficiency and v) ripple factor.

Solution : The diode circuit is as shown in the Fig. 7.48.

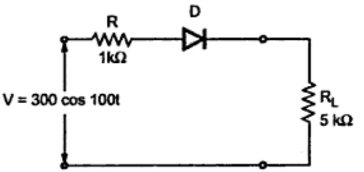


Fig. 7.48

The given voltage is V = 300 cos 100 t volts

$$\text{Compare with,} \quad E = E_{sm} \sin \omega t$$

$$E_{sm} = 300 \text{ volts}$$

$$R = 1 \text{ k}\Omega = \text{Resistance in series with diode}$$

$$R_L = 5 \text{ k}\Omega,\ R_s = R_f = 0\ \Omega$$

$$\text{i)} \qquad I_m = \frac{E_{sm}}{R + R_L + R_f + R_L} = \frac{300}{6 \times 10^3}$$

$$= 50 \text{ mA}$$

$$\text{ii)} \qquad I_{DC} = \frac{I_m}{\pi} = \frac{50}{\pi} = 15.9154 \text{ mA}$$

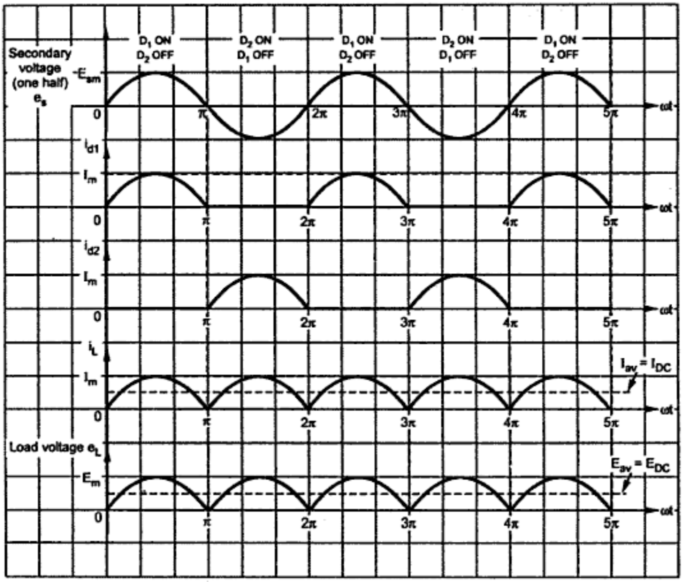


Fig. 7.52 Load current and voltage waveforms for full wave rectifier

Thus the full wave rectifier circuit essentially consists of two half wave rectifier circuits working independently (working in alternate half cycles of a.c.) of each other but feeding a common load. The output load current is still pulsating d.c. and not pure d.c.

7.25.2 Maximum Load Current

Let R_f = Forward resistance of diodes

R_s = Winding resistance of each half of secondary

R_L = Load resistance

e_s = Instantaneous a.c. voltage across each half of secondary

$$\therefore \quad \boxed{e_s = E_{sm} \sin \omega t}$$

$$\omega = 2\pi f$$

E_{sm} = Maximum value of a.c. input voltage across each half of secondary winding

Hence we can write the expression for the maximum value of the load current, looking at equivalent circuit shown in the Fig. 7.53.

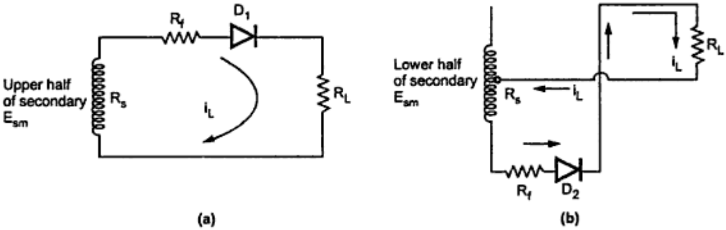


Fig. 7.53

$$\therefore \quad \boxed{I_m = \frac{E_{sm}}{R_s + R_f + R_L}}$$

where I_m = Maximum value of load current i_L

7.25.3 Average D.C. Load Current (I_{DC})

Consider one cycle of load current i_L from 0 to 2π to obtain the average value which is d.c. value of load current.

$$i_L = I_m \sin\omega t \qquad 0 \le \omega t \le \pi$$

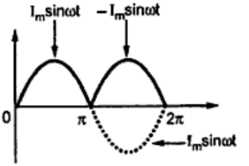


Fig. 7.54

But for π to 2π, the current i_L is again positive while sin ωt term is negative during π to 2π . Hence in the region π to 2π the positive i_L can be represented as negative of I_m sin(ω t).

$$\therefore \quad i_L = -I_m \sin \omega t \qquad \pi \le \omega t \le 2\pi$$

$$\therefore \quad I_{av} = I_{DC} = \frac{1}{2\pi}\int_0^{2\pi} i_L \, d(\omega t)$$

$$= \frac{1}{2\pi}\left[\int_0^{\pi} I_m \sin \omega t \, d(\omega t) + \int_{\pi}^{2\pi} -I_m \sin \omega t \, d(\omega t)\right]$$

$$= \frac{I_m}{2\pi}\left[\int_0^{\pi} \sin \omega t \, d(\omega t) - \int_{\pi}^{2\pi} \sin \omega t \, d(\omega t)\right]$$

$$= \frac{I_m}{2\pi}[(-\cos \omega t)_0^{\pi} - (-\cos \omega t)_{\pi}^{2\pi}]$$

$$= \frac{I_m}{2\pi}[-\cos \pi + \cos 0 + \cos 2\pi - \cos \pi]$$

but $\cos \pi = -1$

$$= \frac{I_m}{2\pi}[-(-1)+1+1-(-1)] = \frac{4I_m}{2\pi}$$

$\therefore$ $\boxed{I_{DC} = \frac{2I_m}{\pi}}$ for full wave rectifier

For half wave it is I_m/π and full wave rectifier is the combination of two half wave circuits acting alternately in two half cycles of input. Hence obviously the d.c. value for full wave circuit is $2 I_m/\pi$.

7.25.4 Average D.C. Load Voltage (E_{DC})

The d.c. load voltage is,

$$E_{DC} = I_{DC}R_L = \frac{2I_m R_L}{\pi}$$

Substituting value of I_m, $E_{DC} = \frac{2 E_{sm} R_L}{\pi [R_f + R_s + R_L]} = \frac{2 E_{sm}}{\pi \left[1 + \frac{R_f + R_s}{R_L}\right]}$

But as R_f and $R_s << R_L$ hence $\frac{R_f + R_s}{R_L} << 1$

$\therefore$ $\boxed{E_{DC} = \frac{2E_{sm}}{\pi}}$

7.25.5 R.M.S. Load Current (I_{RMS})

The R.M.S. value of current can be obtained as follows :

$$I_{RMS} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} i_L^2 d(\omega t)}$$

Since two half wave rectifier are similar in operation we can write,

$$I_{RMS} = \sqrt{\frac{2}{2\pi}\int_0^{\pi}[I_m \sin \omega t]^2 d(\omega t)}$$

$$= I_m\sqrt{\frac{1}{\pi}\int_0^{\pi}\left[\frac{1-\cos 2\omega t}{2}\right]d(\omega t)} \qquad \text{as } \sin^2 \omega t = \frac{1-\cos 2\omega t}{2}$$

$$\therefore \quad I_{RMS} = I_m\sqrt{\frac{1}{2\pi}\left[[\omega t]_0^{\pi} - \left(\frac{\sin 2\omega t}{2}\right)_0^{\pi}\right]} = I_m\sqrt{\frac{1}{2\pi}[\pi - 0]}$$

$$= I_m\sqrt{\frac{1}{2\pi}(\pi)} \qquad \text{as } \sin(2\pi) = \sin(0) = 0$$

$$\therefore \quad \boxed{I_{RMS} = \frac{I_m}{\sqrt{2}}}$$

7.25.6 D.C. Power Output (P_{DC})

$$\text{D.C. Power output} = E_{DC}I_{DC} = I_{DC}^2 R_L$$

$$\therefore \quad P_{DC} = I_{DC}^2 R_L = \left(\frac{2I_m}{\pi}\right)^2 R_L$$

$$\therefore \quad P_{DC} = \frac{4}{\pi^2} I_m^2 R_L$$

Substituting value of I_m we get,

$$\therefore \quad \boxed{P_{DC} = \frac{4}{\pi^2}\frac{E_{sm}^2}{(R_s + R_f + R_L)^2} \times R_L}$$

Key Point: *Instead of remembering this formula, students can use the expression* $E_{DC}I_{DC}$ *or* $I_{DC}^2 R_L$ *to calculate* P_{DC} *while solving the problems.*

7.25.7 A.C. Power Input (P_{AC})

The a.c. power input is given by,

$$\therefore \quad \boxed{P_{AC} = I_{RMS}^2 (R_f + R_s + R_L) = \left(\frac{I_m}{\sqrt{2}}\right)^2 (R_f + R_s + R_L)}$$

$$\therefore \quad P_{AC} = \frac{I_m^2 (R_f + R_s + R_L)}{2}$$

$$\text{Ripple factor} = \sqrt{\left[\frac{I_m/\sqrt{2}}{2I_m/\pi}\right]^2 - 1} = \sqrt{\frac{\pi^2}{8} - 1}$$

$$\therefore \quad \boxed{\text{Ripple factor} = \gamma = 0.48}$$

Key Point: *This indicates that the ripple contents in the output are 48 % of the d.c. component which is much less than that for the half wave circuit.*

7.25.10 Load Current (i_L)

The Fourier series for the load current is obtained by taking the sum of the series for the individual rectifier current. The two diodes conduct in alternate half cycles, i.e. there is a phase difference of π radians between two diode currents. Hence,

$$i_{d_1} = I_m\left[\frac{1}{\pi} + \frac{1}{2}\sin\omega t - \frac{2}{3\pi}\cos 2\omega t - \frac{2}{15\pi}\cos 4\omega t \ldots\right]$$

and $\quad i_{d_2} = i_{d_1}$ with ωt replaced by $(\omega t + \pi)$

$$\therefore \quad i_{d_2} = I_m\left[\frac{1}{\pi} + \frac{1}{2}\sin(\omega t+\pi) - \frac{2}{3\pi}\cos 2(\omega t+\pi) - \frac{2}{15\pi}\cos 4(\omega t+\pi) \ldots\right]$$

$$= I_m\left[\frac{1}{\pi} - \frac{1}{2}\sin\omega t - \frac{2}{3\pi}\cos(2\omega t+2\pi) - \frac{2}{15\pi}\cos(4\omega t+4\pi) \ldots\right]$$

$$= I_m\left[\frac{1}{\pi} - \frac{1}{2}\sin\omega t - \frac{2}{3\pi}\cos 2\omega t - \frac{2}{15\pi}\cos 4\omega t \ldots\right]$$

Then the Fourier series for the load current is,

$$i_L = i_{d_1} + i_{d_2} = I_m\left[\frac{2}{\pi} - \frac{4}{3\pi}\cos 2\omega t - \frac{4}{15\pi}\cos 4\omega t \cdots\right]$$

The first term in the above series represents the average or d.c. value, while the remaining terms "ripple". It is seen that the lowest frequency of the ripple is 2f, i.e. twice the supply frequency of a.c. supply. The lowest ripple frequency in the load current of the full wave connection, is double than that in the half wave connection.

As seen from Fig. 7.50 and Fig 7.51 the individual diode currents are flowing in opposite directions through the two halves of the secondary winding. Hence the net secondary current will be difference of individual diode currents.

Thus, $\quad \boxed{i_{sec} = i_{d_1} - i_{d_2}}$

The Fourier series of i_{sec} is obtained by the difference between the series of individual diode currents. Using above relations we can write,

$$i_{sec} = I_m \sin\omega t$$

$$\boxed{\% R = \frac{\frac{2E_{sm}}{\pi} - I_{DC}R_L}{I_{DC}R_L} \times 100}$$

Now $$I_m = \frac{E_{sm}}{R_f + R_L + R_s}$$

$\therefore$ $$E_{sm} = I_m(R_f + R_L + R_s)$$

and $$I_{DC} = \frac{2I_m}{\pi}$$

$\therefore$ $$\% R = \frac{\frac{2I_m}{\pi}[R_f + R_L + R_s] - \frac{2I_m}{\pi}R_L}{\frac{2I_m}{\pi}R_L} \times 100$$

$$= \frac{R_f + R_L + R_s - R_L}{R_L} \times 100$$

$\therefore$ $$\boxed{\% R = \frac{R_f + R_s}{R_L} \times 100}$$

Neglecting winding resistance R_s, the regulation can be expressed as,

$$\boxed{\% R \approx \frac{R_f}{R_L} \times 100}$$

where R_f = forward resistance of the diode.

The regulation characteristics is drooping, as discussed earlier in case of half wave rectifier as **output voltage decreases as load increases from no load to full load.**

7.25.14 Comparison of Full Wave and Half Wave Circuit

For comparison, we assume that the full wave and half wave circuits use identical diodes, identical load resistances and the voltage across half the secondary winding of transformer used in full wave circuit is the same as the voltage across the secondary winding of the transformer used in half wave circuit.

1. The d.c. load current in case of full wave circuit is twice to that in half wave circuit; similarly the D.C. load voltage in full wave circuit is twice that in half wave circuit.
2. The lowest ripple frequency in full wave circuit is twice that in half wave circuit. Now to remove ripple, the additional circuits called filter circuits are used along with rectifier circuits. But as the frequency is more in full wave, the capacitor values required in capacitance filter are much less hence smaller elements are sufficient in filter circuits used with full wave circuit to reduce ripple.

3. Because there is no net d.c. current through windings of the transformer used in full wave circuit, the losses are less as compared to losses in transformer used in half wave circuit.
4. The full wave connection gives d.c. power output four times as large, when compared with half wave connection.
5. The efficiency of rectification in a full wave connection is twice that for half wave connection.
6. The ripple factor is less for full wave, i.e. rectification is more nearly complete for full wave as compared to half wave.

Example 7.17 : *A full wave rectifier circuit is fed from a transformer having a center-tapped secondary winding. The r.m.s. voltage from either end of secondary to center tap is 30 V. If the diode forward resistance is 2 Ω and that of the half secondary is 8 Ω, for a load of 1 kΩ, calculate,*

a) Power delivered to load, *b) % Regulation at full load,*

c) Efficiency of rectification, *d) T.U.F. of secondary.*

Solution : Given : $E_s = 30\text{V},\ R_f = 2\,\Omega,\ R_s = 8\,\Omega,\ R_L = 1\,\text{k}\Omega$

$$E_s = E_{RMS} = 30\text{ V}$$

$$E_{sm} = E_s\sqrt{2} = 30\sqrt{2}\ \text{volt} = 42.426\text{ V}$$

$$I_m = \frac{E_{sm}}{R_f + R_L + R_s} = \frac{30\sqrt{2}}{2+1000+8}\ \text{A}$$

$$= 42\text{ mA}$$

$$I_{DC} = \frac{2}{\pi} I_m = 26.74\text{ mA}$$

a) Power delivered to the load $= I_{DC}^2\, R_L = (26.74 \times 10^{-3})^2\ (1\,\text{k}\Omega)$

$$= \mathbf{0.715\ W}$$

b)

$$V_{DC}\text{, no load} = \frac{2}{\pi} E_{sm} = \frac{2}{\pi} \times 30\sqrt{2} = 27\text{ V}$$

$$V_{DC}\text{, full load} = I_{DC}\, R_L = (26.74\text{ mA})\ (1\text{ k}\Omega)$$

$$= 26.74\text{ V}$$

$$\%\ \text{Regulation} = \frac{V_{NL} - V_{FL}}{V_{FL}} \times 100 = \frac{27 - 26.74}{26.74} \times 100$$

$$= \mathbf{0.97\ \%}$$

c) Efficiency of rectification $= \dfrac{\text{D.C. output}}{\text{A.C. input}}$

$$= \frac{8}{\pi^2} \times \frac{1}{1 + \frac{R_f + R_s}{R_L}} = \frac{8}{\pi^2} \times \frac{1}{1 + \frac{(2+8)}{1000}}$$

$$= 0.802 \quad \text{i.e.} \quad 80.2\%$$

d) Transformer secondary rating $= E_{RMS}\ I_{RMS} = [30\ V]\left[\frac{42\ mA}{\sqrt{2}}\right]$

$$= 0.89\ W$$

$\therefore$ $$\text{T.U.F.} = \frac{\text{D.C. power output}}{\text{A.C. rating}}$$

$$= \frac{0.715}{0.89} = 0.802$$

Example 7.18 : *For the full wave rectifier circuit shown in the Fig. 7.56, V is a sinusoidal voltage. If the maximum allowable average d.c. current in each diode is 1 A, calculate the maximum allowable peak-to-peak value of V. Assume two diodes to be identical, and neglect diode resistance in forward direction.*

Solution : Given : F.W. rectifier with $R_L = 100\ \Omega$, A.C. input voltage is V.

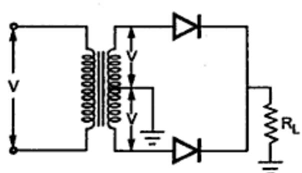


Fig. 7.56 (a)

Let V_m is maximum value or amplitude of sinusoidal voltage, across each half of the secondary winding.

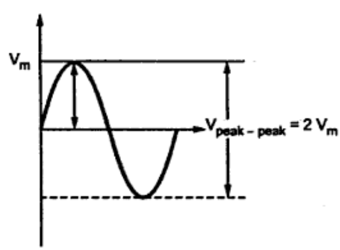


Fig. 7.56 (b)

$$\therefore \quad E_{DC}(\text{on load}) = I_{DC}R_L = 100\times10^{-3}\times84.0317$$

$$= \mathbf{8.4031\ V}$$

iii) $$\%\ \text{Regulation} = \frac{E_{DC}(NL) - E_{DC}(\text{on load})}{E_{DC}(\text{on load})}\times100$$

$$= \frac{9.0031 - 8.4031}{8.4031}\times100$$

$$= \mathbf{7.14\ \%}$$

Example 7.20 : *What is the necessary A.C. input power from the transformer secondary used in a half wave rectifier to deliver 500 W of D.C. power to the load ? What would be the A.C. input power for the same load in a full-wave rectifier ?*

Solution : P_{DC} = 500 W, Half wave rectifier

For half wave rectifier, $\%\ \eta = 40.6\%$ ... (Assuming maximum)

$$\therefore \quad 40.6 = \frac{P_{DC}}{P_{AC}}\times 100$$

$$\therefore \quad 40.6 = \frac{500}{P_{AC}}\times 100$$

$$\therefore \quad P_{AC} = \mathbf{1231.527\ W}$$

For the same load, with full wave rectifier the maximum rectifier efficiency is 81.2%.

$$\therefore \quad 81.2 = \frac{500}{P_{AC}}\times 100$$

$$\therefore \quad P_{AC} = \mathbf{615.76355\ W}$$

7.26 Bridge Rectifier

The bridge rectifier circuits are mainly used as,

a) a power rectifier circuit for converting a.c. power to d.c. power, and

b) a rectifying system in rectifier type a.c. meters, such as a.c. voltmeter, in which the a.c. voltage under measurement is first converted into d.c. and measured with conventional meter. In this system, the rectifying elements are either copper oxide type or selenium type.

The basic bridge rectifier circuit is shown in Fig. 7.57.

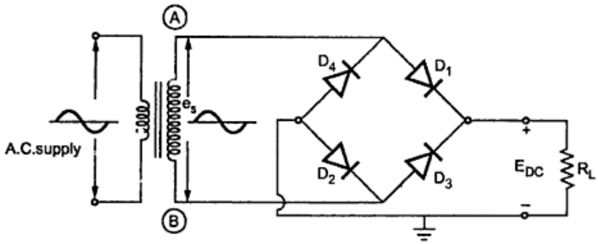


Fig. 7.57 Bridge rectifier circuit

The bridge rectifier circuit is essentially a full wave rectifier circuit, using four diodes, forming the four arms of an electrical bridge. To one diagonal of the bridge, the a.c. voltage is applied through a transformer if necessary, and the rectified d.c. voltage is taken from the other diagonal of the bridge. The main advantage of this circuit is that it does not require a center tap on the secondary winding of the transformer. Hence wherever possible, a.c. voltage can be directly applied to the bridge.

7.26.1 Operation of the Circuit

Consider the positive half of a.c. input voltage. The point A of secondary becomes positive. The diodes D_1 and D_2 will be forward biased, while D_3 and D_4 reverse biased. The two diodes D_1 and D_2 conduct in series with the load and the current flows as shown in Fig. 7.58.

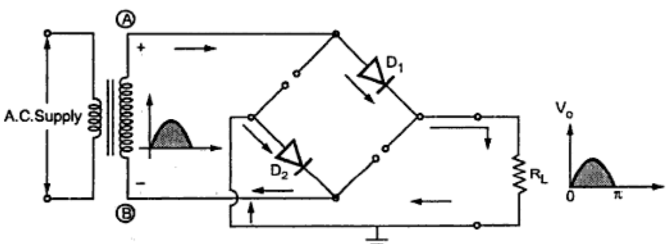


Fig. 7.58 Current flow during positive half cycle

In the next half cycle, when the polarity of a.c. voltage reverses hence point B becomes positive diodes D_3 and D_4 are forward biased, while D_1 and D_2 reverse biased. Now the diodes D_3 and D_4 conduct is series with the load and the current flows as shown in Fig. 7.59.

Solution : The given values are,

$$R_f = 0.1\ \Omega,\ I_{DC} = 10\ A,\ R_s = 0\ \Omega,\ E_s(R.M.S.) = 30\ V$$

Now $$E_{sm} = E_{sm}(R.M.S.) \times \sqrt{2} = \sqrt{2} \times 30$$

$$= 42.4264\ V$$

$$I_{DC} = \frac{2I_m}{\pi}$$

$\therefore$ $$I_m = \frac{\pi \times I_{DC}}{2} = \frac{\pi \times 10}{2} = 15.7079\ A$$

Now $$I_m = \frac{E_{sm}}{2R_f + R_s + R_L}$$

$\therefore$ $$15.7079 = \frac{42.4264}{2 \times 0.1 + R_L}$$

$\therefore$ $$R_L + 0.2 = 2.7$$

$\therefore$ $$R_L = \mathbf{2.5\ \Omega}$$

Now $$P_{DC} = I_{DC}^2 R_L = (10^2) \times 2.5 = 250\ W$$

$$P_{AC} = I_{RMS}^2 (2R_f + R_s + R_L)$$

and $$I_{RMS} = \frac{I_m}{\sqrt{2}} = \frac{15.7079}{\sqrt{2}} = 11.1071\ A$$

$\therefore$ $$P_{AC} = (11.1071)^2\ [2 \times 0.1 + 2.5] = 333.092\ W$$

$\therefore$ $$\%\ \eta = \frac{P_{DC}}{P_{AC}} \times 100$$

$$= \frac{250}{333.092} \times 100$$

$$= \mathbf{75.05\ \%}$$ ... Rectifier efficiency

Example 7.22 : *A 5 kΩ load is fed from a bridge rectifier connected across a tranformer secondary whose primary is connected to 460 V, 50 Hz supply. The ratio of number of primary turns to secondary turns is 2:1. Calculate d.c. load current, d.c. load voltage, ripple voltage and P.I.V. rating of diode.*

Solution : $R_L = 5\ k\Omega = 5 \times 10^3\ \Omega$, $N_1 : N_2$ is 2:1

$$E_p = 460 \text{ V} \quad \text{R.M.S. value}$$

$$\therefore \quad \frac{E_s}{E_p} = \frac{N_2}{N_1} = \frac{1}{2}$$

$$\therefore \quad E_s = \frac{1}{2} \times E_p = 230 \text{ V}$$

$$\therefore \quad E_{sm} = \sqrt{2} \times E_s$$

$$= 230 \times \sqrt{2} = 325.269 \text{ V.}$$

Now $$I_{DC} = \frac{2\, I_m}{\pi}$$ where $I_m = \frac{E_{sm}}{R_L}$ neglecting R_f

$$\therefore \quad I_{DC} = \frac{2\, E_{sm}}{\pi\, R_L}$$

$$= \frac{2 \times 325.269}{\pi \times 5 \times 10^3} = \mathbf{41.41 \text{ mA}}$$

D.C. load voltage $E_{DC} = I_{DC} \times R_L = 41.41 \times 10^{-3} \times 5 \times 10^3$

$$= \mathbf{207.072 \text{ V}}$$

Ripple voltage = Ripple factor $\times V_{DC}$

Ripple factor for bridge rectifier is 0.482.

$\therefore$ Ripple voltage = $0.482 \times 207.072 = \mathbf{99.8 \text{ V}}$

P.I.V. rating of each diode = E_{sm} for bridge rectifier = **325.27 V**

➡ **Example 7.23 :** *A full wave bridge rectifier is supplied from 230 V, 50 Hz and uses a transformer of turns ratio of 15 : 1. It uses load resistance of* 50 Ω. *Calculate load voltage and ripple voltage. Assume ideal diode and transformer. Assume standard value of ripple factor for full wave rectifier.*

Solution :

$$E_p(\text{r.m.s.}) = 230 \text{ V}, \frac{N_2}{N_1} = \frac{1}{15}, R_L = 50\ \Omega$$

$$R_f = R_s = 0\ \Omega \quad \text{as ideal}$$

Now $$\frac{E_p\ (\text{r.m.s.})}{E_s\ (\text{r.m.s.})} = \frac{N_1}{N_2}$$

$$\therefore \quad E_s(\text{r.m.s.}) = \frac{N_2}{N_1} \times E_p(\text{r.m.s.})$$

$$= \frac{1}{15} \times 230 = 15.333 \text{ V}$$

$$\therefore \quad E_{sm} = \sqrt{2} \quad E_s(\text{r.m.s.}) = 21.684 \text{ V}$$

$$\therefore \quad I_m = \frac{E_{sm}}{R_s + 2 \ R_f + R_L}$$

$$= \frac{21.684}{50} = 0.4336 \quad \text{A}$$

$$\therefore \quad I_{DC} = \frac{2 \ I_m}{\pi} = \frac{2 \times 0.4336}{\pi} = 0.276 \ \text{A}$$

$$\therefore \quad E_{DC} = \text{Load voltage} = I_{DC} \ R_L = 0.276 \times 50 = \mathbf{13.8} \ \text{V}$$

$$\text{Ripple factor} = 0.482 \qquad \text{... For full wave rectifier}$$

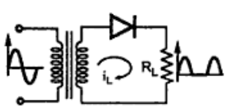
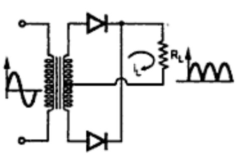
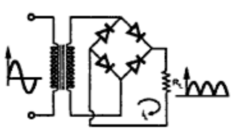
$$\text{Ripple factor} = \frac{\text{a.c. r.m.s. output}}{\text{d.c. output}}$$

$$= \frac{\text{ripple voltage}}{E_{DC}}$$

$$\therefore \quad 0.482 = \frac{\text{ripple voltage}}{13.8}$$

$$\therefore \quad \text{Ripple voltage} = 13.8 \times 0.482 = \mathbf{6.6516} \ \text{V}$$

7.27 Comparison of Rectifier Circuits

Circuit Diagrams		
Half Wave	**Full Wave**	**Bridge**
		

Sr. No.	Parameter	Half Wave	Full Wave	Bridge
1.	Number of diodes	1	2	4
2.	Average D.C. current (I_{DC})	$\frac{I_m}{\pi}$	$\frac{2I_m}{\pi}$	$\frac{2I_m}{\pi}$

$$\therefore \quad \% \eta = \frac{P_{DC}}{P_{ac}} \times 100 = \mathbf{39.7346\ \%}$$

$$\% R = \frac{V_{NL} - V_{FL}}{V_{FL}} \times 100 = \frac{\frac{E_{sm}}{\pi} - E_{DC}}{E_{DC}} \times 100$$

$$= \frac{\frac{155.5635}{\pi} - 48.5464}{48.5464} \times 100 = \mathbf{2\%}$$

Example 7.25 : *Show that maximum D.C. output power $P_{DC} = V_{DC} \times I_{DC}$ in a half wave single phase circuit occur when the load resistance equals diode resistance R_f.*

[JNTU, Nov./Dec.-2004 (Set-3)]

Solution : For a half wave rectifier,

$$I_m = \frac{E_{sm}}{R_f + R_L}$$

and $$I_{DC} = \frac{I_m}{\pi} = \frac{E_{sm}}{\pi [R_f + R_L]}$$

and $$V_{DC} = I_{DC} \times R_L$$

$$\therefore \quad P_{DC} = V_{DC} \times I_{DC} = I_{DC}^2 R_L$$

$$= \frac{E_{sm}^2 R_L}{\pi^2 [R_f + R_L]^2}$$

For this power to be maximum,

$$\frac{d P_{DC}}{d R_L} = 0$$

$$\frac{d}{d R_L}\left\{\frac{E_{sm}^2 R_L}{\pi^2 [R_f + R_L]^2}\right\} = \frac{E_{sm}^2}{\pi^2}\left\{\frac{(R_f + R_L)^2 - R_L \times 2[R_f + R_L]}{(R_f + R_L)^4}\right\} = 0$$

$$\therefore \quad [R_f + R_L]^2 - 2R_L [R_f + R_L] = 0$$

$$\therefore \quad R_f^2 + 2R_f R_L + R_L^2 - 2R_f R_L - 2R_L^2 = 0$$

$$\therefore \quad R_f^2 - R_L^2 = 0$$

$$\therefore \quad R_L^2 = R_f^2 \quad \text{i.e.} \quad \boxed{R_L = R_f}$$

Thus the power output is maximum if $R_L = R_f$.

$\therefore$ VA rating of transformer $= I_{rms} E_{rms} = 22.2143 \times 0.2221$

$= \mathbf{4.9337\ VA}$

The waveforms are as shown in the Fig. 7.62 (b).

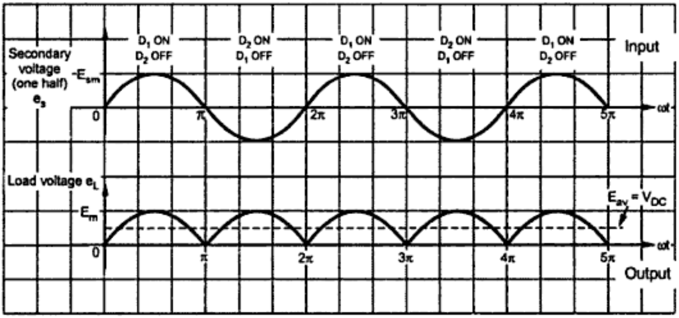


Fig. 7.62 (b)

Example 7.27 : *A 230 V, 60 Hz voltage is applied to the primary of a 5 : 1 step down transformer used in a bridge rectifier having a load of 900 Ω. If the diode resistance and secondary coil resistance together has a resistance of 100 Ω, determine,*

i) D.C. voltage across the load

ii) D.C current flowing through the load

iii) PIV of each diode iv) Ripple voltage and its frequency. **[JNTU, Nov.-2008, (Set-2)]**

Solution : $E_p(\text{r.m.s.}) = 230\ V,\ N_1 : N_2 = 5 : 1,\ R_L = 900\ \Omega,\ R_f + R_s = 100\ \Omega$

$$\frac{E_p(\text{r.m.s.})}{E_s(\text{r.m.s.})} = \frac{N_1}{N_2} \quad \text{i.e.} \quad \frac{230}{E_s(\text{r.m.s})} = \frac{5}{1}$$

$$\therefore \quad E_s(\text{r.m.s.}) = \frac{230}{5} = 46\ V$$

$$\therefore \quad E_{sm} = \sqrt{2} \times E_s(\text{r.m.s.}) = 65.0538\ V$$

$$\therefore \quad I_{sm} = \frac{E_{sm}}{R_L + R_f + R_s} = \frac{65.0538}{900 + 100} = 65.0538\ mA$$

$$I_{DC} = \frac{2I_m}{\pi} = \frac{2 \times 65.0538}{\pi} = 41.4145\ mA$$

i) $$E_{DC} = I_{DC} R_L = 41.4145 \times 10^{-3} \times 900 = \mathbf{37.273\ V}$$

$$P_{AC} = I_{RMS}^2 [R_L] \quad \text{where} \quad I_{RMS} = \frac{I_m}{2} \quad \text{as half wave}$$

$$\therefore \quad P_{AC} = \left(\frac{I_m}{2}\right)^2 \times R_L = \left(\frac{0.8132}{2}\right)^2 \times 200 = 33.0647 \text{ W}$$

$$\therefore \quad \% \eta = \frac{P_{DC}}{P_{AC}} \times 100 = \frac{13.3954}{33.0647} \times 100 = \mathbf{40.512\ \%}$$

Ripple frequency = f = **50 Hz**

Example 7.29 : *If the reverse saturation current is 10 μA then calculate the forward current for the voltages of 0.1, 0.2 and 0.3 V, for a silicon diode.*

[JNTU, Nov.-2008 (Set-3)]

Solution : For a silicon diode, $\eta = 2$, $I_0 = 10\ \mu A$, $V_T = 26$ mV

$$I = I_0[e^{V/\eta V_T} - 1]$$

i) $$I = 10 \times 10^{-6}[e^{0.1/2 \times 26 \times 10^{-3}} - 1] = \mathbf{58.4197\ \mu A}$$

ii) $$I = 10 \times 10^{-6}[e^{0.2/2 \times 26 \times 10^{-3}} - 1] = \mathbf{458.1266\ \mu A}$$

iii) $$I = 10 \times 10^{-6}[e^{0.3/2 \times 26 \times 10^{-3}} - 1] = \mathbf{3.1929\ mA}$$

Example 7.30 : *In the reverse bias region the saturation current of a silicon diode is about 0.1 μA at 20 °C. Determine its value if the temperature is increased to 40 °C.*

[JNTU, Nov.-2007(Set-2)]

Solution : $$I_{01} = 0.1\ \mu A, \ T_1 = 20\ °C, \ T_2 = 40\ °C$$

$$\therefore \quad I_{02} = I_{01}(2^{\Delta T/10}) = 0.1 \times 2^{\frac{(40-20)}{10}}$$

$$= \mathbf{0.4\ \mu A}$$

Example 7.31 : *Compute the conductivity of a silicon semiconductor, which is doped with accepter impurity to a density of 10^{22} atoms/m^3. Given that = $n_i = 1.4 \times 10^{16}/m^3$, $\mu_n = 0.145\ m^2/V\text{-}s$, $\mu_p = 0.05\ m^2/V\text{-}s$.* **[JNTU, Nov.-2007 (Set-4)]**

Solution : As the impurity is accepter, it forms a p-type material.

$$\therefore \quad N_A = 10^{22}/m^3 = p_p$$

Now $$p_p \times n_p = n_i^2 \quad \text{i.e.} \quad 10^{22} \times n_p = (1.4 \times 10^{16})^2$$

$$\therefore \quad n_p = 1.96 \times 10^{10}/m^3$$

$$\therefore \quad \sigma_p = (n_p\mu_n + p_p\mu_p)\, e = [1.96 \times 10^{10} \times 0.145 + 10^{22} \times 0.05] \times 1.6 \times 10^{-19}$$

$$= \mathbf{80\ (\Omega\text{-}m)^{-1}}$$

26. *State and explain the law of mass action.*
27. *Explain the carrier concentrations in extrinsic semiconductors.*
28. *Find the concentration of holes and electrons in a n-type Silicon at 300 °K assuming resistivity of 0.02 Ω-cm.*
    Assume $\mu_n = 1500\ \text{cm}^2/V\text{-}s$ *and* $n_i = 1.45 \times 10^{10}$ *per* cm^3
    (Ans. : 2.08×10^{23} per m^3, 1.009×10^9 per m^3)
29. *Find the concentration of the holes and electrons in p-type germanium at 300 °K if its conductivity is 100* $(\Omega\text{ - cm})^{-1}$
    Assume : n_i *at 300 °K* $= 2.5 \times 10^{13}$ *per* cm^3, $\mu_p = 1800\ \text{cm}^2/V\text{-}s.$
    Atom per $\text{cm}^3 = 4.4 \times 10^{22}$ **(Ans. : 3.468×10^{17} per cm^3, 1.802×10^9 per cm^3)**
30. *Explain the equation of charge neutrality.*
31. *A sample of germanium is doped to the extent of* 10^{14} *donor atoms per* cm^3 *and acceptor atoms of* 7×10^{13} *per* cm^3. *At the same temperature the resistivity of pure germanium is observed as 60 Ω-cm. If the applied electric field is 2 V/cm, calculate the total current density.*
    Hint : Use pure resistivity to get n_i *and then* $N_D + p = N_A + n$.
    Use law of mass action to get n and p.
    Use $J = [n\,\mu_n + p\,\mu_p\,] \times q\,E$.
    Assume $\mu_n = 3800\ cm^2/V\text{-}s$ *and* $\mu_p = 1800\ cm^2/V\text{-}s$ **(Ans. : 52.32 mA/cm²)**
32. *What is ohmic contact ? What are the important properties of ohmic contact ?*
33. *Explain the operation of forward biased diode.*
34. *Explain the effect of the barrier potential in a forward biased diode.*
35. *Explain the operation of reverse biased diode.*
36. *Explain the breakdown mechanisms in a reverse biased diode.*
37. *Explain the current components in a p-n junction diode.*
38. *Explain the V-I characteristics of a diode.*
39. *Define forward static and dynamic resistances of diode.*
40. *Draw and compare V-I characteristics of typical Ge and Si diodes.*
41. *State the diode current equation explaining the meaning of each term involved in it.*
42. *The current of germanium diode is 100 μA at a voltage of – 1 V, at room temperature. Determine the magnitude of the current for the voltages of ± 0.2 V at room temperature.*
    (Ans. : 291 mA, 99.95 μA)
43. *A silicon diode has a reverse saturation current of 60 nA. Calculate the voltage at which 1% of the rated current will flow through the diode, at room temperature if diode is rated for 1A.*
    (Ans. : 0.6252 V)
44. *What is rectifier ?*
45. *What is a rectifier ? Show that a PN diode acts as a rectifier.* **[Dec. - 2003 (Set-1)]**
46. *Which are the important characteristics of a rectifier circuit ?*
47. *Explain why diode can be used as a rectifier ?*

48. *Draw the circuit diagram of half wave rectifier and explain its operation with the help of waveforms.* **[Dec. - 2003 (Set -)]**
49. *Draw the circuit diagram of a full rectifier. Explain the operation of the circuit with relevant waveform.* **[May - 2004 (Set-1)]**
50. *Derive the expressions for the following parameters of the half wave rectifier circuit :*
    a. Average d.c. current (I_{DC}) *b. Average d.c. voltage (E_{DC})*
    c. R.M.S. value of current (I_{RMS}) *d. D.C. power output (P_{DC})*
    e. A.C. power input (P_{AC}) *f. Rectifier efficiency (η)*
    g. Ripple factor (γ)
51. *Explain the following terms.* **[Dec. - 2003 (Set-2); May - 2003 (Set-4); May - 2004 (Set-2)]**
    i) Ripple factor ii) Peak inverse voltage
    iii) Efficiency iv) Transformer utilization factor
    v) Form factor vi) Peak factor
52. *List the following parameters for half wave rectifier in terms of maximum current :*
    a. I_{DC} *b.* I_{RMS} *c.* E_{DC} *d. Ripple factor.*
53. *What is ripple factor ? What is the requirement of a rectifier in terms of ripple factor ? How is it achieved ?*
54. *Define the following terms*
    i) Transformer utilization factor
    ii) Ripple factor of HWR with resistive load and derive the expression for the same.
    [May-2003 (Set-2)]
55. *What are the disadvantages of a half wave rectifier circuit ?*
56. *Prove that the voltage regulation for a half wave rectifier is $[(R_s + R_f)/R_L] \times 100$.*
57. *Describe the action of a full wave bridge rectifier with the aid of input-output waveforms.*
    [May - 2003 (Set-4)]
58. *Prove that the ripple factor for the full wave rectifier circuit is 0.48.*
59. *Compare the full wave rectifier with half wave rectifier circuit.*
60. *A full wave rectifier circuit is fed from a transformer having a center-tapped secondary winding. The r.m.s. voltage from either end of secondary to center tap is 20 V. If the diode forward resistance is 3 Ω and that of the half secondary is 5 Ω, for a load of 1 kΩ, calculate*
    a) Power delivered to load,
    b) % Regulation at full load,
    c) Efficiency at full load,
    d) T.U.F. of secondary. **(Ans. : a) P_{dc} = 0.3191 W, b) % Regulation = 0.7469 %, c) Efficiency of full load = 0.8041, d) T.U.F. = 0.804)**
61. *A full wave single phase rectifier makes use of two diodes, the internal forward resistance of each of which can be considered to be constant and equal to 30 Ω. The load resistance is 1kΩ. The transformer secondary voltage is 200-0-200 V(r.m.s.).*

Calculate :

i) The D.C. load current *ii) The D.C. output voltage*

iii) PIV of diode *iv) R.M.S. voltage across each diode.*

(Ans. : (i) I_{dc} = 175 mA, (ii) V_{dc} = 175 V, (iii) PIV = 565.60 V, (iv) V_{rms} = 200 V)

62. *A full wave rectifier uses diodes with forward resistance of 10 ohms each. The secondary of the transformer is center tapped with output voltage 12-0-12 volts (r.m.s.), and has a resistance of 5 ohm for each half winding.*

    i) No load d.c. voltage *ii) D.C. voltage when the load current is 100 mA*

    iii) % Regulation at 100 mA load current *iv) Peak inverse voltage &*

    v) Ripple factor. **(Ans. : (i) V_{NLdc} = 10.8 V, (ii) V_{dcFL} = 9.8 V, (iii) % Reg. = 10.2 %, (iv) PIV = 33.94 V, (v) R_f = 0.482)**

63. *In a bridge rectifier, by mistake, the input and output terminals are interchanged. Explain what would happen.*
64. *State the advantages and disadvantages of bridge rectifier circuit.*
65. *What are the advantages of bridge rectifier over center tapped rectifiers ?* **[May-2003 (Set-3)]**

❑❑❑

8 Transistors

8.1 Introduction to BJT

In 1951, William Schockley invented the first junction transistor, a semiconductor device that can amplify electronic signals such as radio and television signals. It is essential ingredient of every electronic circuit; from the simplest amplifier or oscillator to the most elaborate digital computer. Thus a proper understanding of transistor is very important.

Before transistor, the amlification was achieved by using vacuum tubes as an amplifier. Now a days vacuum tubes are replaced by transistors because of following advantages of transistors.

- Low operating voltage
- Higher efficiency
- Small size and ruggedness and
- Does not require any filament power

Transistor is a three terminal device : Base, emitter and collector, can be operated in three configurations common base, common emitter and common collector. According to configuration it can be used for voltage as well as current amplification. The input signal of a small amplitude is applied at the base to get the magnified output signal at the collector. Thus provides an amplification of the signal. The amplification in the transistor is achieved by passing input current signal from a region of low resistance to a region of high resistance. This concept of transfer of resistance has given the name TRANSfer-resISTOR (TRANSISTOR).

There are two types of transistors : **Unipolar junction transistor** and **bipolar junction transistor**. In unipolar transistor the current conduction is only due to one type of carriers, majority carriers. The current conduction in bipolar transistor is because of both the types of charge carriers, holes and electrons. Hence this is called **Bipolar junction transistor**, hereafter referred to as **BJT**.

The BJTs are of two types :

- n-p-n type
- p-n-p type

8.1.1 Structure of Bipolar Junction Transistor (BJT)

When a transistor is formed by sandwiching a single p-region between two n-regions, as shown in the Fig. 8.1 (a), it is an n-p-n type transistor. The p-n-p type transistor has a single n region between two p-regions, as shown in Fig. 8.1(b).

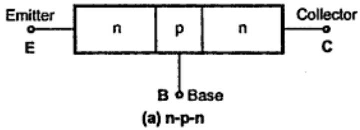


Fig. 8.1 Bipolar transistor construction

The middle region of each transistor type is called the base of the transistor. This region is very thin and lightly doped. The remaining two regions are called **emitter** and **collector**. The emitter and collector are heavily doped. But the doping level in emitter is slightly greater than that of collector and the collector region-area is slightly more than that of emitter.

Fig. 8.2 (a) and (b) shows the symbols of npn and pnp transistors.

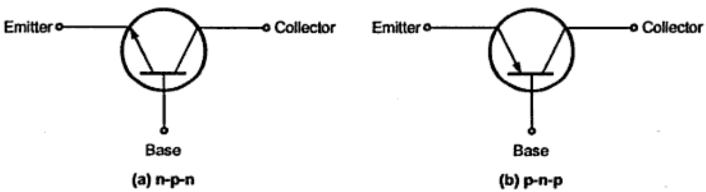


Fig. 8.2 Standard transistor symbols

A transistor has two p-n junctions. One junction is between the emitter and the base, and is called the **emitter base junction,** or simply the **emitter junction** J_E. The other junction is between the base and the collector, and is called **collector-base junction,** or simply **collector junction** J_C. Thus transistor is like two pn junction diodes connected back-to-back as shown in the Fig. 8.3 (a) and (b).

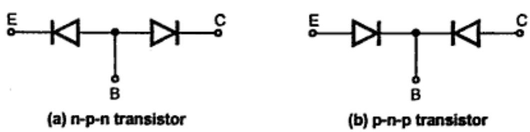


Fig. 8.3 Two-diode transistor analogy

However, we cannot replace transistor by back to back connected diodes because of the following reasons :

1. *Relative doping levels in the base, emitter and collector junctions must be satisfied to work that device as a transistor. Two normal p-n junction diodes cannot satisfy this requirement.*
2. *In a transistor, emitter to base junction is forward biased while base to collector junction is reversed biased. But due to* ***diffusion process*** *almost entire emitter current reaches to collector and base current is negligibly small. Thus due to diffusion, device works as a transistor. While in back to back connected diodes there are two separate diodes, one forward biased and one reverse biased and diffusion cannot take place. Thus maximum series current which can flow is reverse saturation current of a reverse biased diode. Hence the combination of back to back connected diodes can not be used as transistor.*

Another important point is that, the emitter area in the transistor is considerably smaller than the collector area. This is because the collector region has to handle more power than the emitter and more surface area is required for heat dissipation.

8.1.2 Unbiased Transistor

An unbiased transistor means a transistor with no external voltage (biasing) is applied. Obviously, there will be no current flowing from any of the transistor leads. Since transistor is like two pn junction diodes connected back to back, there are depletion regions at both the junctions, emitter junction and collector junction, as shown in the Fig. 8.4.

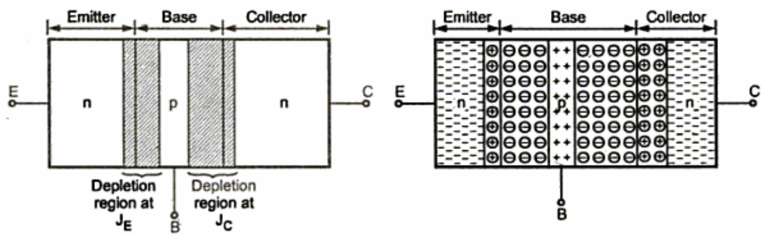


Fig. 8.4 Unbiased npn transistor

During diffusion process, depletion region penetrates more deeply into the lightly doped side in order to include an equal number of impurity atoms in the each side of the junction. As shown in the Fig. 8.4, depletion region at emitter junction penetrates less in the heavily doped emitter and extends more in the base region. Similarly, depletion region at collector junction penetrates less in the heavily doped collector and extends more in the base region. As collector is slightly less doped than the emitter, the depletion layer width at the collector junction is more than the depletion layer width at the emitter junction.

8.1.3 Biased Transistor

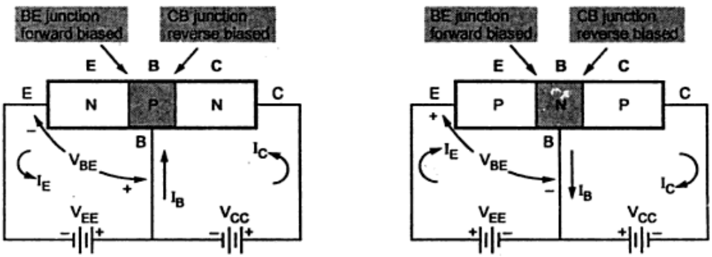
In order to operate transistor properly as an amplifier, it is necessary to correctly bias the two pn junctions with external voltages. Depending upon external bias voltage polarities used, the transistor works in one of the three regions, viz.

1) *Active region* 2) *Cut-off region* and 3) *Saturation region.*

Region	Emitter base junction	Collector base junction
Active	Forward biased	Reverse biased
Cut-off	Reverse biased	Reverse biased
Saturation	Forward biased	Forward biased

To bias the transistor in its active region, the emitter base junction is forward biased, while the collector-base junction in reverse-biased as shown in Fig. 8.5.

The Fig. 8.5 show the circuit connections for active region for both npn and pnp transistors.



(a) npn (b) pnp

Fig. 8.5 Transistor forward-reverse bias

The externally applied bias voltages are V_{EE} and V_{CC}, as shown in Fig. 8.5, which bias the transistor in its active region. The operation of the pnp is the same as for the npn except that the roles of the electrons and holes, the bias voltage polarities, and the current directions are all reversed. Note that in both cases the base-emitter (J_E) junction is forward biased and the collector-base junction (J_C) is reversed biased. With these biasing conditions, what happens inside the transistor, is discussed in the next section.

8.1.4 Transistor Operation

8.1.4.1 Operation of NPN Transistor

Let us consider the npn transistor for our discussion. The base to emitter junction is forward biased by the dc source V_{EE}. Thus, the depletion region at this junction is reduced. The collector to base junction is reverse biased, increasing depletion region at collector to base junction as shown in Fig. 8.6.

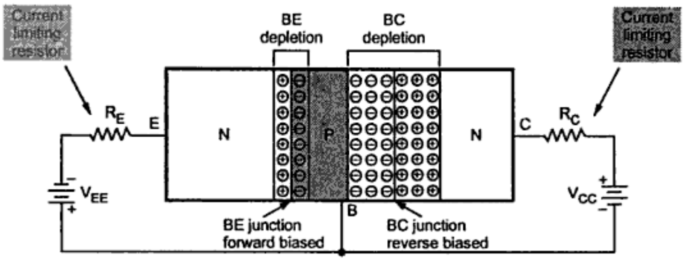


Fig. 8.6 Internal effect of forward biased EB junction and reverse biased CB junction

The forward biased EB junction causes the electrons in the n-type emitter to flow towards the base. This constitutes the emitter current I_E. As these electrons flow through the p-type base, they tend to combine with holes in p-region (base).

We know that, the base region is very thin and lightly doped. The light doping means that the free electrons have a long lifetime in the base region. The very thin base region means that the free electrons have only a short distance to go to reach the collector. For these two reasons, very few of the electrons injected into the base from the emitter recombine with holes to constitute base current, I_B (Refer Fig. 8.7) and the remaining large number of electrons cross the base region and move through the collector region to the positive terminal of the external dc source as shown in Fig. 8.8.

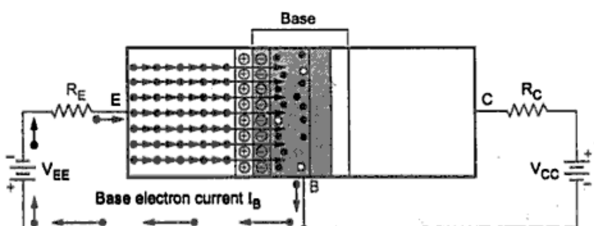


Fig. 8.7 Electron flow across emitter-base junction

8.2 Transistor Voltages and Currents

8.2.1 Transistor Voltages

NPN Transistor

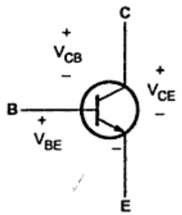


Fig. 8.12 npn transistor voltage and polarities

The Fig. 8.12 shows the terminal voltages and its polarities for an npn transistor. The voltage between base and emitter is denoted as V_{BE}. For V_{BE}, base is positive than emitter because for npn transistor, the base is biased positive with respect to the emitter.

The voltage between the collector and the emitter is denoted as V_{CE} and the voltage between the collector and the base is denoted as V_{CB}. Since collector is positive with respect to base and emitter the polarities are as shown in the Fig. 8.12.

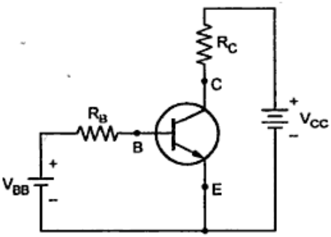


Fig. 8.13 Voltage source connections for npn transistor

The Fig. 8.13 shows the npn transistor with voltage source connections. The voltage sources are connected to the transistor with series resistors. These resistors are called **current limiting resistors.** The base supply voltage V_{BB} is connected via resistor R_B, and the collector supply voltage, V_{CC} is connected via resistor R_C. The negative terminals of both the supply voltages are connected to emitter terminal of the transistor. To make CB junction reverse biased, the supply voltage V_{CC} is always much larger than supply voltage V_{BB}.

PNP Transistor

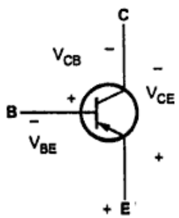


Fig. 8.14 pnp transistor voltages and polarities

The Fig. 8.14 shows the terminal voltages and its polarities for a pnp transistor. For a pnp transistor, the base is biased negative with respect to the emitter, and the collector is made more negative than the base.

The Fig. 8.15 shows the pnp transistor with voltage source connections. Like npn transistor voltage sources are connected with series resistors. The source voltage positive terminals are connected at the emitter with V_{CC} larger than V_{BB} to keep collector-base junction reverse biased.

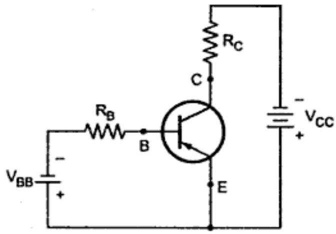


Fig. 8.15 Voltage source connection for pnp transistor

Junction Voltages

In different conditions such as active, saturation and cutoff there are different junction voltages. The junction voltages for a typical npn transistor at 25 °C are given in the Table 8.1.

Type	$V_{CE\,sat}$	$V_{BE\,sat}$	$V_{BE\,active}$	$V_{BE\,cut\text{-}in}$	$V_{BE\,cut\text{-}off}$
Si	0.2	0.8	0.7	0.5	0.0
Ge	0.1	0.3	0.2	0.1	– 0.1

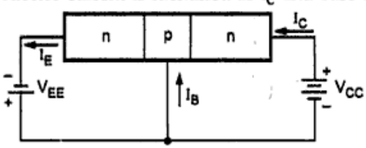
Table 8.1 Typical npn transistor junction voltages at 25 °C

The entries in the table are appropriate for an npn transistor. For pnp transistor the signs of all entries should be reversed.

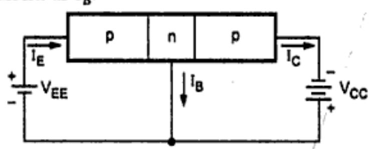
8.2.2 Transistor Currents

The directions of conventional currents in an npn transistor are as shown in Fig. 8.16 (a) and Fig. 8.16 (c) and those for a pnp are shown in Fig. 8.16 (b) and 8.16 (d). Figures show the conventional currents using the schematic symbols of npn and pnp transistors, respectively. It can be noticed that the arrow at the emitter of the transistor's symbol points in the direction of conventional current.

Let us consider pnp transistor. The current flowing into the emitter terminal is referred to as the emitter current and identified as I_E. The currents flowing out of the collector and base terminals are referred to as collector current and base current, respectively. The collector current is identified as I_C and base current as I_B.



(a)



(b)

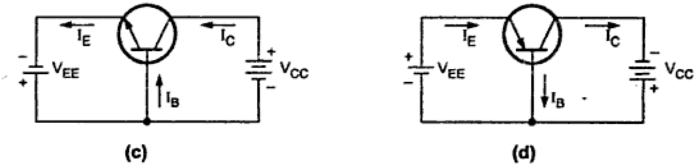


Fig. 8.16 Transistor conventional current directions

We have seen, for an n-p-n transistor, electrons are injected into the base. These electrons constitute the emitter current, I_E. For sake of explanation, assume that 100 electrons are injected into the base region. Since the base is very thin, very few of them, say 2 in number, recombine with holes. This constitutes the base current, I_B. The remaining electrons, 98 in this case, cross the base-collector reverse biased p-n junction and appear on the collector side, constituting the collector current I_C. Thus we see that the emitter current I_E is always equal to the sum of base and collector currents, I_B and I_C respectively. This is true for both types of transistor. Hence

$$I_E = I_B + I_C$$

Example 8.1 : *In a certain transistor, the emitter current is 1.02 times as large as the collector current. If the emitter current is 12 mA, find the base current.*

Solution : Given : $I_E = 12$ mA $I_E = 1.02\ I_C$

$$\therefore \quad 1.02\ I_C = 12 \times 10^{-3}$$

$$I_C = 11.765 \text{ mA}$$

$$I_E = I_B + I_C$$

$$\therefore \quad I_B = I_E - I_C = (12 - 11.765) \text{ mA}$$

$$\therefore \quad I_B = 0.235 \text{ mA} = 235\ \mu\text{A}$$

Actually, there is one more current component flows inside the transistor, called the reverse saturation current (I_{CBO}). This reverse saturation current flows across the reverse biased collector junction when emitter is open circuited. Hence, the collector current is constituted by two components, namely the curent due to injected charge carriers from emitter to collector crossing the base and current due to reverse saturation current.

8.2.3 Definition of α_{dc} and β_{dc}

α_{dc} : It is defined as the ratio of the collector current resulting from carrier injection to the total emitter current.

$$\therefore \quad \alpha_{dc} = \alpha = \frac{I_C}{I_E}$$

Since $I_C < I_E$ the value of α_{dc} is always less than unity. It ranges from 0.95 to 0.995. It represents the current gain in the CB configuration.

β_{dc} : It is defined as the ratio of the collector current to the base current.

$$\therefore \qquad \beta_{dc} = \beta = \frac{I_C}{I_B}$$

8.2.4 Relationship between α_{dc} and β_{dc}

We know that,

$$\beta_{dc} = \frac{I_C}{I_B} \quad \text{and} \quad \alpha_{dc} = \frac{I_C}{I_E}$$

We have, $I_E = I_C + I_B$ i.e. $I_B = I_E - I_C$

$$\beta_{dc} = \frac{I_C}{I_E - I_C} \qquad \because I_B = I_E - I_C$$

Dividing the numerator and denominator of R.H.S. of above equation by I_E, we get,

$$\beta_{dc} = \frac{I_C/I_E}{I_E/I_E - I_C/I_E}$$

$$\therefore \qquad \beta_{dc} = \frac{\alpha_{dc}}{1-\alpha_{dc}} \qquad \because \alpha_{dc} = \frac{I_C}{I_E}$$

Dividing the R.H.S and L.H.S. by $1 + \beta_{dc}$ we get,

$$\frac{\beta_{dc}}{1+\beta_{dc}} = \frac{\dfrac{\alpha_{dc}}{1-\alpha_{dc}}}{1+\beta_{dc}}$$

$$\frac{\beta_{dc}}{1+\beta_{dc}} = \frac{\dfrac{\alpha_{dc}}{1-\alpha_{dc}}}{1+\dfrac{\alpha_{dc}}{1-\alpha_{dc}}} \qquad \because \beta_{dc} = \frac{\alpha_{dc}}{1-\alpha_{dc}}$$

$$\frac{\beta_{dc}}{1+\beta_{dc}} = \frac{\dfrac{\alpha_{dc}}{1-\alpha_{dc}}}{\dfrac{1-\alpha_{dc}+\alpha_{dc}}{1-\alpha_{dc}}}$$

Cancelling common denominator terms we get,

$$\frac{\beta_{dc}}{1+\beta_{dc}} = \frac{\alpha_{dc}}{1-\alpha_{dc}+\alpha_{dc}} = \alpha_{dc}$$

$$\therefore \qquad \boxed{\alpha_{dc} = \frac{\beta_{dc}}{1+\beta_{dc}}}$$

Example 8.2 : *a) Find α_{dc} for each of the following values of β_{dc} = 50 and 190.*
b) Find β_{dc} for each of the following values of α_{dc} = 0.995 and 0.9765.

Solution :

a) $$\alpha_{dc} = \frac{\beta_{dc}}{1+\beta_{dc}}$$

For $$\beta_{dc} = 50,\ \alpha_{dc} = \frac{50}{1+50} = \mathbf{0.9804}$$

For $$\beta_{dc} = 190,\ \alpha_{dc} = \frac{190}{1+190} = \mathbf{0.9947}$$

b) $$\beta_{dc} = \frac{\alpha_{dc}}{1-\alpha_{dc}}$$

For $$\alpha_{dc} = 0.995,\quad \beta_{dc} = \frac{0.995}{1-0.995} = \mathbf{199}$$

For $$\alpha_{dc} = 0.9765,\ \beta_{dc} = \frac{0.9765}{1-0.9765} = \mathbf{41.55}$$

Example 8.3 : *If the base current in a transistor is 20 μA when the emitter current is 6.4 mA, what are the values of α_{dc} and β_{dc} ? Also calculate the collector current.*

Solution : Given : $I_B = 20\ \mu A$ $I_E = 6.4\ mA$

$$I_E = I_B + I_C = I_B + I_B\,\beta_{dc}$$
$$= I_B\,(1+\beta_{dc})$$
$$\beta_{dc}+1 = \frac{I_E}{I_B} = \frac{6.4\times10^{-3}}{20\times10^{-6}} = 320$$

∴ $$\beta_{dc} = \mathbf{319}$$

$$\alpha_{dc} = \frac{\beta_{dc}}{1+\beta_{dc}} = \frac{319}{1+319} = \mathbf{0.9968}$$

$$I_C = \beta_{dc}\ I_B = (319)\ (20\ \mu A) = 6380\ \mu A = \mathbf{6.38\ mA}$$

Also, $$I_C = \alpha_{dc}\ I_E = (0.9968)\ (6.4\ mA) = \mathbf{6.379\ mA}$$

Example 8.4 : *For a transistor in common emitter configuration, the reverse leakage current is 21 μA, whereas when the same transistor is connected in common base configuration, it reduces to 1.1 μA. Calculate values of α_{dc} and β_{dc} of the transistor.*

Solution : Given : $I_{CO} = 1.1\ \mu A,\ I_E = 21\ \mu A$

$$I_{CEO} = (1+\beta_{dc})\ I_{CO}$$

∴ $$1+\beta_{dc} = \frac{I_{CEO}}{I_{CO}} = \frac{21\ \mu A}{1.1\ \mu A} = 19$$

$$\therefore \qquad \beta_{dc} = 18$$

$$\alpha_{dc} = \frac{\beta_{dc}}{1+\beta_{dc}} = \frac{18}{1+18}$$

$$= 0.947$$

8.3 BJT Configurations

In the previous section we have seen that base is taken as common point/terminal to connect transistor in common-base configuration. Similarly, we can use emitter and collector as a common points/terminals to connect transistor in common emitter and common collector configurations, respectively. Thus, the transistor can be connected in a circuit in the following three configurations.

1. Common base configuration
2. Common emitter configuration
3. Common collector configuration

Key Point : *Regardless of circuit configuration, the base emitter junction is always forward biased while the collector-base junction is always reverse biased, to operate transistor in active region.*

8.3.1 Common Base Characteristics

To understand complete electrical behaviour of a transistor it is necessary to study the interrelation of the various currents and voltages. These relationships can be plotted graphically which are commonly known as the **characteristics of transistor.** The most important characteristics of transistor in any configuration are input and output characteristics.

The Fig. 8.17 shows the common base configuration. As shown in Fig. 8.17, in this configuration input is applied between emitter and base and output is taken from the collector and base. Here, base of the transistor is common to both input and output circuits and hence the name common base configuration. Common base configurations for both npn and pnp transistors are shown in Fig. 8.17 (a) and 8.17 (b), respectively.

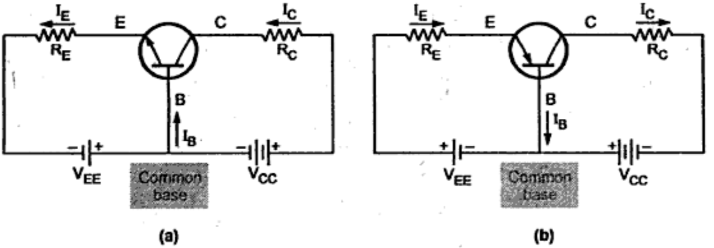


Fig. 8.17 Common base configuration

Input Characteristics :

It is the curve between input current I_E (emitter current) and input voltage V_{EB} (emitter-base voltage) at constant collector-base voltage V_{CB}. The emitter current is taken along Y-axis and emitter base voltage along X-axis. Fig. 8.18 shows the input characteristics of a typical transistor in common-base configuration.

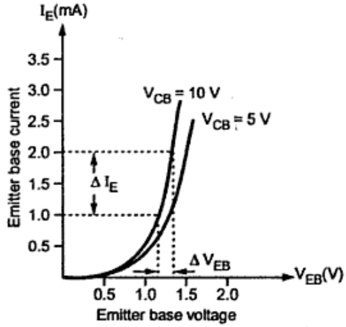


Fig. 8.18 Input characteristics of transistor in CB configuration

From this characteristics we can observe the following important points :

1. After the cut-in voltage (barrier potential, normally 0.7 V for silicon and 0.3 V for Germanium), the emitter current (I_E) increases rapidly with small increase in emitter-base voltage (V_{EB}). It means that input resistance is very small. Because input resistance is a ratio of change in emitter-base voltage (ΔV_{EB}) to the resulting change in emitter current (ΔI_E) at constant collector-base voltage (V_{CB}), this resistance is also known as the dynamic input resistance of the transistor in CB configuration.

$$r_i = \left.\frac{\Delta V_{EB}}{\Delta I_E}\right|_{V_{CB} = \text{constant}}$$

2. It can be observed that there is slight increase in emitter current (I_E) with increase in V_{CB}. This is due to change in the width of the depletion region in the base region under the reverse biased condition.

As shown in Fig. 8.19, when reverse bias voltage V_{CB} increases, the width of depletion region also increases, which reduces the electrical base width. Due to reduction of the electrical base width, now there are more charge particles per unit area. In other words, due to reduction of the electrical base width, concentration of the charge gradient increases in the base region. This increase in concentration of charge carriers causes more diffusion of electrons from n-type emitter to p-type base increasing emitter current slightly.

From this characteristics we observe following points :

1. The output characteristics has three basic regions : Active, cut-off and saturation.
2. **Active Region :** For the operation in the active region, the emitter base junction is forward biased while collector base junction is reverse biased. In this region collector current I_C is approximately equal to the emitter current (I_E) and transistor works as an amplifier.

   Consider first that the emitter current is zero. Then the collector current is small and equals the reverse saturation current (I_{CO}) of the collector junction considered as a diode. The current I_{CO} is the current because of minority current component and hence it is also known as **leakage current.** This current is so small in magnitude compared to the vertical scale of I_C that it virtually appears on the same horizontal line as $I_C = 0$. However, in Fig. 8.20, this has been shown on the exaggerated scale for understanding purpose. Suppose now that a forward emitter current I_E is caused to flow in the emitter circuit, then a fraction of current I_E ($\alpha\, I_E$) will reach the collector, and I_C is therefore given by,

   $$I_C = \alpha I_E + I_{CO} \qquad ...(1)$$

   In the active region, the collector current is essentially almost constant, and the graph is almost parallel to x-axis. The collector current I_C is almost independent on collector base voltage V_{CB} and the transistor can be said to work as constant-current source. This provides very high dynamic output resistance. Dynamic output resistance is the ratio of change in collector base voltage (ΔV_{CB}) to the resulting change in collector current (ΔI_C) at constant emitter current (I_E)

   $$R_o = \left.\frac{\Delta V_{CB}}{\Delta I_C}\right|_{I_E = \text{constant}}$$

   In the active region, the I_C depends only upon the I_E. Because α is less than, but almost equal to, unity, the magnitude of the collector current is slightly less than that of the emitter current.
3. **Saturation Region :** The saturation region is that region of the characteristics which is to the left of V_{CB}=0 V and above the $I_E = 0$. In the Fig. 8.20, the horizontal scale is expanded to clearly show the change in characteristics in the region. Note the exponential increase in collector current as the voltage V_{CB} increases towards 0 V. In this saturation region, the emitter-base and collector-base junctions are both forward biased.
4. **Cut-off Region :** The region below the curve $I_E = 0$ is known as cut-off region, where the collector current is nearly zero and the collector-base and emitter-base junctions of a transistor are reverse biased.
5. **Punch-through Effect :** In the active region, the collector-base junction is reversed-biased. For every transistor there is limit on the maximum value for this reverse bias voltage.

As shown in Fig. 8.21, collector base junction is reverse biased. As mentioned earlier, this reverse bias voltage must be within the maximum safe limits specified by the manufacturer. If this maximum limit is exceeded, transistor breakdown occurs (same as the diode breakdown). Fig. 8.21 shows this breakdown condition.

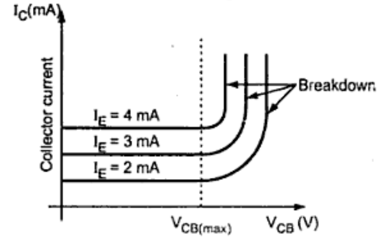
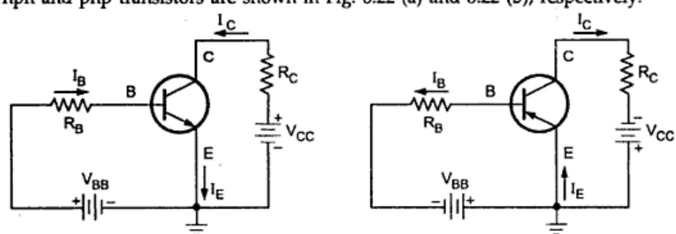


Fig. 8.21 Output characteristics of a transistor showing maximum collector base voltage rating in CB configuration

The curves shown at the right side of dotted line ($V_{CB\,max}$ is exceeded) represent the breakdown condition. When collector to base voltage increases, width of the depletion region at the junction increases. Therefore, when V_{CB} increases above the $V_{CB\,max}$, increase in depletion region is such that it penetrates into the base until it makes contact with emitter-base depletion region. This condition is called **'punch-through'** or **'reach through'** effect. When this situation occurs, breakdown occurs. i.e. large collector current flows which destroys the transistor. To avoid this punch-through effect V_{CB} should always be kept below the maximum safe limit specified by the manufacturer.

8.3.2 Common Emitter Characteristics

The Fig. 8.22 shows the common-emitter configuration. As shown in Fig. 8.22, in this configuration input is applied between base and emitter, and output is taken from collector and emitter. Here, emitter of the transistor is common to both, input and output circuits, and hence the name common emitter configuration. Common emitter configurations for both npn and pnp transistors are shown in Fig. 8.22 (a) and 8.22 (b), respectively.



(a) npn **(b) pnp**

Fig. 8.22 Common emitter configurations

2. After the cut-in voltage the base current (I_B) increases rapidly with small increase in base emitter voltage (V_{BE}). It means that dynamic input resistance is small in CE configuration. It is the ratio of change in base-emitter voltage (ΔV_{BE}) to the resulting change in base current (ΔI_B) at constant collector emitter voltage V_{CE}. It is given by

$$r_i = \left.\frac{\Delta V_{BE}}{\Delta I_B}\right|_{V_{CE} = \text{constant}}$$

3. For a fixed value of V_{BE}, I_B decreases as V_{CE} is increased. A larger value of V_{CE} results in a large reverse bias at collector-base p-n junction. This increases the depletion region and reduces the effective width of the base. Hence, there are fewer recombinations in the base region, reducing the base current I_B.

B) Output Characteristics :

1. This characteristics shows the relation between the collector current I_C and collector voltage V_{CE}, for various fixed values of I_B. This characteristics is often called collector characteristics. A typical family of output characteristics for an n-p-n transistor in CE configuration is shown in Fig. 8.25.

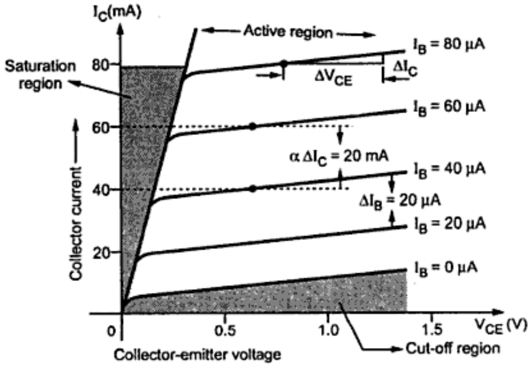


Fig. 8.25 Output characteristics of the transistor in CE configuration

2. The value of β_{dc} of the transistor can be found at any point on the characteristics by taking the ratio I_C to I_B at that point, i.e. $\beta_{dc} = \frac{I_C}{I_B}$. This is known as DC beta for the transistor. For a fixed value of V_{CE}, if we take the ratio of small change in I_C, ΔI_C to small change in I_B, ΔI_B, we get AC beta;

$$\beta_{ac} = \left.\frac{\Delta I_C}{\Delta I_B}\right|_{\Delta V_{CE}=0}$$

Generally dc and ac values of beta of the transistor are nearly equal.

3. From the output characteristics we can see that change in collector-emitter voltage (ΔV_{CE}) causes the little change in the collector current (ΔI_C) for constant base current I_B. Thus the output dynamic resistance is high in CE configuration.

$$r_o = \left.\frac{\Delta V_{CE}}{\Delta I_C}\right|_{I_B=\text{constant OR } \Delta I_B=0}$$

4. The output characteristics of common emitter configuration consists of three regions : Active, Saturation, and Cut-off.

5. **Active region** : The region where the curves are approximately horizontal is the "active" region of the CE configuration. In the active region, the collector **junction** is reverse biased. As V_{CE} is increased, reverse bias increases. This causes **depletion** region to spread more in base than in collector, reducing the chances of recombinations in the base. This increases the value of α_{dc}. This early effect causes collector current to rise more sharply with increasing V_{CE} in the linear region of output characteristics of CE transistor.

From Kirchhoff's current law (KCL) applied to Fig. 8.23,

We have, $\qquad I_E = I_C + I_B \qquad$ ... (2)

Substituting value of I_E in equation (1) we get,

$$I_C = \alpha(I_C+I_B)+I_{CO}$$

$$\therefore \quad I_C(1-\alpha) = \alpha I_B + I_{CO}$$

$$\therefore \quad I_C = \frac{\alpha I_B}{1-\alpha}+\frac{I_{CO}}{1-\alpha}$$

$$\therefore \quad \boxed{I_C = \beta I_B + (1+\beta) I_{CO}} \qquad \because \beta = \frac{\alpha}{1-\alpha} \qquad \text{... (3)}$$

Note that usually $I_B >> I_{CO}$, and hence $IC \approx \beta I_B$ in the active region.

6. **Saturation region** : If V_{CE} is reduced to a small value such as 0.2 V, then collector-base junction becomes forward biased, since the emitter base junction is already forward biased by 0.7 V. The input junction in CE configuration is base to emitter junction, which is always forward biased to operate transistor in active region. Thus input characteristics of CE configuration is similar to forward characteristics of p-n junction diode. (considering silicon transistor). When both the junctions are forward biased, the transistor operates in the saturation region, which is indicated on the output characteristics. The saturation value of V_{CE}, designated $V_{CE(sat)}$, usually ranges between 0.1 V to 0.3 V.

7. **Cut-off region** : When the input base current is made equal to zero, the collector current is the reverse leakage current I_{CEO}. The region below $I_B = 0$ is the cut-off region of operation for the transistor. In this region, both the junctions of the transistor are reverse biased.

Since $I_B = 0$,

$I_C = I_E$

From equation (3), we have,

$$\boxed{I_C = (1+\beta)I_{CO} = \frac{I_{CO}}{1-\alpha} = I_{CEO}} \quad ...(4)$$

The actual collector current with collector junction reverse biased and base open-circuited is designated by the symbol I_{CEO}.

8. **Reverse collector saturation current** : The collector current in a physical transistor when the emitter current is zero is designated by the symbol I_{CBO}. The $|I_{CBO}|$ is larger than $|I_{CO}|$, since it also constitutes the leakage current around the junction and across the surfaces. Another reason why $|I_{CBO}|$ exceeds $|I_{CO}|$, is that new carriers may be generated by collision in the collector-junction transition region, leading to avalanche multiplication of current and eventual breakdown.

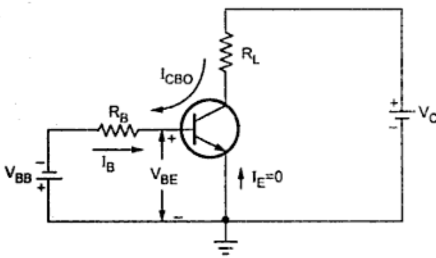


Fig. 8.26 Existence of I_{CBO}

The I_{CBO} is a temperature sensitive. It approximately doubles for every 10 °C increase in temperature for both Ge and Si.

9. In the active region, the collector base junction is reverse biased. For every transistor there is limit on the maximum value for this reverse bias voltage. If this limit is exceeded as shown in Fig. 8.27, the breakdown occurs in the transistor. This effect is commonly known as **punch through effect.**

This large current may damage transistor. Hence, in practice, maximum collector emitter voltage rating, V_{CE} (max) should never be exceeded for the safe operation of the transistor.

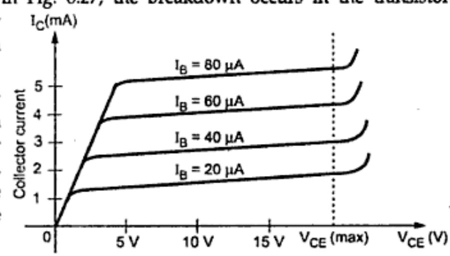


Fig. 8.27 Maximum collector-emitter voltage rating

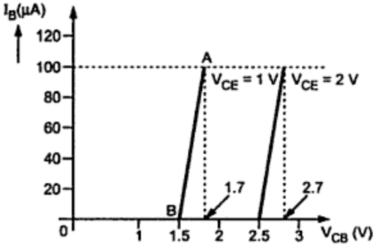


Fig. 8.29 Input characteristics of transistor in CC configuration

The common collector input characteristics are quite different from either common base or common emitter input characteristics. This difference is due to the fact that the input voltage V_{CB} is largely determined by the level of collector to emitter voltage V_{CE}. Looking at Fig. 8.29 we can write,

$$V_{CE} = V_{CB} - V_{BE}$$

or

$$V_{CB} = V_{CE} + V_{BE}$$

When collector base junction is reverse biased and emitter base junction is forward biased, V_{BE} remains around 0.7 V for a silicon transistor and 0.3 V for a germanium transistor. From the characteristics we can observe that V_{CE} may be larger than 0 V.

Consider the characteristic for V_{CE} = 1 V. At I_B = 100 μA (point A), V_{CB} = 1.7 and

$$V_{BE} = V_{CB} - V_{CE}$$
$$= 1.7 - 1 = 0.7 \text{ V}$$

Now suppose V_{CE} is maintained constant at 1 V while the input voltage V_{CB} is decreased to 1.5 V (point B). The base emitter voltage then becomes,

$$V_{BE} = 1.5 - 1$$
$$= 0.5 \text{ V}$$

From the characteristics we can see that as V_{BE} is reduced from 0.7 V to 0.5 V, I_B is reduced from 100 μA to zero.

Fig. 8.30 shows the part of our interest of the input characteristics of the common collector configuration on exaggerated scale. This indicates that when transistor is biased, base current is very sensitive to collector base voltage.

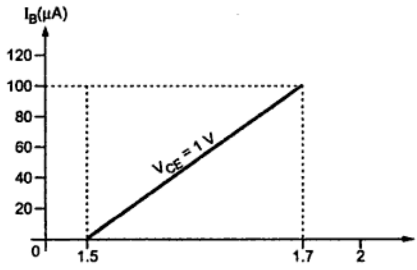


Fig. 8.30 Input characteristics of common collector configuration

When we bias a transistor we establish a certain current and voltage conditions for the transistor. These conditions are known as operating conditions or **d.c. operating point** or **quiescent point**. The operating point must be stable for proper operation of the transistor. However, the operating point shifts with changes in transistor parameters such as β, I_{CO} and V_{BE}. As transistor parameters are temperature dependent, the operating point also varies with changes in temperature.

8.6.1 Purpose of Biasing

In section 8.1.2 we have seen the status of unbiased transistor. Its of no use. To operate transistor in any of three regions we have to establish proper voltage levels at the two junctions of the transistor. When transistor is in the active region it can be used as an amplifier. When it is operated in cut-off and saturation regions it can be used as a switch. Therefore, depending on the application we have to provide proper biasing for transistor.

8.6.2 The D.C. Operating Point

Consider a common emitter circuit shown in the Fig. 8.32. The transistor in the Fig. 8.34 is biased with a common supply such that the base emitter junction is forward biased and the collector base junction is reverse biased, i.e. transistor is in the active region.

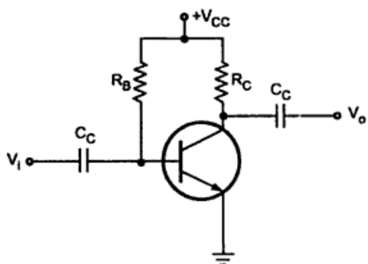


Fig. 8.32 Common emitter amplifier

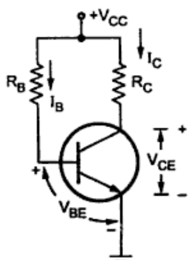


Fig. 8.33

In the absence of a.c. signal, the capacitors provide very high impedance, i.e. open circuit. Therefore, the equivalent circuit for common emitter amplifier becomes, as shown in the Fig. 8.33.

Applying Kirchhoff's voltage law to the collector circuit shown in the Fig. 8.33, we get,

$$V_{CC} - I_C (R_C) - V_{CE} = 0$$

$$\therefore \quad V_{CC} = I_C (R_C) + V_{CE} \quad \ldots (1)$$

where I_C (R_C) is the voltage drop across R_C, and V_{CE} is the collector to emitter voltage.

$$I_C = \frac{V_{CC} - V_{CE}}{R_C}$$

Rearranging the terms in above equation we get,

$$I_C = \left[-\frac{1}{R_C}\right] V_{CE} + \frac{V_{CC}}{R_C} \qquad \dots (2)$$

If we compare this equation with equation of straight line y = mx + c, where m is the slope of the line and c is the intercept on Y-axis, then we can draw a straight line on the graph of I_C versus V_{CE} which is having slope $-1/R_C$ and Y-intercept V_{CC}/R_C. To determine the two points on the line we assume $V_{CE} = V_{CC}$ and $V_{CE} = 0$.

a) When $V_{CE} = V_{CC}$; $I_C = 0$ and we get a point A and

b) When $V_{CE} = 0$; $I_C = V_{CC}/R_C$ and we get a point B

The Fig. 8.34 shows the output characteristics of a common emitter configuration with points A and B, and line drawn between them. The line drawn between points A and B is called **dc load line.** The 'dc' word indicates that only dc conditions are considered, i.e. input signal is assumed to be zero.

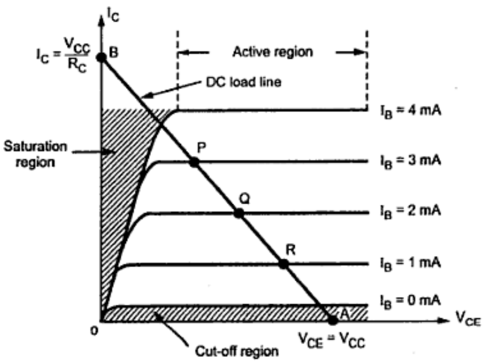


Fig. 8.34 Common emitter output characteristics with d.c. load line

The d.c. load line is a plot of I_C versus V_{CE} for a given value of R_C and a given level of V_{CC}. Thus, it represents all collector current levels and corresponding collector-emitter voltages that can exist in the circuit. Knowing any one of I_C, I_B or V_{CE}, it is easy to determine the other two from the load line. The slope of the d.c. load line depends on the value of R_C. It is negative and equal to reciprocal of the R_C.

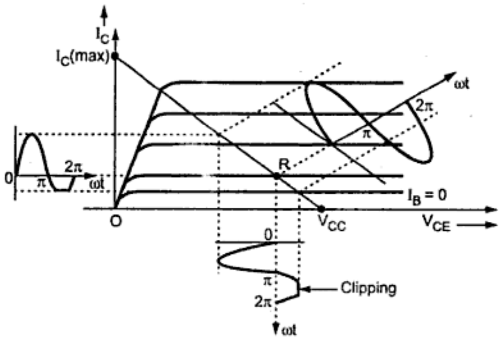


Fig. 8.36 Operating point near cut-off region gives clipping at the negative peaks

Case 3 : Biasing circuit is designed to fix a Q-point at point Q as shown in Fig. 8.37. The output signal is sinusoidal waveform without any distortion. Thus point Q is the best operating point.

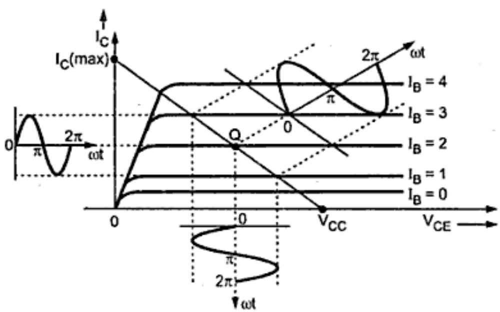


Fig. 8.37 Operating point at the centre of active region is most suitable

8.6.4 Typical Junction Voltages and Conditions for Operating Region

We have seen that, we can operate transistor configurations in different operating regions by selecting appropriate operating point. For the analysis and design of such circuits we should know the typical junction voltages in different operating regions.

The Table 8.4 shows the typical junction voltages for cut-off, active and saturation regions for n-p-n silicon and germanium transistors. For pnp transistors we have to reverse the polarities of voltages given in Table 8.4.

Transistor	$V_{CE\,(sat)}$	$V_{BE\,(sat)}$	$V_{BE\,(active)}$	$V_{BE\,(cut-in)}$	$V_{BE\,(cut-off)}$
Si	0.2 V	0.8 V	0.7 V	0.5 V	0 V
Ge	0.1 V	0.3 V	0.2 V	0.1 V	– 0.1 V

Table 8.4 Typical junction voltages

To identify the operating region of transistor we can observe certain conditions. These are :

For saturation : $I_B > \dfrac{I_C}{\beta_{dc}}$

For active region : $V_{CE} > V_{CE\,(sat)}$

8.7 BJT as a Voltage Amplifier

The Fig. 8.38 shows the transistor circuit with a base bias and signal generator. Let us assume that the transistor, Q_1 in the circuit has β_{dc} = 100. The base bias of 0.7 V forward biases the base-emitter junction of transistor. An ac signal source, v_i in series with V_{BB} provides ± 10 mV input voltage.

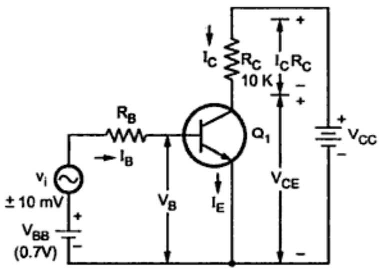


Fig. 8.38

Now let us observe the changes in I_B and I_C due to changes in v_i from the V-I characteristics of transistor given in Fig. 8.39.

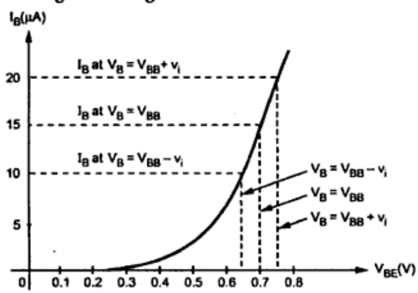


Fig. 8.39 V_{BE} versus I_B characteristics of transistor

Condition 1 : When $v_i = 0$, $V_B = V_{BB} = 0.7$ and $I_B = 15\ \mu A$.

$\therefore \quad I_C = 15\ \mu A \times 100 = 1.5$ mA

Condition 2 : When $v_i = -10$ mV,

$$V_B = V_{BB} - v_i = 0.7 - 10 \text{ mV} = 0.69 \text{ V}$$

$$I_B = 10\ \mu A$$

$$\therefore \quad I_C = 10\ \mu A \times 100 = 1 \text{ mA}$$

Condition 3 : When $v_i = 10$ mV

$$V_B = V_{BB} + v_i = 0.7 + 10 \text{ mV} = 0.71 \text{ V}$$

$$I_B = 20\ \mu A$$

$$\therefore \quad I_C = 20\ \mu A \times 100 = 2 \text{ mA}$$

The Fig. 8.40 shows the output characteristics of a common emitter configuration showing the conditions 1, 2, 3.

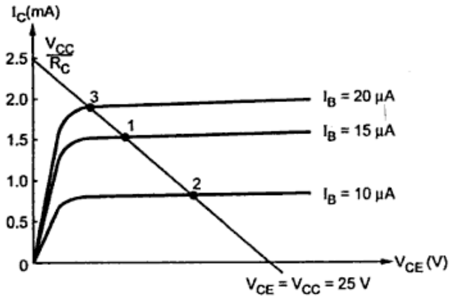


Fig. 8.40 CE output characteristics with DC load line

The output voltage V_C is given by

$$V_C = V_{CC} - I_C R_C$$

For condition 1 : $V_{CC} - I_C R_C = 25 - 1.5 \text{ mA} \times 10 \text{ K} = 10$ V

For condition 2 : $V_{CC} - I_C R_C = 25 - 1.0 \text{ mA} \times 10 \text{ K} = 15$ V

For condition 3 : $V_{CC} - I_C R_C = 25 - 2 \text{ mA} \times 10 \text{ K} = 5$ V

Therefore, change in V_C (ΔV_C) = 15 − 5 = 10 V due to ± 10 mV change in v_i. The output voltage v_o is the change in V_C. Therefore, the ac voltage gain which is the ratio of change in the output voltage to change in the input voltage is given by,

$$A_v = \frac{v_o}{v_i} = \frac{\Delta V_C}{\Delta V_B} = \frac{15-5}{0.71-0.69} = 500$$

Since the output voltage is greater than the input voltage, the circuit has a voltage gain and it is a voltage amplifier.

8.8 Single Stage BJT Amplifier

8.8.1 Common Emitter Amplifier Circuit

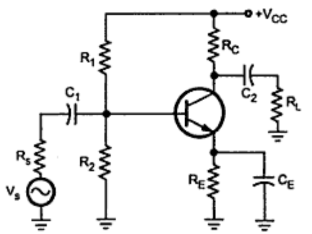


Fig. 8.41 Practical common emitter amplifier circuit

Fig. 8.41 shows the practical circuit of common emitter transistor amplifier. It consists of different circuit component. The functions of these components are as follows :

1. Biasing Circuit

The resistances R_1, R_2 and R_E forms the voltage divider biasing circuit for the CE amplifier. It sets the proper operating point for the CE amplifier.

2. Input Capacitor C_1

This capacitor couples the signal to the base of the transistor. It blocks any dc component present in the signal and passes only ac signal for amplification. Because of this biasing conditions are maintained constant.

3. Emitter Bypass Capacitor C_E

An emitter bypass capacitor C_E is connected in parallel with the emitter resistance, R_E to provide a low reactance path to the amplified ac signal. If it is not inserted, the amplified ac signal passing through R_E will cause a voltage drop across it. This will reduce the output voltage, reducing the gain of the amplifier.

4. Output Coupling Capacitor C_2

The coupling capacitor C_2 couples the output of the amplifier to the load or to the next stage of the amplifier. It blocks dc and passes only ac part of the amplified signal.

Need for C_1, C_2 and C_E

We know that, the impedance of capacitor is given as

$$X_C = \frac{1}{2\pi fC}$$

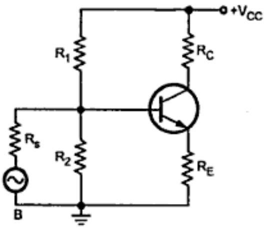


Fig. 8.42

Thus, at signal frequencies all the capacitors have extremely small impedance and it can be treated as an ac short circuit. For bias/dc conditions of the transistor all the capacitors act as a dc open circuit. With this knowledge we will see the importance of C_1, C_2 and C_E.

Consider that the signal source is connected directly to the base of the transistor as shown in Fig. 8.42.

Looking at the Fig. 8.42 we can immediately notice that source resistance R_s is in parallel with R_2. This will reduce the bias voltage at the transistor base and, consequently alter the collector current, which is not desired. Similarly, by connecting R_L directly, the dc levels of V_C and V_{CE} will change. To avoid this and maintain the stability of bias condition coupling capacitors are connected. As mentioned earlier, coupling capacitors act as open circuits to dc, maintain stable biasing conditions even after connection of R_s and R_L. Another advantage of connecting C_1 is that any dc component in the signal is opposed and only ac signal is routed to the transistor amplifier.

The emitter resistance R_E is one of the component which provides bias stabilization. But it also reduces the voltage swing at the output. The emitter bypass capacitor C_E provides a low reactance path to the amplified a.c. signal increasing the output voltage swing.

For the proper operation of the circuit, polarities of the capacitors must be connected correctly. The curve bar which indicates negative terminal must always be connected at a dc voltage level lower than (or equal to) the dc level of the positive terminal (straight bar). For example, C_1 in Fig. 8.41 has its negative terminal at dc ground level, because it is grounded through the signal source resistance R_s. The positive terminal of C_1 is at + V_B with respect to ground.

Phase Reversal

The phase relationship between the input and output voltages can be determined by considering the effect of a positive half cycle and negative half cycle separately. Consider the positive half cycle of input signal in which terminal A is positive w.r.t. B. Due to this, two voltages, ac and dc will be adding each other, increasing forward bias on base emitter junction. This increases base current. The collector current is β times the base current, hence the collector current will also increase. This increases the voltage drop across R_C. Since $V_C = V_{CC} - I_C R_C$, the increase in I_C results in a drop in collector voltage V_C, as V_{CC} is constant. Thus, as V_i increases in a positive direction, V_o goes in a negative direction and we get negative half cycle of output voltage for positive half cycle at the input.

In the negative half cycle of input, in which terminal A becomes negative w.r.t. terminal B, the ac and dc voltages will oppose each other, reducing forward bias on

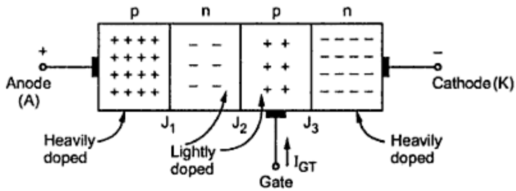


Fig. 8.46 Construction of SCR

Anode must be positive with respect to cathode to forward bias the SCR. But this is not sufficient criterion to turn SCR ON. To make it ON, a current is to be passed through the gate terminal denoted as I_{GT}. Thus it is a **current operated device.**

Three types of constructions are used to manufacture SCR,

1) Planar type 2) Mesa type 3) Press pack type

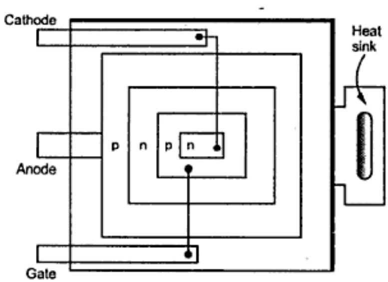


Fig. 8.47 Planar type construction

1. Planar type : This construction is used for low current SCRs. In this type, all the p-n junctions come to the same surface on the cathode side. This is shown in the Fig. 8.47. All the junctions are diffused in this type of construction. The disadvantage of this type is more silicon per ampere current is required. The advantage is that the mass production is possible and large number of SCRs can be manufactured with uniform characteristics.

2. Mesa type : In this construction, the junction J_2 is diffused while the outer layers are alloyed to it. This is shown in the Fig. 8.48. To handle the large currents, the molybdenum or tungsten plates are braced to p-n-p-n silicon pellet. This provides the additional mechanical strength. When SCR is ON, the area around the gate starts the conduction first. But in this construction, area around the gate is small hence this construction is not suitable for high di/dt ratings.

8.9.2 Operation of SCR

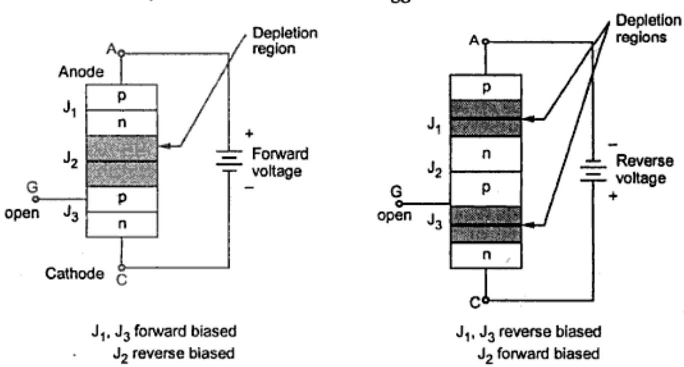
The operation of SCR is divided into two categories,

1. When gate is open :

Consider that the anode is positive with respect to cathode and gate is open. The junctions J_1 and J_3 are forward biased and junction J_2 is reverse biased. There is depletion region around J_2 and only leakage current flows which is negligibly small. Practically the SCR is said to be OFF. This is called **forward blocking state** of SCR and voltage applied to anode and cathode with anode positive is called **forward voltage**. This is shown in the Fig. 8.51 (a).

With gate open, if cathode is made positive with respect to anode, the junctions J_1, J_3 become reverse biased and J_2 forward biased. Still the current flowing is leakage current, which can be neglected as it is very small. The voltage applied to make cathode positive is called **reverse voltage** and SCR is said to be in **reverse blocking state.** This is shown in the Fig. 8.51 (b).

In forward blocking state, if the forward voltage is increased, the current remains almost zero upto certain limit. At a particular value, the reverse biased junction J_2 breaks down and SCR conducts heavily. This voltage is called **forward breakover voltage** V_{BO} of SCR. In such condition, SCR is said to be ON or triggered.



(a) (b)

Fig. 8.51 Operation of SCR when gate is open

2. When gate is closed :

Consider that the voltage is applied between gate and cathode when the SCR is in forward blocking state. The gate is made positive with respect to the cathode. The electrons from n-type cathode which are majority in number, cross the junction J_3 to reach to positive of battery.

7. Critical rate of rise of current, di/dt : Another important parameter of SCR is a critical rate of current rise. If the anode current tries to rise faster than this, a local hot spots will be formed near the gate connection due to high current density. This causes the junction temperature to rise above the safe limit and SCR may be damaged permanently. We know that, an inductor opposes change in current, connecting inductor in series with SCR, as shown in Fig. 8.56, reduces the rate of current rise.

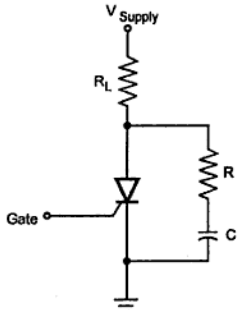


Fig. 8.55 SCR with snubber circuit

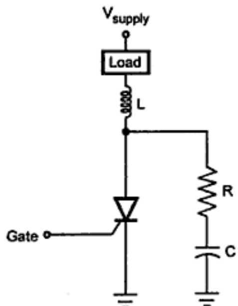


Fig. 8.56 Series inductor

Typical value of di/dt rating of SCR is 100 A/μsec

8. Current rating : It is the maximum current carrying ability of the SCR.

9. Minimum gate trigger current (I_{GTmin}) : The minimum value of gate current which can trigger SCR is defined as I_{GTmin}.

10. Maximum gate current (I_{GTmax}) : It is the peak value of gate current which must not be exceeded to avoid damage to the SCR.

11. Gate power loss (P_G) : It is the mean power loss due to gate current between the gate and the main terminal.

12. Turn on time (t_{on}) : The time required by SCR to reach full conduction after triggering is called 'turn on time'. The turn on time consists of :

- Time required for charging gate to cathode capacitance and
- Time required for reaching latching current value.
- Typically, turn on time of SCR is of the order of 2-4 μsec.

13. Turn off time (t_{off}) : It is time required from the zero current point to the time when the SCR regains its full blocking voltage in positive direction after the application of reverse voltage across it. Typically, the turn off time of SCR is of the order of 10-50 μsec. For high frequency SCRs it is 10-20 μsec. For satisfactory operation of the circuit, circuit turn off-time must always be greater than the turn off time of SCR.

8.9.6 Methods of Turning ON SCR

There are five basic methods of triggering of SCR.

1. Thermal triggering : We know that the width of depletion layer of a semiconductor decreases as temperature increases. Thus in a SCR if applied voltage is very near the break down voltage, the increase in temperature can trigger the SCR.

2. Radiation triggering : The SCR can be triggered by bombarding photons which results electron-hole pairs. Such triggering is called radiation triggering and such SCR are called LASCR (Light-Activated SCRs).

3. Voltage triggering : An increase in forward biased voltage causes, the electrons and holes to concentrate at reverse biased junction. This increases the blocking current and the SCR is triggered.

4. dV/dt triggering : If the rate of rise of voltage exceeds the critical rate of rise of voltage SCR is triggered.

5. Gate triggering : This is the most easy and useful method of turning ON SCR. While designing the gate control circuit the following points must be considered.

i) When the SCR is forward biased, appropriate gate-to-cathode voltage must be applied to turn ON SCR.

ii) To reduce losses and higher junction temperatures, gate signal should be removed after the SCR is turned ON.

iii) No gate signal should be applied when the SCR is reverse biased.

iv) To improve the characteristics of the SCR, negative voltage should be applied between gate and cathode when the SCR is in the OFF state.

There are three ways to turn SCR ON by gate control which are,

a. By d.c. gate signal : In this method, a d.c. voltage of proper polarity and magnitude is applied between gate and cathode.

b. By a.c. gate signal : In this method, a.c. voltage is applied between gate and cathode. It is shifted a.c. voltage which is derived from the mains supply.

c. By pulsed gate signal : In this method, a pulsed waveform is applied to gate. The pulse occurs periodically at the gate. Due to pulse signal, the gate current is not continuous hence losses are less and gate isolation is achieved.

8.9.7 Turn OFF Mechanism

The SCR can be brought back to the forward blocking state from the conduction state only by reducing the forward current to a level below that of the holding current. In AC circuits where the current goes through a natural zero value, the SCR will be automatically turned off.

8.9.8 Applications of SCR

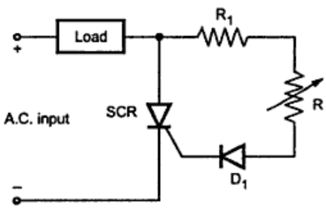


Fig. 8.57 Variable half wave rectifier

The SCRs conduct in one direction hence can be used in the rectifier circuits. And controlling the instant of turning ON the SCR, the average power delivered to the load also can be controlled.

1. Single phase half wave rectifier : The Fig. 8.57 shows circuit for single phase half wave rectifier using SCR. As shown in the Fig. 8.57, triggering circuit consists of a diode and resistor in series with the gate. By changing resistor R it is possible to change gate current at specific point on the sine wave. Recall that the SCR is a current operated device and it is the gate current (injected carriers) that turns on the SCR.

Therefore by changing resistor R, the point on the sine wave at which the SCR fires can be changed, as shown in the Fig. 8.57. This makes it possible to change the average power delivered to the load.

In the negative half of cycle, SCR is in the reverse blocking state. The diode D_1 in the gate circuit blocks the reverse voltage on the gate.

As shown in the Fig. 8.58, α is a **firing angle** and ϕ is a **conduction angle.** With this gate circuit maximum firing angle we can get is 90°.

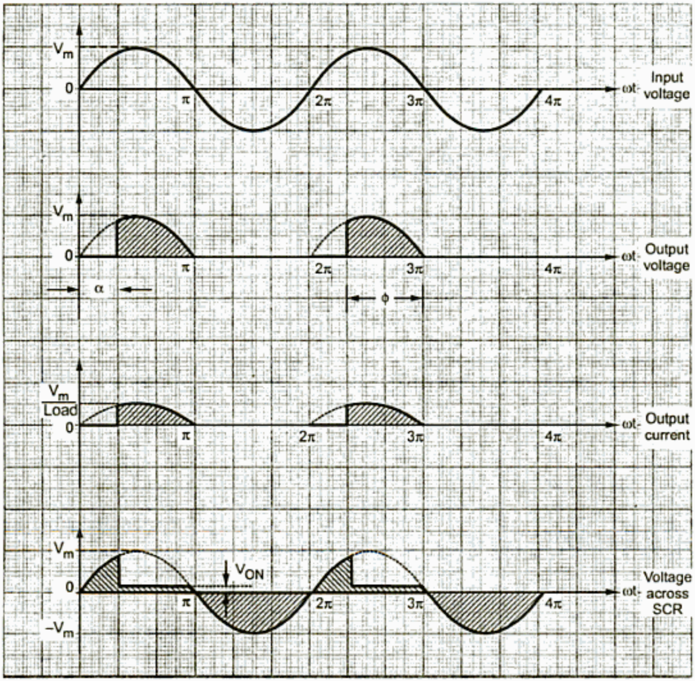


Fig. 8.58 Voltage and current waveforms with a half wave circuit at firing angle α

Average output voltage

The average value of the output voltage can be derived as

$$V_{dc} = \frac{1}{2\pi}\int_0^{2\pi} V_o\,(\omega t)\, d\omega t = \frac{1}{2\pi}\int_\alpha^{2\pi} V_m \sin(\omega t)\, d\omega t = \frac{V_m}{2\pi}(1+\cos\alpha)$$

The r.m.s. output voltage

The r.m.s. value of the output voltage can be derived as

$$V_{o(r.m.s.)} = \left[\frac{1}{2\pi}\int_\alpha^{\pi} V_m^2 \sin(\omega t)^2\, d\omega t\right]^{1/2} = V_m\left[\frac{\pi-\alpha}{4\pi}+\frac{\sin 2\alpha}{8\pi}\right]^{1/2}$$

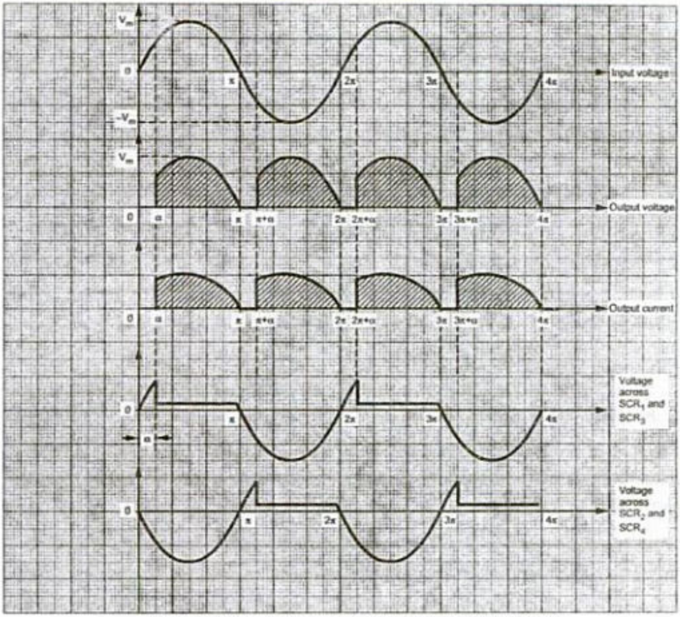


Fig. 8.61 Waveforms of full-wave rectifier load

The comparison of SCR half wave and full wave is given in the Table. 8.5.

Sr. No	Half wave rectifier	Full wave rectifier
1.	One SCR is used.	Two or four SCRs are required.
2.	Only part of positive half cycle is available to the load.	The part of positive half cycle and complete negative half cycle is available to the load.
3.	The centre tap transformer is not required.	The centre tap transformer is required which makes the circuit bulky and heavier.
4.	The output voltage is less compared to full wave.	The output voltage is high compared to half wave.
5.	$V_{d.c.(av)} = \frac{V_m}{2\pi}(1+\cos\alpha)$	$V_{d.c.(av)} = \frac{V_m}{\pi}(1+\cos\alpha)$
6.	The ripple contents are high.	The ripple contents are less.
7.	Simple to design as only one gate signal is necessary.	Two or four gate signals must be properly controlled to get successful operation hence complicated to design.

Table 8.5

3. SCR crowbar : Crowbar circuits provide protection against over voltage conditions for entire circuit. The Fig. 8.62 shows crowbar circuit using SCR. The zener diode in the circuit is selected such that at normal output voltage it acts as an open switch. Hence voltage across R is zero and SCR remains open.

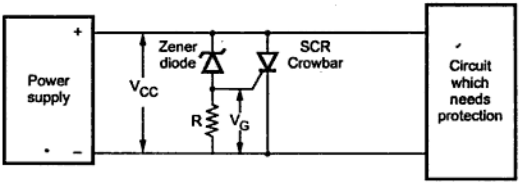


Fig. 8.62 Crowbar circuit using SCR

When output voltage of the supply increases than the normal supply voltage by any reason, the zener diode conducts and a voltage appears across R. This voltage is sufficient to turn ON the SCR. The conduction of SCR reduces the voltage drop across it and thus protects the circuit from large over voltage.

➡ **Example 8.7 :** *A SCR half-wave rectifier circuit operates for a resistive load of value 100 Ω. The forward breakover voltage of SCR is 100 V for a gate current of 2 mA. If input applied is 200 V peak, find*

(i) Firing angle α *(ii) Conduction angle β*

(iii) Average output voltage $V_{o(d.c.)}$ *(iv) Average output power*

Solution : V_m = Peak value of input = 200 V

V_{BO} = 100 V with I_G = 2 mA and R_L = 100 Ω

i) Firing angle $\alpha = \sin^{-1}\frac{V_{BO}}{V_m} = \sin^{-1}\frac{100}{200} = \mathbf{30^\circ}$

ii) Conduction angle $\beta = 180^\circ - \alpha = \mathbf{150^\circ}$

iii) Average output voltage $V_{d.c.(av)} = \frac{V_m}{2\pi}(1+\cos\alpha) = \frac{200}{2\pi}(1+\cos 30^\circ) = \mathbf{59.39\ V}$

iv) $P_{d.c.(av)} = \frac{V^2_{d.c.(av)}}{R_L} = \frac{(59.39)^2}{100} = \mathbf{35.2717\ W}$

➡ **Example 8.8 :** *In a particular application single phase half wave rectifier using SCR is used. The average load voltage is 80 V. If supply voltage is 230 V, 50 Hz a.c., find the firing angle of the SCR.*

Solution : $V_{d.c.\,(av)}$ = 80 V, $V_{r.m.s.}$ = 230 V

Note that given a.c. mains voltage is r.m.s. unless and otherwise specified to be peak.

$$\therefore \quad V_m = \sqrt{2}V_{r.m.s.} = \sqrt{2}\times 230 = 325.269 \text{ V}$$

Now $$V_{d.c.(av)} = \frac{V_m}{2\pi}(1+\cos\alpha)$$

$$\therefore \quad 80 = \frac{325.269}{2\pi}(1+\cos\alpha)$$

$$\therefore \quad \cos\alpha = 0.5453$$

$$\therefore \quad \alpha = 56.95^\circ \quad \text{... Required firing angle}$$

Example 8.9 : *In a particular application single phase half wave rectifier using SCR is used. The supply voltage is 325 sin ωt where ω = 100π rad/sec. Find the time for which SCR remains OFF if forward breakover voltage is 125 V.*

Solution : For half wave rectifier, the SCR operates as shown in the Fig. 8.63.

$$V_{in} = 325 \sin\omega t$$

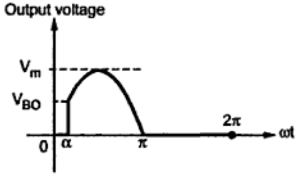


Fig. 8.63

$$= V_m \sin\omega t$$

$$\therefore \quad V_m = 325 \text{ V}$$

$$\omega = 100\pi \text{ rad/sec.}$$

$$V_{BO} = 125 \text{ V}$$

$$\therefore \quad \alpha = \sin^{-1}\frac{V_{BO}}{V_m} = \sin^{-1}\frac{125}{325} = 22.619^\circ$$

$$\therefore \quad \alpha = 22.619 \times \frac{\pi}{180} \text{ radians} = 0.3947 \text{ rad}$$

$$\therefore \text{Time of } \alpha = \frac{\alpha}{\omega} = \frac{0.3947}{100\,\pi} = 0.00125 \text{ sec} = 1.25 \text{ msec}$$

For this period SCR remains OFF in positive half cycle.

While for entire negative half cycle i.e for π radians (180°) it remains OFF. Thus corresponding time is $\frac{\text{angle}}{\omega}$.

i.e. $$\frac{\pi}{100\pi} = 10 \text{ msec}$$

Total time for which SCR is OFF = 10 + 1.25 = **11.25 msec**

Example 8.10 : *A sinusoidal voltage V_i = 200 sin 314 t is applied to an SCR whose forward breakdown voltage is 150 V. Determine the time during which SCR remains OFF.*

[JNTU : Nov.-2008 (Set - 3)]

19. *Derive the relationship between α_{dc} and β_{dc}.*
20. *What is early effect ?*
21. *What is punch through effect ?*
22. *Explain the transfer characteristics of CB configuration.*
23. *Draw and explain input and output characteristics of a transistor CB configuration.*
24. *Sketch the family of common base output characteristics for a transistor. Indicate the active, cut-off and saturation regions. Explain the shapes of the curves qualitatively.*
25. *Explain the various current components of the transistor.*
26. *What is reverse saturation current ?*
27. *What is I_{CBO} ? Why is it greater than I_{CO} ?*
28. *Define I_{CEO}.*
29. *Draw the output characteristics of BJT in CE configuration. Indicate all the three regions of operation on it.*
30. *Draw and explain input and output characteristics for transistor CE configuration.*
31. *Draw and explain the input and output characteristics of a transistor in CC configuration.*
32. *List the comparison between CB, CE and CC amplifiers.*
33. *What is biasing ?*
34. *State the biasing conditions required for the three regions of operation of a BJT.*
35. *Why biasing is needed in a transistor ?*
36. *What is d.c. load line ? Derive its equation for a CE amplifier.*
37. *What do you understand by D.C. load line and D.C. bias point? Explain their significance.*
38. *Explain the criteria for selection of the operating point.*
39. *Explain how transistor can be used as an amplifier with the help of D.C. load line approach.*
40. *Draw the circuit diagram of a common emitter amplifier. State the function of each component used in the above circuit.*
41. *Explain how phase reversal of the signal takes place when it is amplified using common emitter amplifier.*
42. *Draw a neat circuit diagram of a single stage CE amplifier.*
43. *Draw a neat circuit diagram of common base R-C coupled amplifier and indicate all necessary components. Also sketch the input and output waveforms for sinusoidal signal. Give the applications of this configuration.*
44. *Give the constructional details of SCR.*
45. *Draw the SCR characteristics and explain :*
    (a) Holding current, (b) Latching current, (c) Reverse breakdown voltage and
    (d) Forward breakover voltage
46. *Explain two transistor analogy of SCR.*
47. *Explain the different methods of turning ON SCR.*
48. *Explain the mechanism of turning OFF the SCR.*

9 Cathode Ray Oscilloscope

9.1 Motion of Charge in Constant Electric Field

Let us consider the motion of a charged particle in the uniform constant electric field. Such an uniform constant electric field is the field produced between the two plates of a parallel plate capacitor as shown in the Fig. 9.1. The potential difference V is applied between the two plates. The direction of the electric field between the plates is shown in the Fig. 9.1.

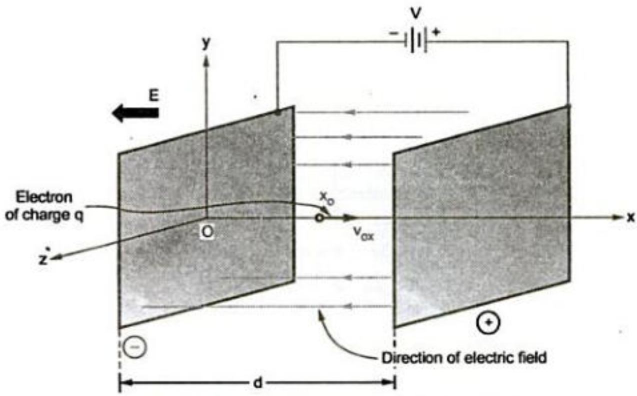


Fig. 9.1 Motion of charge in constant electric field

The following assumptions are made while doing the analysis.

1. The plates are placed in evacuated envelope i.e. in high vacuum so that no collisions of an electron with gas atoms or ions occur.
2. The electron is point mass as its mass is extremely small. Hence gravitational force can be neglected.

3. The distance d between the plates is very small compared with the dimensions of the plates. Hence electric field E can be considered to be uniform. The lines of force are pointing towards negative direction of x axis.
4. The initial conditions are,

$$v_x = v_{ox}, \quad x = x_0 \quad \text{when } t = 0 \qquad \text{... (1)}$$

Thus the initial velocity of an electron is chosen along E, the lines of force and the initial position x_0 of the electron is along the x-axis.

Now as E is pointing along only x direction, E_y and E_z which are components of E in y and z direction are zero. Thus there is no force along y and z direction. According to Newton's law of motion, as force is zero, the acceleration along y and z axes is zero. The zero acceleration means constant velocity. But initial velocity of an electron is along x axis and is zero along y and z axes. Hence **electron will not move along y and z direction.**

Key Point: *As electron moves along x axis only, the motion is one dimensional.*

The equations of motion according to Newton's law are,

$$a_x = \frac{dv_x}{dt} = \text{Acceleration in x direction}$$

$$a_y = a_z = 0$$

But $$F = qE = m\,a \qquad \text{... (magnitude)}$$

$\therefore$ $$qE = m\,a_x$$

$\therefore$ $$\boxed{a_x = \frac{qE}{m} = \text{constant}} \qquad \text{... (2)}$$

where E = Magnitude of the electric field in V/m

Key Point: *This indicates that an electron or a charged particle moves with a constant acceleration in the uniform electric field.*

The velocity of an electron in x direction at any time t can be obtained by an integration as,

$$a_x = \frac{qE}{m} = \frac{dv_x}{dt}$$

$\therefore$ $$v_x = \int \frac{qE}{m}\,dt + v_{ox}$$

For constant uniform electric field,

$\therefore$ $$\boxed{v_x = \frac{qE}{m}t + v_{ox} = a_x\,t + v_{ox}} \qquad \text{... (3)}$$

The position of an electron along x direction can be obtained as,

$$v_x = \frac{dx}{dt} \quad \text{where } x = \text{position along x direction}$$

$$\therefore \quad x = \int v_x \, dt = \int [a_x t + v_{ox}] dt + c_1 = \int a_x t \, dt + \int v_{ox} \, dt + c_1$$

$$\therefore \quad x = \frac{a_x t^2}{2} + v_{ox} t + c_1$$

At $x = x_o$, $t = 0$ hence $c_1 = x_o$ is initial position.

$$\therefore \quad \boxed{x = \frac{1}{2} a_x t^2 + v_{ox} t + x_o} \quad \text{... (4)}$$

Key Point: *The equations (3) and (4) are applicable only if acceleration a_x is constant and independent of time.*

Time varying field : If the voltage applied to the plates is time varying such as $V_m \sin\omega t$ the field intensity E is also time varying and the function of time. This function must be used to find acceleration a_x. Thus acceleration a_x will also be the function of time and no longer will be a constant. In such a case the equations (3) and (4) are not valid. Thus for time varying field, the velocity and position of an electron must be obtained by carrying out actual integration of the equations,

$$a_x = \frac{dv_x}{dt} \quad \text{and} \quad v_x = \frac{dx}{dt}$$

where $a_x = \frac{qE}{m}$ and E as a function of time which must be used to find a_x.

Example 9.1 : *An electron at rest is on one plate of a parallel plate capacitor. The distance between the plates is 2 cm and a potential of 10 V is applied across the plates. Find the velocity and position of the electron after 1 nsec. How much time an electron will take to reach to the second plate ?*

Solution : The arrangement is shown in the Fig. 9.2.

Let x = 0 at negative plate and $x = 2 \times 10^{-2}$ m at positive plate.

The E is constant and its magnitude is given by,

$$E = \frac{V}{d} = \frac{10}{2 \times 10^{-2}}$$

$$= 500 \text{ V/m}$$

The electron will move with constant acceleration as field is uniform,

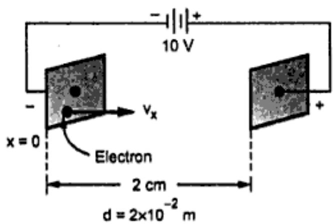


Fig. 9.2

$$a_x = \frac{qE}{m} = \frac{1.6 \times 10^{-19} \times 500}{9.107 \times 10^{-31}} = 8.7844 \times 10^{13} \text{ m / sec}^2$$

The velocity v_x is given by,

$$v_x = a_x t + v_{ox} \qquad \text{... } v_{ox} = 0 \text{ as electron is at rest}$$

i) After t = 1 nsec = 1×10^{-9} sec,

$$v_x = a_x t = 8.7844 \times 10^{13} \times 1 \times 10^{-9} = 8.7844 \times 10^4 \text{ m/sec}$$

and $$x = \frac{1}{2} a_x t^2 + v_{ox} t + x_o \text{ ...... } v_{ox} = x_o = 0$$

$$\therefore \quad x = \frac{1}{2} \times 8.7844 \times 10^{13} \times (1 \times 10^{-9})^2 = 4.3922 \times 10^{-5} \text{ m} = \mathbf{0.00439 \text{ cm}}$$

ii) When electron reaches to second plate, $x = 2 \times 10^{-2}$ m.

$$\therefore \quad x = \frac{1}{2} a_x t^2$$

$$\therefore \quad 2 \times 10^{-2} = \frac{1}{2} \times 8.7844 \times 10^{13} t^2$$

$$\therefore \quad t^2 = 4.5535 \times 10^{-16}$$

$$\therefore \quad t = 2.1338 \times 10^{-8} \text{ sec} = \mathbf{21.338 \text{ nsec}}$$

9.2 Potential

The **potential** V of a point x with respect to the point x_o is defined as the work done against the field in taking unit positive charge from the position x_o to x.

Mathematically potential can be expressed as,

$$\therefore \quad \boxed{V = -\int_{x_o}^{x} E_x \, dx} \qquad \text{... (1)}$$

The negative sign indicates against the direction of electric field.

Relation between V and E :

Integrating the equation (1),

$$V = -E_x [x]_{x_o}^{x} = -E_x [x - x_o]$$

$$\therefore \quad E_x = -\frac{V}{x - x_o}$$

If the plate 1 is at x_o and plate 2 is at x where distance $d = x - x_o$, then for voltage V applied across the plates,

$$\therefore \quad \boxed{E_x = -\frac{V}{d}} \quad \text{... For constant fields}$$

While differentiating both sides of equation (1),

$$\therefore \quad \boxed{E_x = -\frac{dV}{dx}} \quad \text{... For fields varying with distance}$$

Thus electric field E is the negative of the ratio of the applied potential difference between the two points and the distance between those two points. The negative sign indicates that it is directed from region of higher potential to the lower potential.

9.2.1 Energy Acquired by an Electron

Consider an electron moving in the field between two parallel plates, against the direction of field. According to Newton's second law of motion,

$$-\frac{q E_x}{m} = \frac{d v_x}{dt} \quad \text{...} \frac{d v_x}{dt} = a_x \text{ and } F = -q E$$

The velocity of an electron in x direction is,

$$v_x = \frac{dx}{dt}$$

$$\therefore \quad dx = v_x \, dt$$

Multiplying both sides by $dx = v_x \, dt$,

$$-\frac{q E_x}{m} dx = \frac{dv_x}{dt} dx = \frac{dv_x}{dt} [v_x \, dt]$$

$$\therefore \quad -\frac{q E_x}{m} dx = v_x \, d v_x$$

Integrating both sides, with initial values as x_o and v_{ox}

$$-\frac{q}{m} \int_{x_o}^{x} E_x \, dx = \int_{v_{ox}}^{v_x} v_x \, d v_x$$

Using result (1),

$$-\int E_x \, dx = V$$

$$\frac{+q V}{m} = \int_{v_{ox}}^{v_x} v_x \, d v_x = \left[\frac{v_x^2}{2}\right]_{v_{ox}}^{v_x}$$

$$\therefore \quad \boxed{qV = +\frac{1}{2}\, m \left[v_x^2 - v_{ox}^2\right]} \quad \text{... (2)}$$

where qV = Energy in joules

This equation (2) states that when electron falls through a potential of V volts while travelling from x_o and x then it acquires a kinetic energy and velocity.

Key Point : *The velocity and kinetic energy values are not dependent on the type of distribution of the electric field intensity between x_o and x but only* ***depends on the potential difference V between x_o and x.***

Note :

1. The equation (2), though derived assuming constant electric field intensity, it is applicable if the electric field intensity is function of distance.
2. The derivation assumes that electric field intensity is one dimensional (only in x direction) but it is applicable for multidimensional electric field intensities.
3. The equation (2) is nothing but the **law of conservation of energy.** The left hand side is the rise in the potential energy while right hand side is the drop in the kinetic energy. According to law of conservation of energy, **rise in potential energy is equal to drop in the kinetic energy. Thus total energy remains unchanged.**

So if two points A and B are considered in space and B is at higher potential than A by V_{BA} then equation (2) becomes,

$$\therefore \quad \boxed{q\,V_{BA} = \frac{1}{2} m\left[v_A^2 - v_B^2\right] = \frac{1}{2}\, m\, v_A^2 - \frac{1}{2}\, m\, v_B^2} \quad \text{... (3)}$$

where q = Charge in coulombs

$q\,V_{BA}$ = Potential energy in joules

v_A = Initial speed in m/sec

v_B = Final speed in m/sec

The product of the potential between two points and the charge moving is the **potential energy** between the two points. So equation (3) states that,

Rise in potential energy = Drop in kinetic energy

Key Point : *The equation (3) is* ***not applicable for the time varying fields.***

From equation (2)

$$qV = \frac{1}{2} m \left[v_f^2 - v_{ox}^2\right]$$

$$\therefore \quad v_f^2 = v_{ox}^2 + \frac{2qV}{m}$$

$$\therefore \quad \boxed{v_f = \sqrt{v_{ox}^2 + \frac{2qV}{m}}} \quad \text{... (8)}$$

Let t_{ka} = time of transit from cathode to anode

$$\therefore \quad \boxed{t_{ka} = \frac{\text{distance travelled by electron}}{\text{average speed of electron}}}$$

Now d = distance travelled = separation of plates

$$\text{Average speed} = \frac{1}{2}\,[\text{initial speed + final speed}] = \frac{1}{2}\left[v_{ox} + v_f\right]$$

$$\therefore \quad \boxed{t_{ka} = \frac{d}{\frac{v_{ox}+v_f}{2}} = \frac{2d}{v_{ox}+v_f}} \quad \text{... (9)}$$

Example 9.2 : *An electron from rest is accelerated by a potential of 200 V. Find the final velocity of the electron.*

Solution : Initially electron is at rest and V = 200 V

$$\therefore \quad v = \sqrt{\frac{2qV}{m}} = 5.94 \times 10^5 \sqrt{V}$$

$$= 5.94 \times 10^5 \times \sqrt{200} = \mathbf{8.4 \times 10^6 \ m/sec}$$

Example 9.3 : *In a parallel plate diode, the cathode and anode are spaced 5 mm apart and the anode is kept at 220 V d.c. with respect to cathode. The initial velocity of an electron is 2×10^6 m/s in the direction towards anode. Calculate,*

i) velocity and time of electron at midway between cathode and anode

ii) velocity and time of electron when it reaches anode.

Solution : d = 5 mm , V = 200 V, $v_{ox} = 2 \times 10^6$ m/s

$$\therefore \quad E = \frac{V}{d} = \frac{220}{5 \times 10^{-3}} = 4.4 \times 10^4 \ \text{V/m} \quad \text{... Uniform field}$$

9.3 Introduction to C.R.O.

In studying the various electronic, electrical networks and systems, signals which are functions of time, are often encountered. Such signals may be periodic or non periodic in nature. The device which allows, the amplitude of such signals, to be displayed primarily as a function of time, is called **cathode ray oscilloscope**, commonly known as C.R.O. The C.R.O. gives the visual representation of the time varying signals. The oscilloscope has become an universal instrument and is probably most versatile tool for the development of electronic circuits and systems. It is an integral part of electronic laboratories.

The oscilloscope is, in fact, a voltmeter. Instead of the mechanical deflection of a metallic pointer as used in the normal voltmeters, the oscilloscope uses the movement of an electron beam against a fluorescent screen, which produces the movement of a visible spot. The movement of such spot on the screen is proportional to the varying magnitude of the signal, which is under measurement.

The electron beam can be deflected in two directions : the horizontal or x-direction and the vertical or y-direction. Thus an electron beam producing a spot can be used to produce two dimensional displays. Thus C.R.O. can be regarded as a fast x-y plotter. The x-axis and y-axis can be used to study the variation of one voltage as a function of another. Typically the x-axis of the oscilloscope represents the time while the y-axis represents variation of the input voltage signal. Thus if the input voltage signal applied to the y-axis of C.R.O. is sinusoidally varying and if x-axis represents the time axis, then the spot moves sinusoidally and the familiar sinusoidal waveform can be seen on the screen of the oscilloscope. The oscilloscope is so fast device that it can display the periodic signals whose time period is as small as microseconds and even nanoseconds.

Key Point: *The C.R.O. basically operates on voltages, but it is possible to convert current, pressure, strain, acceleration and other physical quantities into the voltage using transducers and obtain their visual representations on the C.R.O.*

9.4 Cathode Ray Tube (CRT)

The cathode ray tube (CRT) is the heart of the C.R.O. The CRT generates the electron beam accelerates the beam, deflects the beam and also has a screen where beam becomes visible as a spot.

The main parts of the CRT are :

1. Electron gun
2. Deflection system
3. Fluorescent screen
4. Glass tube or envelope
5. Base

A schematic diagram of CRT, showing its structure and main components is shown in the Fig. 9.3.

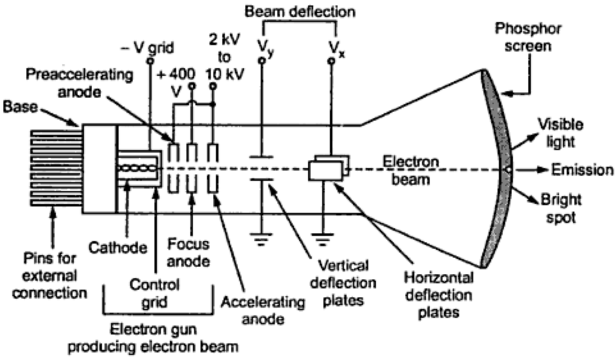


Fig. 9.3 Cathode ray tube

9.4.1 Electron Gun

The electron gun section of the cathode ray tube provides a sharply focused electron beam directed towards the fluorescent - coated screen. This section starts from thermally heated cathode, emitting the electrons. The control grid is given negative potential with respect to cathode. This grid controls the number of electrons in the beam, going to the screen.

The momentum of the electrons (their number × their speed) determines the intensity, or brightness, of the light emitted from the fluorescent screen due to the electron bombardment. The light emitted is usually of the green colour. Because the electrons are negatively charged, a repulsive force is created by applying a negative voltage to the control grid (in CRT, voltages applied to various grids are stated with respect to cathode, which is taken as common point).This negative control voltage can be made variable. A more negative voltage results in less number of electrons in the beam and hence decreases brightness of the beam spot.

Since the electron beam consists of many electrons, the beam tends to diverge. This is because the similar (negative) charges on the electron repel each other. To compensate for such repulsion forces, an adjustable electrostatic field is created between two cylindrical anodes, called the focusing anodes. The variable positive voltage on the second anode is used to adjust the focus or sharpness of the bright beam spot.

The high positive potential is also given to the preaccelerating anodes and accelerating anodes, which results into the required acceleration of the electrons.

Both focusing and accelerating anodes are cylindrical in shape having small openings located in the centre of each electrode, co-axial with the tube axis. The preaccelerating and accelerating anodes are connected to a common positive high voltage which varies between 2 kV to 10kV. The focusing anode is connected to a lower positive voltage of about 400 V to 500 V.

9.4.2 Deflection System

When the electron beam is accelerated it passes through the deflection system, with which beam can be positioned any where on the screen.

The deflection system of the cathode-ray-tube consists of two pairs of parallel plates, referred to as the vertical and horizontal deflection plates. One of the plates in each set is connected to ground (0V).To the other plate of each set, the external deflection voltage is applied through an internal adjustable gain amplifier stage. To apply the deflection voltage externally, an external terminal, called the Y input or the X input, is available.

As shown in the Fig. 9.3, the electron beam passes through these plates. A positive voltage applied to the Y input terminal (V_y) causes the beam to deflect vertically upwards due to the attraction forces, while a negative voltage applied to the Y input terminal will cause the electron beam to deflect vertically downwards, due to the repulsion forces.

Similarly, a positive voltage applied to X-input terminal (V_x) will cause the electron beam to deflect horizontally towards the right; while a negative voltage applied to the X-input terminal will cause the electron beam to deflect horizontally towards the left of the screen. The amount of vertical or horizontal deflection is directly proportional to the corresponding applied voltage.

When the voltages are applied simultaneously to vertical and horizontal deflecting plates, the electron beam is deflected due to the resultant of these two voltages.

The face of the screen can be considered as an x-y plane. The (x,y) position of the beam spot is thus directly influenced by the horizontal and the vertical voltages applied to the deflection plates V_x and V_y, respectively.

The horizontal deflection (x) produced will be proportional to the horizontal deflecting voltage V_x, applied to X-input.

$$\therefore \quad x \propto V_x$$

$$\therefore \quad \boxed{x = K_x V_x \quad \text{where } K_x \text{ is constant of proportionality.}}$$

The deflection produced is usually measured in cm or as number of divisions on the scale, in the horizontal direction.

Then $K_x = \frac{x}{V_x}$ where K_x expressed as cm/volt or division/volt, is called **horizontal sensitivity** of the oscilloscope.

Similarly, the vertical deflection (y) produced will be proportional to the vertical deflecting voltage , V_y , applied to the y-input.

$$\therefore \quad y \propto V_y$$

$$\therefore \quad \boxed{y = K_y V_y}$$

where $K_y = y/V_y$ and K_y is the **vertical sensitivity,** which is expressed as cm/volt, or division/volt.

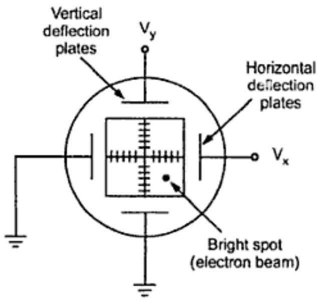


Fig. 9.4

The values of vertical and horizontal sensitivities are selectable and adjustable through multipositional switches on the front panel that controls the gain of the corresponding internal amplifier stages. The bright spot of the electron beam can thus trace (or plot) the x-y relationship between the two voltages, V_x and V_y.

The schematic arrangement of the vertical and the horizontal plates, controlling the position of the spot on the screen is shown in the Fig. 9.4.

9.4.3 Fluorescent Screen

The light produced by the screen does not disappear immediately when bombardment by electrons ceases, i.e., when the signal becomes zero. The time period for which the trace remains on the screen after the signal becomes zero is known as **"persistence or fluorescence"**. The persistence may be as short as a few microsecond, or as long as tens of seconds or even minutes.

Medium persistence traces are mostly used for general purpose applications.

Long persistence traces are used in the study of transients. Long persistence helps in the study of transients since the trace is still seen on the screen after the transient has disappeared.

Short persistence is needed for extremely high speed phenomena.

The screen is coated with a fluorescent material called phosphor which emits light when bombarded by electrons. There are various phosphors available which differ in colour, persistence and efficiency. One of the common phosphor is Willemite, which is zinc, orthosilicate, $ZnO + SiO_2$, with traces of manganese. This produces the familiar greenish trace. Other useful screen materials include compounds of zinc, cadmium, magnesium and silicon.

9.4.3.1 Types of Phosphors

The various types of phosphors are :

The type P1, P2, P11 or P31 are the short persistence phosphors and are used for the general purpose oscilloscopes.

Medical oscilloscopes require a longer phosphor decay and hence like P7 and P39 are preferred for such applications.

Very slow displays like radar require long persistence phosphors to maintain sufficient flicker free picture. Such phosphors are P10 , P26 and P33.

The phosphors P10, P26, P33 have low burn resistance. The phosphors P1, P2, P4, P7, P11 have medium burn resistance while P15, P31 have high burn resistance.

9.4.3.2 Advantages of P31

Out of these varieties, the material P31 is used commonly for general purpose oscilloscopes due to following characteristics :

1. it gives colour to which human eye response is maximum.
2. it gives short persistence required to avoid multiple image display.
3. it has high burn resistance to avoid the accidental damage.
4. its illumination level is high.
5. it provides high writing speed.

The light output of a fluorescent screen is proportional to the number of bombarding electrons, i.e. to the beam current.

9.4.3.3 Functions of Aluminium Layer

The phosphor screen is provided with an aluminium layer called **aluminizing** the cathode ray tube. This is shown in the Fig. 9.5.

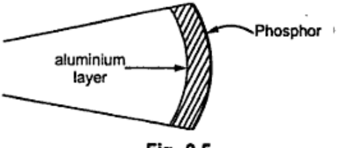


Fig. 9.5

Such a layer serves three functions :

1. To avoid build up of charges on the phosphor which tend to slow down the electrons and limits the brightness.
2. It serves as a light scatter. When the beam strikes the phosphor with aluminized layer, the light emitted back into the tube is reflected back towards the viewer which increases the brightness.

zero in a short interval of time, known as **flyback period**. Hence the beam suddenly jumps back to the original positions at the extreme left hand side. Then again it starts moving to the right during the next cycle of sawtooth waveform. The fly back of the beam is blanked out by a suitable voltage and is not visible on the screen.

Thus for a selected trace time T_r , the spot moves horizontally across the face of the screen along the x-axis from left to right, with a constant speed, restarts again from the left and repeats such traces. Depending on the speed of the bright spot and the persistence of vision, the trace produced by the spot will look like a horizontal straight line. Thus the horizontal axis is now converted into a time axis.

When a periodically varying voltage say sinusoidal voltage is applied to the y terminal of the scope and internally generated sawtooth voltage is applied to the horizontal deflection plates, then sawtooth voltage keeps on shifting the spot horizontally while the applied voltage shifts the spot vertically proportional to its magnitude. Hence finally due to the effect of both the voltage, a familiar sinusoidal waveform can be observed on the screen.

When the sweep and signal frequencies are equal, a single cycle appears on the screen. When the sweep is lower than the signal, several cycles appear on the screen. In such case, the number of cycles depends on the ratio of the two frequencies. When the sweep is higher than the signal, less than one cycle appears on the screen.

The display of spot on the screen appears stationary only when the two frequencies i.e. sweep and signal are same or are integral multiples of each other. For any other frequencies the trace on the screen keeps on drifting horizontally. Thus for the trace to appear stationary, the sawtooth voltage is synchronized with signal applied to the vertical input. For the vertical input signal, the triggering pulses are derived for the synchronization.

9.5.1 Requirements of Sweep Generator

There are two important requirements of a sweep generator :

1. The sweep must be linear in nature, for all screen horizontal deflection.
2. To move the spot in one direction only, the sweep voltage must drop to zero suddenly, after reaching to its maximum value. Otherwise the return sweep will trace the signal backwards.

9.6 Block Diagram of Oscilloscope

The block diagram of oscilloscope is shown in the Fig. 9.7.

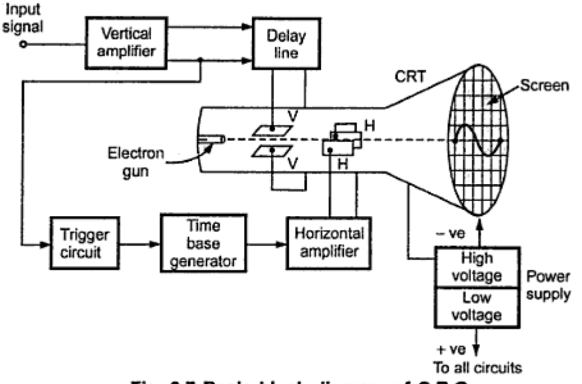


Fig. 9.7 Basic block diagram of C.R.O.

The various blocks of block diagram of simple oscilloscope are as follows :

9.6.1 CRT

This is the cathode ray tube which is heart of C.R.O. It is used to emit the electrons required to strike the phosphor screen to produce the spot for the visual display of the signals.

9.6.2 Vertical Amplifier

The input signals are generally not strong to provide the measurable deflection on the screen. Hence the vertical amplifier stage is used to amplify the input signals. The amplifier stages used are generally wide band amplifiers so as to pass faithfully the entire band of frequencies to be measured.

Similarly it contains the attenuator stages as well. The attenutors are used when very high voltage signals are to be examined, to bring the signals within the proper range of operation.

9.6.3 Delay Line

The delay line is used to delay the signal for some time in the vertical sections. When the delay line is not used, the part of the signal gets lost. Thus the input signal is not applied directly to the vertical plates but is delayed by some time using a delay line circuit.

The signal applied to horizontal section passes through trigger circuit, sweep generator and horizontal amplifier before it reaches to deflection plates. So it takes some time to reach horizontal plates. The signal in vertical section must be delayed by same amount so

The electrostatic focusing uses two parallel plates called **deflecting plates.** These are spaced at a distance d from each other. The plates are subjected to a potential difference V_d due to which an electric field E exists between the two plates. While electrons are emitted by the hot cathode K, which are accelerated towards the anode by the potential V_a. Those electrons which are not collected by anode pass through small hole at the anode. This is shown in the Fig. 9.9. These electrons are then focused with the help of vertical deflecting plates on the screen coated with fluorescent material. Properly focused electrons strike the screen and positions where the electrons strike become visible to an eye. This is called **electrostatic deflection** of the electrons.

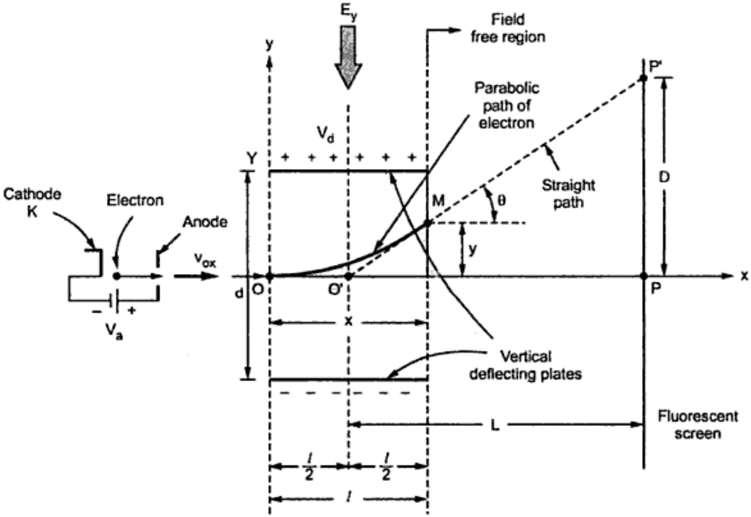


Fig. 9.9 Electrostatic deflection

The electrons are emitted by cathode and emerge from anode hole with a velocity determined by voltage V_a given by,

$$v_{ox} = \sqrt{\frac{2\,q\,V_a}{m}} \qquad \text{... (1)}$$

This is initial velocity with which an electron enters into an electric field E_y given by,

$$E_y = \frac{V_d}{d} \qquad \text{... (2)}$$

The field E_y is in the y-direction, towards negative y axis while velocity v_{ox} of an electron is directed towards positive x-axis. This is **two dimensional** motion. Hence electron moves along the **parabolic path** between the plates from point O to M as shown in the Fig. 9.9.

$$\therefore \quad y = \left[\frac{1}{2}\frac{a_y}{v_{ox}^2}\right] x^2 \qquad \text{(Refer section 1.8)} \quad \ldots (3)$$

$$\text{where} \quad a_y = \frac{E_y\, q}{m} = \frac{q\, V_d}{d\, m} \quad \ldots (4)$$

Once an electron leaves the region of field, it moves linearly in a **straight line** path till it reaches to the screen. This is path MP′ shown in the Fig. 9.9.

The initial velocity is v_{ox} and there is no force acting in x direction hence v_{ox} remains constant in x direction.

$$\therefore \quad x - v_{ox}\, t \quad \ldots (5)$$

The slope at any point along the parabolic path is,

$$\frac{dy}{dx} = \frac{d}{dx}\left[\left(\frac{1}{2}\frac{a_y}{v_{ox}^2}\right)x^2\right] \quad \ldots \text{from equation (3)}$$

$$\therefore \quad \frac{dy}{dx} = \frac{1}{2}\frac{a_y}{v_{ox}^2}(2\,x) = \frac{x\, a_y}{v_{ox}^2} \quad \ldots (6)$$

From the Fig. 9.9, $\tan\theta = \frac{dy}{dx}$

Hence the slope at point M which is at a distance $x = l$ is given by,

$$\tan\theta = \left.\frac{dy}{dx}\right|_{x=l} = \frac{l\, a_y}{v_{ox}^2} \quad \ldots (7)$$

From point M onwards electron moves along the straight line which is tangent to the parabola at $x = l$ and intersects x axis at O′. This is called **apparent origin.**

The equation of the straight line MP′ is,

$$y = m\,x + C \text{ where } m = \text{slope} = \left.\frac{dy}{dx}\right|_{x=l}$$

$$\therefore \quad y = \frac{l\, a_y}{v_{ox}^2}\, x + C \quad \ldots (8)$$

$$\text{At } x = l, \quad y = \frac{1}{2}\frac{a_y}{v_{ox}^2}\, l^2 \quad \text{from equation (3)}$$

$$\therefore \qquad \frac{1}{2}\frac{a_y}{v_{ox}^2}\, l^2 = \frac{l^2 a_y}{v_{ox}^2} + C$$

$$\therefore \qquad C = -\frac{a_y}{v_{ox}^2}\frac{l^2}{2}$$

So the equation of straight line,

$$y = \frac{l\,a_y}{v_{ox}^2}\,x - \frac{a_y}{v_{ox}^2}\frac{l^2}{2}$$

$$\therefore \qquad y = \frac{l\,a_y}{v_{ox}^2}\left[x - \frac{l}{2}\right] \qquad \text{... (9)}$$

At O′, y = 0 hence $x = \frac{l}{2}$ from equation (9).

Key Point: *Thus point O′ is at the centre of the plates and is called* ***virtual cathode*** *as electron appears to emerge from O′ regardless of V_a and V_d following the straight line path O′MP′.*

Now at point P′, y = D and $x = L + \frac{l}{2}$ hence from equation (9),

$$D = \frac{l\,a_y}{v_{ox}^2}\left[L + \frac{l}{2} - \frac{l}{2}\right]$$

$$\therefore \qquad \boxed{D = \frac{l\,L\,a_y}{v_{ox}^2}} \qquad \text{... (10)}$$

From equations (4) and (1) we have,

$$v_{ox}^2 = \frac{2\,q\,V_a}{m}$$

$$a_y = \frac{q\,V_d}{d\,m}$$

$$\therefore \qquad D = l\,L\left(\frac{q\,V_d}{d\,m}\right) \times \frac{1}{\left(\frac{2\,q\,V_a}{m}\right)}$$

$$\therefore \qquad \boxed{D = \frac{l\,L\,V_d}{2\,d\,V_a}} \qquad \text{... Required deflection in metres (11)}$$

From above equation (11), following observations can be made,

1. For a given accelerating voltage V_a and for the particular dimensions of cathode ray tube, the deflection of the beam is directly proportional to the deflecting voltage V_d. So cathode ray tube can be used as a linear voltage indicating device.
2. It is assumed that V_d is fixed d.c. voltage but practically it is time varying and deflection follows it in a linear manner.
3. The deflection is independent of the ratio q/m.

9.7.1 Electrostatic Deflection Sensitivity

The **electrostatic deflection sensitivity** is defined as deflection on the screen in metres per volt of the deflection voltage. It is denoted as S.

$$\therefore \quad \boxed{S = \frac{D}{V_d} = \frac{lL}{2d\,V_a} \quad \text{m/V}}$$

Similarly the **deflection factor** G of a cathode ray tube is given by the reciprocal of the sensitivity.

$$\therefore \quad \boxed{G = \frac{1}{S} = \frac{2d\,V_a}{lL} \quad \text{V/m}}$$

Thus it can be observed that decreasing the accelerating voltage, sensitivity can be increased. But this decreases the brightness of spot. On the other hand high V_a values, produces a bright spot. But for high V_a, high V_d is required and such a beam which is highly accelerated is difficult to deflect. Such a beam is called **hard beam.** The typical values of deflection sensitivities are 1.0 mm/V to 0.1 mm/V corresponding to the deflection factors of 10 V/cm to 100 V/cm respectively.

Example 9.4 : *In a C.R.T., the distance between the plates is 1 cm, the length of the deflecting plates is 4.5 cm and the distance of the screen from the centre of the plates is 33 cm. If the accelerating voltage is 300 V and deflecting voltage is 50 V, find*
i) Velocity of electron reaching the field
ii) Deflection produced on the screen
iii) Deflection sensitivity.

Solution : d = 1 cm, L = 33 cm, l = 4.5 cm, V_a = 300 V, V_d = 50 V

i) $$v_{ox} = \sqrt{\frac{2qV_a}{m}} = \sqrt{\frac{2\times1.6\times10^{-19}\times300}{9.107\times10^{-31}}} = \mathbf{1.0267\times10^{7}\ m/s}$$

ii) $$D = \frac{lL\,V_d}{2d\,V_a} = \frac{4.5\times10^{-2}\times33\times10^{-2}\times50}{2\times1\times10^{-2}\times300} = 0.1237\text{ m} = \mathbf{12.37\ cm}$$

iii) $$S = \frac{D}{V_d} = \frac{0.1237}{50} = 2.474\times10^{-3}\ \text{m/V} = \mathbf{2.474\ mm/V}$$

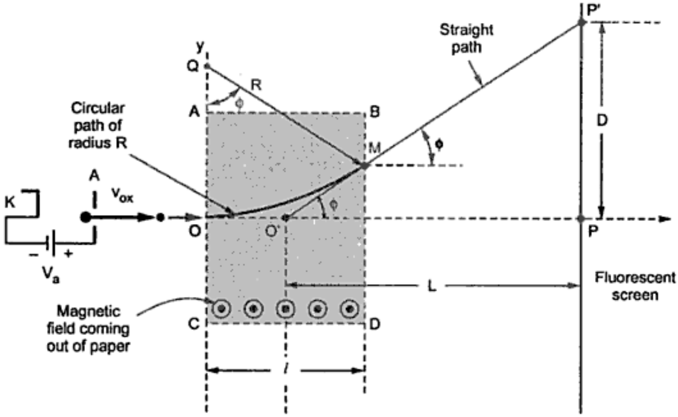


Fig. 9.10 Magnetic deflection

As electron enters the magnetic field, it experiences a constant force and is perpendicular to both, direction of motion and magnetic field. So electron moves along the **circular path** shown by an arc OM having radius R, whose centre is Q.

$$\therefore \qquad R = \frac{m\,v}{q\,B} \qquad \text{... (2)}$$

The angle ϕ subtended by the arc OM at the centre is,

$$\phi = \frac{\text{arc length(OM)}}{R}$$

But for small deflection, arc length OM ≈ l.

$$\therefore \qquad \phi = \frac{l}{R} \qquad \text{... (3)}$$

$$\therefore \qquad \phi = \frac{l\,q\,B}{m\,v} \qquad \text{(Using (2)) ... (4)}$$

Key Point: *This expression gives angle in* ***radians*** *and not in degrees.*

Now after leaving the magnetic field at M, the electron travels along straight line path MP' till it reaches screen at P'. This line is tangent to circular path of an electron at M. If extended backwards it intersects x axis at O' which is **centre of the magnetic field.** This is called apparent origin.

Example 9.7 : *It is found that an electron beam is deflected 8 degrees when it traverses a uniform magnetic field, 3 cm wide, having a density of 0.6 mT. Calculate : a) the speed of the electrons, and*
b) the force on each electron. The direction of the beam is normal to that of the flux.

Solution : $\phi = 8°, \ l = 3 \text{ cm}, \ B = 0.6 \text{ mT} = 0.6\times10^{-3} \text{ Wb/m}^2$

a) $$\phi = \frac{l\,q\,B}{m\,v} \quad \text{where } v = \text{velocity of electrons}$$

But ϕ must in radians

$$\therefore \quad \phi = 8° = \frac{8\times\pi}{180} \text{ radians} = 0.1396 \text{ rad}$$

$$\therefore \quad 0.1396 = \frac{3\times10^{-2}\times1.6\times10^{-19}\times0.6\times10^{-3}}{9.107\times10^{-31}\times v}$$

$$\therefore \quad v = \mathbf{22.65\times10^{6} \ m/s}$$

b) $$f_m = B\,q\,v = \text{force on each electron}$$

$$= 0.6\times10^{-3}\times1.6\times10^{-19}\times22.65\times10^{6}$$

$$= \mathbf{2.174\times10^{-15} \ N}$$

9.9 Comparison between Deflection Methods

The electrostatic deflection and the magnetic deflection methods can be compared on the following points :

Sr. No.	Electrostatic deflection	Magnetic deflection
1.	The deflection is achieved by applying voltage to the plates.	The deflection is obtained by controlling the magnetic field by varying current through the coils.
2.	The deflection is inversely proportional to the accelerating voltage V_a.	The deflection is inversely proportional to the square root of the voltage V_a.
3.	The deflection of the beam is smaller, for given V_a.	The scheme gives wider beam deflection, for given V_a.
4.	The sensitivity is given by $S = \frac{L\,l}{2\,d\,V_a}$	The sensitivity is given by, $S = L\,l\sqrt{\frac{q}{m}}\,\frac{1}{\sqrt{2\,V_a}}$
5.	The deflection is independent of the ratio q/m.	The deflection is dependent on the ratio q/m.
6.	For the given display area longer tubes are necessary.	The shorter tubes can be built for the given display area.

While the r.m.s. value of sinusoidal signal can be obtained as,

$$\boxed{V_{RMS} = \frac{V_m}{\sqrt{2}} = \frac{V_{P-P}}{2\sqrt{2}} \text{ only for sinusoidal signals}}$$

Key Point : *The volts/div is nothing but* ***deflection sensitivity*** *of C.R.O.*

Example 9.8 : *Calculate the amplitude and r.m.s. value of the sinusoidal voltage, the waveform of which is observed on C.R.O. as shown in the Fig. 9.11. The vertical attenuation selected is 2 mV/div.*

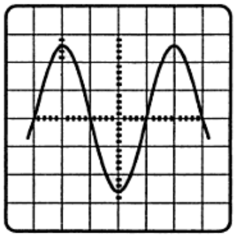


Fig. 9.11

Solution : It can be observed that the screen is divided such that one part is subdivided into 5 units.

$$\therefore \quad 1 \text{ subdivision} = \frac{1}{5} = 0.2 \text{ units}$$

It can be observed that positive peak of signal corresponds to two full divisions and three subdivisions. Hence positive peak is 2 + 3 × 0.2 = 2.6 units while the negative peak also corresponds to 2.6 units.

$$\therefore \quad V_{P-P} = \text{Peak to peak} = 2.6 + 2.6 = 5.2 \text{ divisions}$$

$$\therefore \quad V_{P-P} = \text{Number of divisions} \times \frac{\text{volts}}{\text{division}}$$

$$= 5.2 \times 2 \times 10^{-3} = 10.4 \text{ mV}$$

$$\therefore \quad V_m = \text{Amplitude} = \frac{V_{P-P}}{2} = \frac{10.4}{2}$$

$$= \mathbf{5.2\ mV}$$

and $$V_{RMS} = \frac{V_m}{\sqrt{2}} = \frac{5.2}{\sqrt{2}} = \mathbf{3.6769\ mV}$$

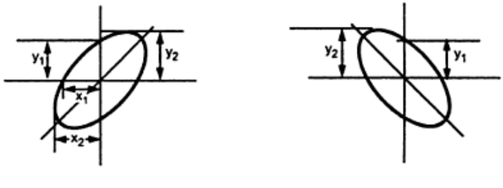


Fig. 9.14

Example 9.10 : *The Lissajous figure obtained on the C.R.O. is shown in the Fig. 9.15. Find the phase difference between the two waves applied.*

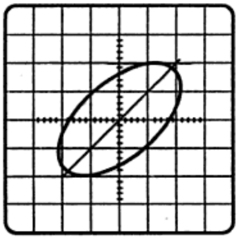


Fig. 9.15

Solution : It can be observed from the Lissajous figures that,

$$y_1 = 8 \text{ units}$$

and $$y_2 = 10 \text{ units}$$

$\therefore$ $$\phi = \sin^{-1}\frac{y_1}{y_2}$$

$$= \sin^{-1}\frac{8}{10} = \mathbf{53.13^\circ}$$

9.11.2 Measurement of Frequency

To measure the unknown frequency, the signal with unknown frequency is applied to vertical deflection plates called f_V. Then signal applied to horizontal deflection plates is obtained from a variable frequency oscillator of known frequency f_H.

Thus, f_H = Frequency of signal applied to horizontal plates which is known.

f_V = Frequency of signal applied to vertical plates which is unknown.

Using the shift control, stationary Lissajous figure is obtained on screen such that to the figure vertical and horizontal axes are tangential to one or more points. The patterns depends on the ratio of two frequencies f_H and f_V as shown in the Fig. 9.16.

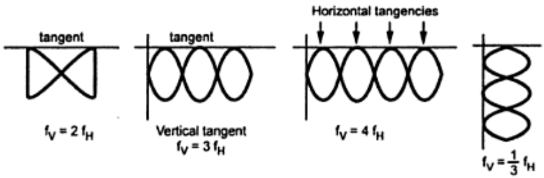


Fig. 9.16

The ratio of two frequencies can be obtained as,

$$\boxed{\frac{f_V}{f_H} = \frac{\text{Number of horizontal tangenicies}}{\text{Number of vertical tangenicies}}}$$

As f_H is known, the unknown frequency can be calculated.

If the ratio of two frequencies is not integral then the pattern is obtained as shown in the Fig. 9.17.

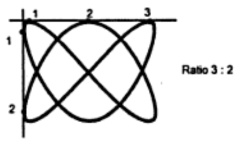


Fig. 9.17

It can be seen that the horizontal tangencies are 3 while vertical tangencies are 2.

Hence, $\frac{f_V}{f_H} = \frac{3}{2} = 1.5$

$\therefore$ $f_V = 1.5\ f_H$

Example 9.11 : *The Lissajous pattern obtained on the screen by applying horizontal signal of frequency 1 kHz, as shown in the Fig. 9.18. Determine the unknown frequency of vertical signal.*

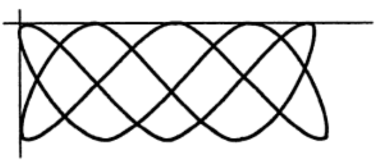


Fig. 9.18

Solution : It can be observed that,

$$\text{Number of vertical tangencies} = 2$$

$$\text{Number of horizontal tangencies} = 5$$

Now, $$\frac{f_V}{f_H} = \frac{5}{2}$$

$\therefore$ $$f_V = \frac{5}{2}$$

$$f_H = \frac{5}{2}\times 1 \text{ kHz}$$

$\therefore$ $$f_V = \mathbf{2.5 \text{ kHz}}$$

This is the unknown frequency.

9.12 Applications of C.R.O.

There are variety of applications in which C.R.O. is used. Some of these applications are :

1. It is used to measure a.c. as well as d.c. voltages and currents. It is useful to calculate the parameters of the voltages as peak to peak value, r.m.s. value, duty cycle etc.
2. In laboratory to measure the frequency, period, phase relationships between the signals and to study periodic as well as nonperiodic signals.
3. In radar, it is used for giving the visual representation of target such as aeroplane, ship etc.
4. In radio applications, it is used to trace and measure a signal through out the RF, IF and AF channels of radio and television receivers. It provides the only effective way of adjusting FM receivers, broadband high frequency RF amplifiers and automatic frequency control circuits.
5. In medical applications, it is used to display the cardiograms which are useful for the diagnosis of heart of the patient. Similarly electromyograms are useful to study muscle condition of the patient.
6. In industry, it is used for many purposes. It is used to observe B-H curves, P-V diagrams and other effects. Used to study the response of various transducers which measure strain, pressure, temperature etc. It is used to observe the radiation pattern generated by transmitting antenna.
7. It is used to determine the modulation characteristics and to detect the standing waves in transmission lines.
8. Curve tracers use the oscilloscope technique for testing the active devices such as vacuum tubes, transistors and integrated circuits.
9. It is used to measure capacitance, inductance and also used to check the diodes. It can be used to check the faulty components in the various circuits.

Examples with Solutions

Example 9.12 : *In a electrostatic deflecting CRT, the length of the deflection plates is 2 cm and spacing between the plates is 0.5 cm. The distance from the centre of the deflecting plates to screen is 20 cm, the deflecting voltage is 25 V. Find the deflection sensitivity, the angle of deflection and the velocity of beam. Assume final anode potential as 1000 V.*

[JNTU, April - 2003 (Set-3)]

Solution : $d = 0.5$ cm, $l = 2$ cm, $L = 20$ cm, $V_a = 1000$ V, $V_d = 25$ V

i) $$D = \frac{lLV_d}{2dV_a} = \frac{2\times10^{-2}\times20\times10^{-2}\times25}{2\times0.5\times10^{-2}\times1000} = 0.01 \text{ m} = \mathbf{1\ cm}$$

$\therefore$ $$S = \frac{D}{V_d} = \frac{0.01}{25} = \mathbf{4 \times 10^{-4}\ m/V}$$

ii) $$\tan\theta = \frac{l\,a_y}{v_{ox}^2}$$

where $$a_y = \frac{qV_d}{dm} = \frac{1.6\times10^{-19}\times25}{0.5\times10^{-2}\times9.107\times10^{-31}} = 8.7844\times10^{14} \text{ m/s}^2$$

$$v_{ox} = \sqrt{\frac{2qV_a}{m}} = \sqrt{\frac{2\times1.6\times10^{-19}\times1000}{9.107\times10^{-31}}} = 18.745\times10^{6} \text{ m/s}$$

$\therefore$ $$\tan\theta = \frac{2\times10^{-2}\times8.7844\times10^{14}}{(18.745\times10^{6})^2} = 0.05$$

$\therefore$ $$\theta = \mathbf{2.862^\circ}$$

iii) $$v_{ox} = \text{Velocity of electrons} = \text{Velocity of beam}$$

$$= \sqrt{\frac{2qV_a}{m}} = \mathbf{18.745\times10^{6}\ m/s}$$

Example 9.13 : *A cathode ray tube has a magnetic deflection field of flux density 10^{-4} Wb/m^2 for a distance of 3 cm, along the tube axis. The distance from the end of the field of the screen is 20 cm. The final anode voltage is 800 V. Calculate the deflection of electron spot in cm and also find the displacement of a charged particle of charge twice that of an electron and mass 7344 times as large as electron.* **[JNTU, April - 2003]**

Solution : $B = 10^{-4}$ Wb/m^2, $l = 3$ cm, $L = 20$ cm, $V_a = 800$ V

$\therefore$ $$D = \frac{lLB}{\sqrt{2V_a}}\times\sqrt{\frac{q}{m}} = \frac{3\times10^{-2}\times20\times10^{-2}\times10^{-4}}{\sqrt{2\times800}}\times\sqrt{\frac{1.6\times10^{-19}}{9.107\times10^{-31}}}$$

$$= \mathbf{6.28\times10^{-3}\ m}$$ ... For electron

For a charged particle, $m' = 7344$ m and $q' = 2q$

Solution : The arrangement is shown in the Fig. 9.19.

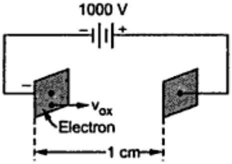


Fig. 9.19

Negative plate is at x = 0

∴ Positive plate is at $x = 1 \times 10^{-2}$ m

$$E = \frac{V}{d} = \frac{1000}{1 \times 10^{-2}} \quad \ldots\ d = 1 \times 10^{-2}\ \text{m}$$

$$= 1 \times 10^{5}\ \text{V/m}$$

As field is uniform, the electron will move with constant acceleration.

$$\therefore \quad x = \frac{1}{2} a_x t^2$$

where $a_x = \frac{qE}{m} = \frac{1.6 \times 10^{-19} \times 1 \times 10^{5}}{9.107 \times 10^{-31}} = 1.7568 \times 10^{16}\ \text{m/s}^2$

So time for electron to reach positive plate is,

$$t^2 = \frac{2x}{a_x} = \frac{2 \times 1 \times 10^{-2}}{1.7568 \times 10^{16}}$$

$$\therefore \quad t = 1.067 \times 10^{-9}\ \text{s} = \mathbf{1.067\ ns}$$

Example 9.16 : *In a cathode ray tube having electric deflection system, the deflecting plates are 2 cm long and have a uniform spacing of 4 mm between them. The fluorescent screen is 25 cm away from the center of the deflection plates. Calculate the deflection sensitivity, if the potential of the final anode is,*

i) 1000 V ii) 2000 V and iii) 3500 V [JNTU : Nov.-2008 (Set-3)]

Solution : l = 2 cm, d = 4 mm , L = 25 cm

$$S = \frac{D}{V_d} = \frac{lL}{2dV_a}\ \text{m/V} \quad \ldots \text{Deflection sensitivity}$$

i) $V_a = 1000$ V

$$\therefore \quad S = \frac{2 \times 10^{-2} \times 25 \times 10^{-2}}{2 \times 4 \times 10^{-3} \times 1000} = 6.25 \times 10^{-4}\ \text{m/V}$$

ii) $V_a = 2000$ V

$$\therefore \quad S = \frac{2 \times 10^{-2} \times 25 \times 10^{-2}}{2 \times 4 \times 10^{-3} \times 2000} = 3.125 \times 10^{-4}\ \text{m/V}$$

iii) $V_a = 3500$ V

$$\therefore \quad S = \frac{2 \times 10^{-2} \times 25 \times 10^{-2}}{2 \times 4 \times 10^{-3} \times 3500} = 1.7857 \times 10^{-4}\ \text{m/V}$$

➡ **Example 9.17 :** *In a vacuum diode, the spacing between the parallel plates of cathode and anode is 5 mm and the potential difference is 250 V. Calculate the time taken by the electron with an initial velocity of 1×10^{6} m/s, to travel from cathode to anode.*

[JNTU : Nov.-2008 (Set-3)]

Solution : D = 5 mm V = 250 V, $v_{ox} = 1\times10^{6}$ m/s

$$E = \frac{V}{d} = \frac{250}{5\times10^{-3}} = 50\times10^{-3}\ \text{V/m}$$

$$a_x = \frac{qE}{m} = \frac{1.6\times10^{-19}\times50\times10^{3}}{9.107\times10^{-31}} = 8.7844\times10^{15}\ \text{m/s}^2$$

Now $$x = \frac{1}{2}a_x t^2 + v_{ox}t + x_o \quad \text{but } x_o = 0 \text{ initially}$$

$\therefore$ $$x = \frac{1}{2}a_x t^2 + v_{ox}t$$

When electron reaches to anode, x = 5 mm.

$\therefore$ $$5\times10^{-3} = \frac{1}{2}(8.7844\times10^{15})t^2 + 1\times10^{6}t$$

$\therefore$ $$t = 9.5916\times10^{-10}\,\text{s}$$ **... Neglecting negative value**

Thus time taken by electron to reach anode from cathode is **0.95916 ns.**

➡ **Example 9.18 :** *A CRT has an anode voltage of 2000 V and parallel deflection plates of 2 cm long and 5 mm apart. The screen is 30 cm from the center of the plates. Find the input voltage required to deflect the beam through 3 cm. The input voltage is applied to the deflection plates through amplifiers having overall gain of 100.* **[JNTU : Nov.-2007 (Set-1)]**

Solution : l = 2 cm, d = 5 mm , L = 30 cm, V_a = 2000 V, D = 3 cm

$$D = \frac{l\,L V_d}{2\,d\,V_a} \qquad \text{i.e.}\quad 3\times10^{-2} = \frac{2\times10^{-2}\times30\times10^{-2}\times V_d}{2\times5\times10^{-3}\times2000}$$

$\therefore$ $$V_d = 100\ \text{V}$$

But it is applied through amplifier of gain 100.

$\therefore$ Input voltage required $= \frac{V_d}{\text{gain}} = \frac{100}{100} = 1\ \text{V}$

➡ **Example 9.19 :** *An electrically deflected cathode ray tube has a final anode voltage of 2000 V and parallel deflection plates of 1.5 cm long and 5 mm apart. If the screen is 50 cm from the center of the deflecting plates, find :*

1. Beam velocity 2. The deflection sensitivity and

3. The deflection factor of the tube. **[JNTU : Nov.-2008 (Set-4)]**

Solution : l = 1.5 cm, d = 5 mm, L = 50 cm, V_a = 2000 V

1. $$v_{ox} = \text{Beam velocity} = \sqrt{\frac{2qV_a}{m}} = \sqrt{\frac{2\times1.6\times10^{-19}\times2000}{9.107\times10^{-31}}}$$

$$= \mathbf{26.5095\times10^{6}\ m/s}$$

2. $$S = \frac{D}{V_d} = \frac{lL}{2dV_a} = \frac{1.5\times10^{-2}\times50\times10^{-2}}{2\times5\times10^{-3}\times2000}$$

$$= \mathbf{3.75\times10^{-4}\ m/v}$$

3. $$G = \frac{1}{S} = \frac{1}{3.75\times10^{-4}} = \mathbf{2666.667\ V/m}$$

Review Questions

1. *Derive the expressions for acceleration, velocity and displacement of a charged particle placed in an electric field E.* **[Dec. - 2003 (Set 1)]**
2. *Derive the expression for the position attained by an electron in an electric field of intensity E.*
3. *Derive the equation for the energy acquired by an electron placed in uniform electric field of intensity E.*
4. *Derive the expression for transit time τ (tow) and final velocity v in the case of an electron traversing in uniform electric field E.* **[Nov./Dec. - 2004 (Set - 1); May/June-2004 (Set 3)]**
5. *Derive the expression for the deflection in an electrostatic deflection system. Hence obtain the expression for electrostatic deflection sensitivity.*
6. *Derive the expression for the deflection in an magnetic deflection system. Hence obtain the expression for magnetic deflection sensitivity.*
7. *Compare the electrostatic and magnetic deflection systems.*
8. *An electrically deflected CRT has an accelerating voltage of 900 V and parallel deflecting plates of 1.5 cm long and 0.5 cm apart. If the screen is 40 cm away from the centre of the plates, calculate*

   i) Beam speed

   ii) Deflection sensitivity of tube

   iii) Deflection factor of tube

   Assume charge of electron as 1.6×10^{-19} C and mass of electron as 9.1×10^{-31} kg.

   (Ans. : i) 17.8×10^{6} m/s, ii) 0.67 mm/V, iii) 1.5 V/mm)
9. *An electrically deflected CRT has a final anode voltage of 2000 V and parallel deflecting plates 1.5 cm long and 5 mm apart. If the screen is 50 cm from the centre of the deflecting plates, find i) The beam speed ii) The deflection sensitivity of tube iii) Deflection factor of tube.*

   (Ans. : 26.5×10^{6} m/s, 0.375 mm / V, 2.66 V / mm)

Q.6 *Three resistors R_1, R_2 and R_3 are connected in series with a constant voltage source of V volts. The voltage across R_1 is 4 V, power loss in R_2 is 16 W and the value of R_3 is 6 Ω. If the current flowing through the circuit is 2 A, find the voltage V.*
[Nov.-2007 (Set-1), 8 Marks]

Ans. : Refer example 1.22.

Q.7 *How the network elements can be classified ? Explain it clearly.*
[Nov.-2007, 2008 (Set-3), 8 Marks]

Ans. : Refer section 1.9.

Q.8 *For the circuits shown in the Fig. 2, determine the current through 6 ohms resistor and the power supplied by the current source.* **[Nov.-2007, 2008 (Set-2, 4), 8 Marks]**

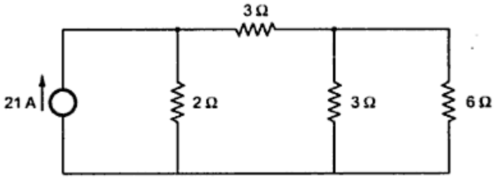


Fig. 2

Ans. : Refer example 1.21.

Q.9 *In the network shown in Fig. 3, find all branch currents and voltage drops across all resistors.* **[Nov.-2007, 2008 (Set-3), 8 Marks]**

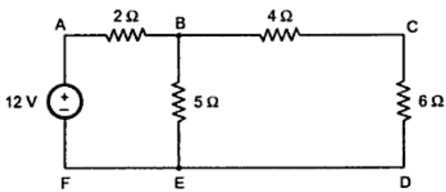


Fig. 3

Ans. : Refer example 1.23.

Q.10 *What is KCL and KVL ? Explain it clearly.* **[Nov.-2007, 2008 (Set-4), 8 Marks]**

Ans. : Refer section 1.19.

Q.11 *Determine the value of resistance R and current in each branch when the total current taken by the circuit in Fig. 4 is 6 amps.* **[Nov.-2008 (Set-1), 8 Marks]**

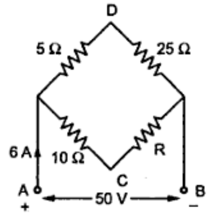


Fig. 4

Ans. : Refer example 1.27.

Q.12 *Differentiate between independent and dependent sources. What is their circuit representation ?* **[Nov.-2008 (Set-1), 8 Marks]**

Ans. : Refer section 1.13.

Q.13 *Define : i) e.m.f. ii) Current iii) Resistance and iv) Conductance.* **[Nov.-2008 (Set-1), 6 Marks]**

Ans. : Refer sections 1.4, 1.10.1.

The conductance is defined as the reciprocal of the resistance.

Q.14 *Find equivalent resistance between points A and B in the network shown in Fig. 5.* **[Nov.-2008 (Set-1), 10 Marks; Nov.-2008 (Set-3), 8 Marks]**

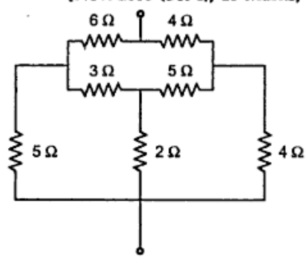


Fig. 5

Ans. : Refer example 1.28.

Q.15 *A voltage waveform shown in the Fig. 6 is applied to a pure resistor of 40 Ω.*

Sketch the waveform of the current passing through the resistance.

[Nov.-2008 (Set-1), 6 Marks]

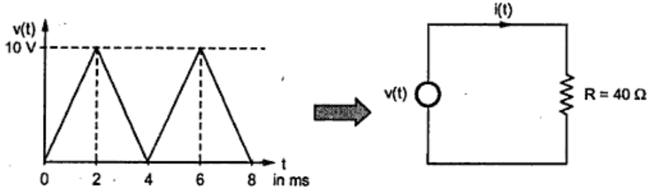


Fig. 6

Ans. : Refer example 1.30.

Q.16 *Solve the network shown in the Fig. 7 below for currents in the various resistors.* **[Nov.-2008 (Set-1), 10 Marks]**

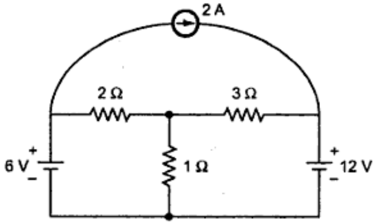


Fig. 7

Ans. : Refer exanple 1.31.

Q.17 *When three inductances of values L_1, L_2 and L_3 Henries are connected in series. Prove that $L_{equivalent} = L_1 + L_2 + L_3$.* **[Nov.-2008 (Set-2), 8 Marks]**

Ans. : Refer section 1.14.2.

Q.18 *In an a.c. circuit, containing pure inductance, the voltage applied is 110 V, 50 Hz while the current is 10 A. Find the value of inductive reactance and inductance.* **[Nov.-2008 (Set-2), 8 Marks]**

Ans. : Refer example 1.25.

Q.19 *Write the difference between active elements and passive elements with examples.* **[Nov.-2008 (Set-2), 8 Marks]**

Ans. : Refer section 1.9.

Q.20 *Give the formulae to convert delta connected resistances into equivalent stat resistances.* **[Nov.-2008 (Set-2), 6 Marks]**

Ans. : Refer section 1.22.1.

Q.21 *Find R_{ab} across a-b terminals of the network shown in the Fig. 8.*
[Nov.-2008 (Set-2), 10 Marks]

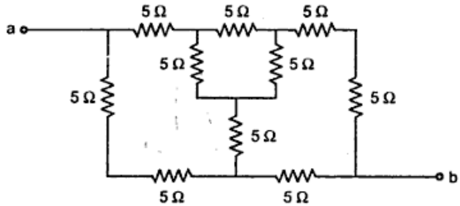


Fig. 8

Ans. : Refer example 1.29.

Q.22 *Calculate the current through the resistance of 5 Ω in the specified direction as shown in the Fig. 9.*
[Nov.-2008 (Set-2), 10 Marks]

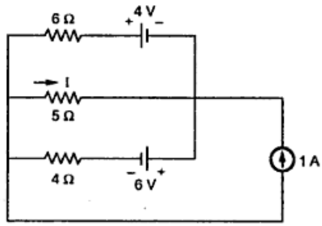


Fig. 9

Ans. : Refer example 1.32.

Q.23 *Determine the reactance of a 50 μF capacitor in a d.c. supply and also in an a.c. supply of 100 Hz.*
[Nov.-2008 (Set-3), 8 Marks]

Ans. : Refer example 1.26.

Q.24 *When three inductances of values L_1, L_2 and L_3 Henries are connected in parallel. Find its equivalent inductance.*
[Nov.-2008 (Set-3), 8 Marks]

Ans. : Refer section 1.15.2.

Q.25 *Explain about star-delta transformation.*
[Nov.-2008 (Set-3), 8 Marks]

Ans. : Refer section 1.22.2.

Q.26 *Calculate the values of two resistances which when connected in series gives 50 Ω and 8 Ω when in parallel.*
[Nov.-2008, (Set-3), 8 Marks]

Ans. : $R_1 + R_2 = 50\ \Omega$ and $\dfrac{R_1 R_2}{R_1 + R_2} = 8\ \Omega$

$$\therefore \qquad \frac{R_1R_3}{50} = 8 \quad \text{i.e.} \quad R_1R_2 = 400 \quad \text{i.e.} \quad R_2 = \frac{400}{R_1}$$

$$\therefore \qquad R_1 + \frac{400}{R_1} = 50 \quad \text{i.e.} \quad R_1^2 - 50\,R_1 + 400 = 0$$

Solving, $R_1 = 40\,\Omega$ or $10\,\Omega$, $R_2 = 10\,\Omega$ or $40\,\Omega$

Q.27 *If a voltage of $V = V_m \sin\theta$ is applied across a pure resistor alone give the expression for current and power.* **[Nov.-2008, (Set-3), 8 Marks]**

Ans. : Refer section 1.25.

Q.28 *A 60 Hz voltage of 115 V r.m.s. is impressed on a 100 ohms resistance.*

i. Write the time equation for the voltage and the resulting current. Let the zero point of the voltage wave be at t = 0.

ii. Show the voltage and current on a time diagram.

iii. Show the voltage and current on a phasor diagram. **[Nov.-2008 (Set-3), 8 Marks]**

Ans. : V = 115 V (r.m.s.), f = 60 Hz.

$$\therefore \qquad V_m = \sqrt{2} \times V = \sqrt{2} \times 115 = 162.6345\text{ V}$$

$$\therefore \qquad v(t) = V_m \sin(2\pi f t) = 162.6345 \sin(120\,\pi\, t)\text{ V}$$

$$\therefore \qquad i(t) = \frac{v(t)}{R} = 1.6263 \sin(120\pi\, t)\text{ A}$$

The voltage and current waveforms are shown in the Fig. 10 (a) while the phasor diagram is shown in the Fig. 10 (b).

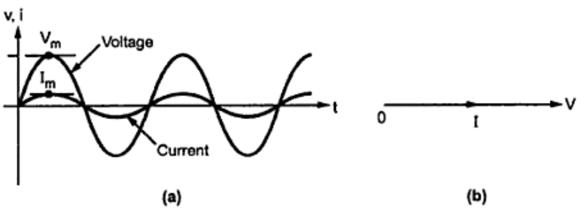


Fig.10

Q.29 *Find the voltage drop across 1 ohm resistor and power loss across 2 ohm resistor in the Fig. 11.* **[Nov.-2008 (Set-4), 16 Marks]**

Q.14 *Write the e.m.f. equation of a d.c. generator and derive it* **[Nov.-2007, (Set-1), 8 Marks]**

Ans. : Refer section 2.8.

Q.15 *A long shunt compound generator delivers a load current of 50 A at 500 V and has armature, series field and shunt field resistances of 0.05 Ω, 0.03 Ω and 250 Ω respectively. Calculate the generated voltage and the armature current. Allow 1 V per brush for contact drop.* **[Nov.-2007, (Set-2), 8 Marks]**

Ans. : Refer example 2.30.

Q.16 *How the d.c. generators are classified. Explain it with neat circuit diagrams.* **[Nov.-2007, (Set-3), Nov.-2008 (Set-1), 8 Marks]**

Ans. : Refer sections 2.12, 2.13, 2.14, 2.15, 2.16, 2.17.

Q.17 *Explain the constructional features of a d.c. machine with the help of a neat sketch.* **[Nov.-2007, (Set-4), 8 Marks]**

Ans. : Refer section 2.6.

Q.18 *The armature of an 8 pole d.c. generator has 960 conductors and runs at 400 r.p.m. The flux per pole is 40 milliweber :*

a) Calculate the induced EMF, if the armature is lap wound.

b) At what speed should it be driven to generate 400 V, if the armature is wave connected. **[Nov.-2008, (Set-1), 16 Marks]**

Ans. : Refer example 2.31.

Q.19 *Explain the differences between lap and wave windings.* **[Nov.-2008, (Set-1), 4 Marks]**

Ans. : Refer section 2.7.3.

Q.20 *Explain the function of the following :*

i. Commutator *ii. Brushes*

iii. Field system and *iv. Armature circuit* **[Nov.-2008, (Set-1), 6 Marks]**

Ans. : Refer section 2.6.

Q.21 *With neat sketches, explain the construction and functions of the various parts of a d.c. machine.* **[Nov.-2008, (Set-1), 10 Marks]**

Ans. : Refer section 2.6.

Q.22 *Explain d.c. motor principle and its working.* **[Nov.-2008, (Set-1), 6 Marks]**

Ans. : Refer section 2.19.

Q.23 *A 4 pole, long shunt, lap wound generator supplies 25 kW at a terminal voltage of 500 V. The armature resistance is 0.03 ohms, series field resistance is 0.04 ohms and shunt field resistance is 200 ohms. The brush drop may be taken as 1 V. Determine :*

a) The e.m.f. generated

3 Transformers

Q.1 *A double wound 1-phase transformer is required to step down from 1900 V to 240 V, 50 Hz. It is to have 1.5 V per turn. Calculate the required number of turns on the primary and secondary windings respectively. The peak value of flux density is required to be not more than 1.5 Wb/m^2. Calculate the required cross sectional area of the steel core. If the output is 10 kVA. Calculate the secondary current.*

[Nov.-2004 (Set-1), May-2005 (Set-3)]

Ans. : Refer example 3.21.

Q.2 *The following readings were obtained from O.C. and S.C. tests on 8 kVA, 400/120 V, 50 Hz transformer.*

O.C. Test	On L.V. side	120 V	4 A	75 W
S.C. Test	On H.V. side	9.5 V	20 A	110 W

Calculate the voltage regulation and efficiency at full load 0.8 p.f. lagging.

[Nov.-2004 (Set-2), Nov.-2005 (Set-3), March-2006 (Set-2), 8 Marks]

Ans. : Refer example 3.26.

Q.3 *The efficiency of a 250 kVA, single phase transformer is 96 % when delivering full load at 0.8 power factor lagging and 97.2 % when delivering half full load at unity power factor. Determine the efficiency at 75 % of full load at 0.8 power factor lagging.*

[Nov.-2004 (Set-4), 8 Marks]

Ans. : Refer example 3.22.

Q.4 *A single phase transformer has 400 primary and 1000 secondary turns. The net constructional area of the core is 60 cm^2. If the primary winding is connected to a 50 Hz supply at 520 V. Calculate*

i) Peak value of flux density in the core.

ii) Voltage induced in the secondary winding.

iii) Transformation ratio.

iv) E.M.F. induced per turn in both the windings.

[May-2005 (Set-2), March-2006 (Set-3), 8 Marks]

Ans. : Refer example 3.25.

Q.14 *Draw and explain no load phasor diagram for a single phase transformer.*
[Nov.-2008 (Set-1), 8 Marks]

Ans. : Refer section 3.8.

Q.15 *A single phase transformer with 10 : 1 turn ratio and rated 50 kVA, 2400/240 V, 50 Hz is used to step down the voltage of a distribution system. The low tension voltage is to be kept constant at 240 V. Find the value of load impendence of the low tension side so that the transformer will be loaded fully. Find also the value of maximum flux inside the core if the low tension side has 23 turns.*
[Nov.-2008 (Set-1), 8 Marks]

Ans. : Refer example 3.18.

Q.16 *Explain the principle of working of 1-ϕ transformer on no load conditions. Also explain the nature of no load current.* **[Nov.-2008 (Set-1), 8 Marks]**

Ans. : Refer sections 3.2, 3.9, 3.8.

Q.17 *Explain how the equivalent circuit parameters can be obtained from open circuit and short circuit tests.* **[Nov.-2008 (Set-1), 8 Marks]**

Ans. : Refer section 3.17.

Q.18 *The wattmeter reads iron losses in O.C. test, and it reads copper losses in S.C. test of a transformer. Why ?* **[Nov.-2008 (Set-1), 8 Marks]**

Ans. : Refer section 3.17.

Q.19 *A 11000/230 V, 150 kVA, 1-Phase, 50 Hz transformer has core loss of 1.4 kW and full load copper loss of 1.6 kW. Determine :*
a) kVA load for maximum efficiency and value of maximum efficency at unity power factor.
b) The efficiency at half full load 0.8 power factor leading.
[Nov.-2008 (Set-2), 16 Marks]

Ans. : Refer example 3.20.

Q.20 *Explain the constructional details of a transformer with neat sketches.*
[Nov.-2008 (Set-2), 8 Marks]

Ans. : Refer section 3.3.

Q.21 *On what factors the induced e.m.f. in the transformer winding depends. Justify the answer with appropriate derivation.* **[Nov.-2008 (Set-2), 8 Marks]**

Ans. : Refer section 3.5.

Q.7 *A 1 MVA, 11 kV, 3 phase star connected alternator has the following OCC test data :*

I_f Amps	50	110	140	180
V_{oc} (Line volts)	7000	12500	13750	15000

The short circuit test yielded full load current at a field current of 40 Amps.

The armature resistance per phase is 0.6 omhs. Find the % regulation of half full load at 0.8 p.f. lagging and at full load, 0.9 p.f. leading. **[Nov.-2005 (Set-1), 16 Marks]**

Ans. : Refer example 4.19.

Q.8 *The effective resistance of a 2200 V, 50 Hz, 440 kVA, 1-phase alternator is 0.5 Ohms. On short circuit a field current of 40 Amps gives the full load current of 200 Amps. The e.m.f. on open circuit with the same field excitation is 1160 V. Calculate*

i) Synchronous impedance

ii) Synchronous reactance

iii) % regulation at 0.707 p.f. leading.
[March-2006 (Set-4), Nov.-2005 (Set-2, 3), May-2005 (Set-1, 2), 8 Marks]

Ans. : Refer example 4.16.

Q.9 *Why do we prefer field winding of an alternator as a rotating element and armature as a stationary element, give reasons.* **[Nov.-2004 (Set-2), Nov.-2008 (Set-4), 6 Marks]**

Ans. : Refer section 4.3.

Q.10 *Explain pessimistic method of finding regulation of a given alternator.*
[Nov.-2005 (Set-2, 3), March-2006 (Set-4), May-2005 (Set-1, 2), Nov.-2008 (Set-2), 8 Marks]

Ans. : Refer section 4.24.

Q.11 *A 550 V, 55 kVA, single phase alternator has an effective resistance of 0.2 ohms. A field current of 10 A produces an armature current of 200 A on short circuit, and an e.m.f. of 450 V on open circuit. Calculate (i) The synchronous reactance (ii) The full-load regulation with power factor 0.8 lagging.* **[March-2006 (Set-2), 8 Marks]**

Ans. : Refer example 4.15.

Q.12 *What are the three voltage drops occurring in an alternator on-load ?*
[March-2006 (Set-2), 8 Marks]

Ans. : Refer section 4.19.

Q.13 *Explain the construction of salient pole alternator.* **[March-2006 (Set-1), 8 Marks]**

Ans. : Refer section 4.6.1.

Q.14 *A 3-phase star connected alternator has an open circuit line voltages of 6599 volts. The armature resistance and synchronous reactance are 0.6 ohms and 6 ohms per phase, respectively. Find terminal voltage and voltage regulation if load current 180 A at power factor of*

i) 0.9 lagging

ii) 0.8 leading. **[March-2006 (Set-1), 8 Marks]**

Ans. : Refer example 4.20.

Q.15 *Explain clearly about open circuit and short circuit tests in alternator.*
[Nov.-2007 (Set-1, 2, 4), Nov.-2008 (Set-1, 3, 4), 8 Marks]

Ans. : Refer section 4.24.

Q.16 *Discuss in detail the predetermination of regulation of an alternator.*
[Nov.-2007 (Set-3), Nov.-2008 (Set-2), 8 Marks]

Ans. : Refer section 4.24.

Q.17 *The coil span for the stator winding of an alternator is 120° (electrial). Find the chording factor of the winding.* **[Nov.-2008 (Set-1), 8 Marks]**

Ans. : Refer example 4.13.

Q.18 *Compare salient pole and non-salient pole synchronous machines and explain the importance of saliency.* **[Nov.-2008 (Set-1), 8 Marks]**

Ans. : Refer section 4.6.3.

Q.19 *State the advantages and disadvantages of using short pitch winding and distributed winding in alternator.* **[Nov.-2008 (Set-2), 8 Marks]**

Ans. : Refer section 4.11.

Q.20 *Deduce the relation between the number of poles, the frequency and the speed of rotation in alternator.* **[Nov.-2008 (Set-2), 8 Marks]**

Ans. : Refer section 4.9.

Q.21 *The data obtained on 100 kVA, 1100 V, 3-phase alternator is :*

DC resistance test : E between lines = 6 V d.c., I in lines = 10 A d.c.

OC test : Field current = 12.5 A, Voltage between lines = 420 V

SC test : Field current = 12.5 A, Line current = rated value.

Calculate the voltage regulation of alternator at 0.8 power factor lagging.
[Nov.-2008 (Set-4), 16 Marks]

Ans. : Refer example 4.17.

Q.9 *Define "slip" of a 3-phase induction motor. Can be motor run at synchronous speed explain ?* **[Nov.-2004 (Set-1), 5 Marks]**

Ans. : Refer sections 5.5, 5.6.

Q.10 *Draw the torque slip characteristics and explain why it is in the shape of rectangular hyperbola.* **[March-2006 (Set-2), 8 Marks]**

Ans. : Refer section 5.12.

Q.11 *Draw the torque slip characteristic and mark the operating region of the motor in regard to its safety.* **[March-2006 (Set-3), 6 Marks]**

Ans. : Refer section 5.12.

Q.12 *State the effects of increasing rotor resistance on starting current, starting torque, maximum torque and full-load slip of an induction motor.* **[Nov.-2004 (Set-4), 8 Marks]**

Ans. : It is known that in slip ring induction motor, externally resistance can be added in the rotor. Let us see the effect of change in rotor resistance on the torque produced.

Let R_2 = Rotor resistance per phase

Corresponding torque, $$T \propto \frac{s E_2^2 R_2}{R_2^2 + (sX_2)^2}$$

Now externally resistance is added in each phase of rotor through slip rings.

Let R_2' = New rotor resistance per phase

Corresponding torque, $$T' \propto \frac{s E_2^2 R_2'}{R_2'^2 + (sX_2)^2}$$

Similarly the starting torque at s = 1 for R_2 and R_2' can be written as

$$T_{st} \propto \frac{E_2^2 R_2}{R_2^2 + X_2^2}$$

and $$T_{st}' \propto \frac{E_2^2 R_2'}{R_2'^2 + X_2^2}$$

Maximum torque $$T_m \propto \frac{E_2^2}{2 X_2}$$

Key Point : *It can be observed that T_m is independent of R_2 hence whatever may be the rotor resistance, maximum torque produced never changes but the slip and speed at which it occurs depends on R_2.*

normal running condition of the motor. So this method is used in practice to achieve higher starting torque hence resistance in rotor is added **only at start.**

Thus good performance at start and in the running condition is ensured.

Key Point : *This is possible only in case of slip type of induction motors as in squirrel cage due to short circuited rotor, extra rotor resistance cannot be added.*

Q.13 *Explain the operation of slip ring induction motor.* **[Nov.-2004 (Set-2), 8 Marks]**

Ans. : Refer section 5.5.

Q.14 *The power input to the rotor of a 440 V, 50 Hz, 3-phase, 6-pole induction motor is 50 kW. It is observed that the rotor e.m.f. makes 120 complete cycles per minute calculate.*

i) Slip *ii) Rotor speed* *iii) Rotor copper loss/phase*

iv) Mechanical power developed, and

v) Rotor resistance/phase if rotor current is 50 A. **[Nov.-2004 (Set-2), 8 Marks]**

Ans. : Refer example 5.21.

Q.15 *Explain the rotating magnetic field developed in an induction motor.*
[Nov.-2005 (Set-3), 9 Marks; May-2006 (Set-1), 8 Marks]

Ans. : Refer section 5.2.

Q.16 *A 12 pole 3-phase alternator coupled to an engine running at 500 r.p.m. It supplies an induction motor which has a full load speed of 1440 r.p.m. Find the % slip and the number of poles of the motor.*
[Nov.-2005 (Set-3), 7 Marks; March.-2006 (Set-1), 8 Marks]

Ans. : Refer example 5.17.

Q.17 *If the electromotive force in the stator of an 8 pole induction motor has a frequency of 50 Hz and that in the rotor is 1.5 Hz. At what speed is the motor running and what is the slip ?* **[March-2006 (Set-2), 8 Marks]**

Ans. : Refer example 5.18.

Q.18 *A 3-phase, 6 pole, 50 Hz induction motor has a slip of 1 % at no load and 3 % at full load. Find*

i) Synchronous *ii) No load speed*

iii) Full load speed *iv) Frequency of rotor current at standstill*

v) Frequency of rotor current at full load. **[March-2006 (Set-3), 10 Marks]**

Ans. : Refer example 5.22.

6 Instruments

Q.1 *List the advantages of gravity control over spring control.* **[Nov.-2008 (Set-1), 8 Marks]**

Ans. : Refer section 6.5.1.

Q.2 *List the different types of materials used in components of spring and gravity control.* **[Nov.-2008 (Set-1), 8 Marks]**

Ans. : Refer section 6.5.

Q.3 *Explain the different methods of supporting the moving system in instruments.* **[Nov.-2008 (Set-2), 8 Marks]**

Ans. : Refer section 6.5.

Q.4 *Explain the advantages and disadvantages of different damping systems.* **[Nov.-2008 (Set-2), 8 Marks]**

Ans. : Refer section 6.6.

Q.5 *Explain the following terms :*

a) Critically damped b) Over damped c) Under damped **[Nov.-2008 (Set-3), 16 Marks]**

Ans. : Refer section 6.6.

Q.6 *Explain the moving coil instruments as ammeters and voltmeters.* **[Nov.-2008 (Set-4), 8 Marks]**

Ans. : Refer sections 6.9, 6.11.

Q.7 *List the advantages of moving coil instruments.* **[Nov.-2008 (Set-4), 8 Marks]**

Ans. : Refer section 6.7.2.

Q.8 *Discuss the classification of electrical measuring instruments employed for measurement of current.* **[Nov.-2008 (Set-1, 2), 6 Marks]**

Ans. : Refer section 6.2.

Q.9 *Explain the significance of controlling torque and damping torque relevent to the operation of indicating instruments.* **[Nov.-2008 (Set-1, 2), 10 Marks]**

Ans. : Refer section 6.3.

Q.10 *With neat diagram, describe a permanent magnet moving coil instrument and indicate how it can be employed as an Ammeter and as a Voltmeter.*

[Nov.-2008 (Set-3), 8 Marks]

Ans. : Refer sections 6.7, 6.9, 6.11.

Q.11 *A Permanent magnet moving coil voltmeter has the following specifications number of turns in the moving coil = 100.*
Depth of the coil = 3 cms
Width of the coil = 2.5 cms
Flux density in the air gap = 0.15 Wb/m^2
When the instrument is used for measuring a voltage the moving coil carries a current of 5 mA. Calculate the deflecting torque produced in the instrument.

[Nov.-2008 (Set-3), 8 Marks]

Ans. :

$$T_d = NBAI$$

$A = 3 \times 2.5 = 7.5 \text{ cm}^2$, $B = 0.15 \text{ Wb/m}^2$, $N = 100$, $I = 5 \text{ mA}$.

$$\therefore \quad T_d = 100 \times 0.15 \times 7.5 \times 10^{-4} \times 5 \times 10^{-3} = \mathbf{5.625 \times 10^{-5} \text{ Nm}}$$

Q.12 *Explain clearly the methods commonly employed to produce controlling torque in an indicating instrument. Bring out their relative merits and demerits.*

[Nov.-2008 (Set-4), 8 Marks]

Ans. : Refer section 6.5.

Q.13 *Describe the working of a permanent magnet moving coil instrument with a neat sketch. Discuss the possible source of errors in such an instrument and indicate the methods to reduce these errors.* **[Nov.-2008 (Set-4), 8 Marks]**

Ans. : Refer section 6.7.

Q.14 *Sketch and describe the construction of a moving coil ammeter and give the principle of operation.* **[Nov.-2008 (Set-1, 2); March-2006 (Set-2), 8 Marks]**

OR

Draw a diagram to show the essential parts of a modern moving coil instrument. Label each part and state its function. **[Nov.-2004 (Set-3), 8 Marks]**

Ans. : Refer section 6.7.

Q.15 *A moving coil instrument gives full scale deflection with 15 mA and has a resistance of 5 Ω. Calculate the resistance of the necessary components in order that the instrument may be used as*
i) A 2 A - Ammeter ii) A 100 V Voltmeter.

[Nov.-2008 (Set-1, 2), March-2006 (Set-4), Nov.-2005 (Set-2), Nov.-2004 (Set-4), 8 Marks]

Ans. : Refer section 6.8.

Q.16 *Why is spring control to be preferred to gravity control in an electrical measuring instrument ?* **[Nov.-2008 (Set-4), Nov.-2005 (Set-1), Nov.-2004 (Set-2), 8 Marks]**

Ans. : Refer example 6.5.

Q.17 *The coil of a moving coil meter has resistance of 5 Ω and given full scale deflection when a current of 15 mA passes through it. What modification must be made to the instrument to convert it into*

i) An ammeter reading to 15 A

ii) A voltmeter reading to 15 V ?

[Nov.-2008 (Set-4), Nov.-2005 (Set-1), Nov.-2004 (Set-2), 8 Marks]

Ans. : Refer example 6.9.

Q.18 *A moving coil instrument which gives full scale deflection with 15 mA has a copper coil having a resistance of 1.5 Ω at 15 °C and a temperature coefficient of 1/234.5 at 0 °C in series with a resistor of 3.5 Ω having a negligible temperature coefficient. Determine i) The resistance of shunt required for a full scale deflection of 20 A and ii) The resistance required for a full scale deflection of 250 V. If the instrument reads correctly at 15 °C, determine the percentage error in each case when the temperature is 25 °C.*

[March-2006 (Set-1), 16 Marks, Nov.-2004, Nov.-2003, 8 Marks]

Ans. : Refer example 6.11.

Q.19 *A moving iron voltmeter in which full scale deflection is given by 100 V, has a coil of 10,000 turns and a resistance 2000 Ω. Calculate the number of turns required on the coil of instrument converted for use as an ammeter reading 20 A full scale deflection.*

[May-2005 (Set-4), 8 Marks]

Ans. : Refer example 6.10.

Q.20 *A voltage of 80.0 V is applied to a circuit comprising two resistors of resistance 105 Ω and 55 Ω respectively. The voltage across the 55 Ω resistor is to be measured by a voltmeter of internal resistance 100 Ω/V. Given that the meter is set to a scale of 0-50 V. Determine the voltage indicated.* **[March-2006 (Set-2), 6 Marks]**

Ans. : Refer example 6.7.

□□□

7

Semiconductor Physics and Diode

Q.1 *Mention the applications of PN junction diode.* **[Nov.-2008 (Set-1), 8 Marks]**

Ans. : The various applications of PN junction diode are rectifiers, clippers, clampers etc.

Q.2 *The turms ratio of the transformer used in a half wave rectifier is 2 : 1 and the primary is connected to 230 V, 50 Hz power mains. Assuming the diodes to be ideal, determine*

a) D.C. voltage across the load.

b) PIV of each diode and

c) Maximum and average values of power delivered to the load having a resistance of 200 Ω.

Also find the efficiency of the rectifier and output ripple frequency.

[Nov.-2008 (Set-2), 16 Marks]

Ans. : Refer example 7.28.

Q.3 *Describe the phenomenon of diffusion of charge carriers in semiconductorcs,*

[Nov.-2008 (Set-3), 8 Marks]

Ans. : Refer section 7.15.

Q.4 *Calculate the conductivity of a pure silicon at room temperature of 300 °K. Given that $n_i = 1.5 \times 10^{16}/m^3$, $\mu_n = 0.13\ m^2/V\text{-}s$, $\mu_p = 0.05\ m^2/V\text{-}s$ and $q = 1.6 \times 10^{-19}$ C. Now the silion is doped 2 in 10^8 of a donor impurity. Calculate its conductivity if there are 5×10^{28} silicon atoms $/m^3$. By what factor does the conductivity increases question* **[Nov.-2008 (Set-4), 8 Marks]**

Ans. : Refer example 7.33.

Q.5 *Derive suitable expression for conductivity of p-type and n-type semiconductors.*

[Nov.-2007 (Set-1), Nov.-2008 (Set-4), 8 Marks]

Ans. : Refer section 7.14.

Q.6 *A crystal of pure germanium is sufficiently added with antimony to produce 1.5×10^{22} antimony atoms$/m^3$. The electron and hole mobility are 0.38 m^2/volt-sec and 0.18 m^2/volt-sec respectively, and the intrinsic charge carrier density is $2.5 \times 10^{19}/m^3$. Calculate,*

i) The density of electrons and holes in the crystal and *ii) The conductivity of the material.* **[Nov.-2007 (Set-1), Nov.-2008 (Set-4), 8 Marks]**

Ans. : Refer example 7.32.

Q.7 *In the reverse bias region the saturation current of a silion diode is about 0.1 μA at 20 °C. Determine its value if the temperature is increased to 40 °C. Can a silicon diode be used as a switch? Justify your answer.* **[Nov.-2007, (Set-2, 6), 6 Marks]**

Ans. : Refer example 7.30.

Q.8 *Explain the operation of a full wave rectifier using neat circuit diagram and waveforms.* **[Nov.-2007 (Set-3), 8 Marks]**

Ans. : Refer section 7.25.

Q.9 *Explain the hole movement in an extrinsic semiconductor with neat sketches, when it is excited by external voltage.* **[Nov.-2007 (Set-4), 6 Marks]**

Ans. : Refer section 7.13.

Q.10 *Compute the conductivity of a silicon semiconductor, which is doped with acceptor impurity to a density of 10^{22} atoms/m^3. Given that $n_i = 1.4 \times 10^{16}/m^3$, $\mu_n = 0.145\, m^2/$ volt-sec and $\mu_p = 0.05\, m^2/$volt-sec* **[Nov.-2007 (Set-4), 6 Marks]**

Ans. : Refer example 7.31.

Q.11 *With the help of neat sketches, explain the circuit operation of a half wave rectifier.* **[Nov.-2008 (Set-1), 8 Marks]**

Ans. : Refer section 7.24.

Q.12 *Show that, for the full wave rectifier, the ratio of rectification is twice that of half wave rectifier.* **[Nov.-2008 (Set-2), 8 Marks]**

Ans. : Refer section 7.25.

Q.13 *A 230 V, 60 Hz, voltage is applied to the primary of a 5 : 1 step down transformer used in a bridge rectifier having a load of 900 Ω. If the diode resistance and secondary coil resistance together has a resistance of 100 Ω, determine*

i) D.C. voltage across the load

ii) D.C. current flowing through the load

iii) PIV of each diode

iv) Ripple voltage and its frequency. **[Nov.-2008 (Set-2), 8 Marks]**

Ans. : Refer example 7.27.

Q.14 *Describe the action of pn junction diode under forward and reverse bias conditions.* **[Nov.-2008 (Set-3), 10 Marks]**

Ans : Refer sections 7.17, 7.18.

Q.15 *If the reverse saturation current is 10 μA, calculate the forward current for voltage of 0.1, 0.2 and 0.3 V respectively, for silicon diode.* **[Nov.-2008 (Set-3), 6 Marks]**

Ans. : Refer example 7.29.

Q.16 *Compare the half wave, full wave and bridge rectifiers in terms of ripple factor efficiency, PIV and number of diodes.* **[Nov.-2008 (Set-1), 8 Marks]**

Ans. : Refer section 7.27.

Q.17 *Discuss the behavior of a pn junction under forward and reverse biasing.* **[Nov.-2008 (Set-2), 8 Marks]**

Ans. : Refer sections 7.17, 7.18.

Q.18 *With neat sketch explain the working of full wave bridge rectifier ? Give its advantages ?* **[Nov.-2008 (Set-3), 8 Marks]**

Ans. : Refer section 7.26.

Q.19 *In bridge type full wave rectifier, the maximum secondary voltage is 136 V. Find the dc load voltage and PIV ? Assume the diodes to be ideal.* **[Nov.-2008 (Set-3), 8 Marks]**

Ans. : $E_{sm} = 136$ V hence $E_{DC} = \frac{2E_{sm}}{\pi} = \mathbf{86.5802\ V}$, PIV = E_{sm} = **136 V.**

Q.20 *Explain*

i. Conductor ii. Insulator and iii. Semiconductor.

[Nov.-2008 (Set-4), 6 Marks]

Ans. : Refer section 7.4.

Q.21 *Why are diodes not operated in the breakdown region in rectifier service ?* **[Nov.-2008 (Set-4), 4 Marks]**

Ans. : Refer section 7.18.

Q.22 *What do you mean by regulation and explain how a full wave rectifier is used for is.* **[Aug.-2008 (Set-1,) 6 Marks]**

Ans. : Refer section 7.25.13.

□□□

8 Transistors

Q.1 *Explain the input and output characteristics of a transistor in CB configuration.* **[Nov.-2008 (Set-1), 12 Marks]**

Ans. : Refer section 8.3.1.

Q.2 *Explain the following :*

a) Firing angle

b) Conduction angle of an SCR and

c) Once the SCR is triggered, the gate looses its control. **[Nov.-2008 (Set-2), 16 Marks]**

Ans. : Refer section 8.9.

Q.3 *A sinusoidal voltage $V_i = 200 \sin 314\ t$ is applied to an SCR whose forward breakdown voltage is 150 V. Determine the time during which SCR remains OFF.* **[Nov.-2008 (Set-3), 8 Marks]**

Ans. : Refer example 8.10.

Q.4 *Describe the working principle of an SCR with V - I characteristics.* **[Nov.-2008 (Set-4), 8 Marks]**

Ans. : Refer section 8.9.

Q.5 *Derive the relationship between α and β from the fundamental equations of transistor. How do you obtain them from the characteristic curves?* **[Nov.-2007 (Set-2), 10 Marks]**

Ans. : Refer sections 8.2, 8.3.

Q.6 *Which configurations is called emitter follower ? With suitable circuit diagram explain how the input and output characteristics are obtained?* **[Nov.-2007 (Set-3), 10 Marks]**

Ans. : Refer section 8.3.3.

Q.7 *Sketch a family of CC input and output characteristics for p-n-p transistor and explain how the characteristics are useful for determining input output resistance of the configuration?* **[Nov.-2007 (Set-4), 10 Marks]**

Ans. : Refer section 8.3.3.

Q.8 *Draw the input and output characteristics of n-p-n transistor in common base configuration and explain how they are obtained?* **[Nov.-2008 (Set-3), 10 Marks]**

Ans. : Refer section 8.3.1.

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